Massachusetts Institute of Technology

Physics 8.03 Fall 2004

Problem Set 5

Due Friday, October 22, 2004 at 4 PM

Reading Assignment

French pages 161-178, 189-196. Bekefi & Barrett pages 165-178.

To do 5.1 and 5.6, you need access to a piano.

Problem 5.1 – Piano galore

For this problem you need a piano. Most of the dormitories and fraternity houses have at least one and there are several in the student center. Below is a picture labelling the keys to which we will refer.

Let 256 Hz equal one unit of frequency, $\nu = 1$. The harmonics of this note are then $\nu = 2,\ 3,\ 4$, etc. Middle C on the piano is $C256$ (if the piano is tuned that way). We call it C_4. The subscript refers to the octave; it increases by one at each higher octave of C. Thus the fundamental of C_3 (called a subharmonic of C_4) is 128 Hz ($\nu = \frac{1}{2}$). To avoid confusion, I will always refer to the fundamental as the first harmonic. Thus the first harmonic of C_3 is 128 Hz and the second harmonic is 256 Hz.

Suppose that piano strings behave ideally. Then the mode frequencies of a given string would consist of the harmonic sequence $\nu_1,\ 2\nu_1,\ 3\nu_1$, etc. The names and frequencies of the first 16 harmonics of string C_4 and also its first two subharmonics ($\nu = 1/3$ *and* $\nu = 1/2$) would be as follows (we underline C_4 and its octaves):

Names:	F_2	C_3	C_4	C_5	G_5	C_6	E_6	G_6	Bb_6	C_7	D_7	E_7	$F\#_7$	G_7	$G\#_7$	Bb_7	B_7	C_8
ν:	$\frac{1}{3}$	$\underline{\frac{1}{2}}$	$\underline{1}$	$\underline{2}$	3	$\underline{4}$	5	6	7	$\underline{8}$	9	10	11	12	13	14	15	$\underline{16}$

We will start this experiment by determining whether your piano belongs in a bar or a concert hall. Strike the notes from C_3 up (one at the time) and listen for beats. Many keys (not all) activate two or three identical strings simultaneously. A Steinway grand piano has a total of 216 strings (88 keys). If the three (or two) strings which make-up one note are not properly tuned, you will hear beats.

Suppose you hit C_5 and you hear maximum sound at 1 second intervals.

(a) What then is the difference in tension between the strings of C_5? The tension in each string in the piano is about 250 Newtons.

(b) What is the approximate total force on the frame of the piano that holds all the strings?

Steadily hold down various keys (one at the time) so as to lift their dampers without sounding the notes. Then, while you are still holding down a key of your choice, strike the G_5 sharply, hold it for a few seconds and release it (you still hold the other key down). Listen carefully. You clearly hear sound in case you had chosen C_4 or G_6.

(c) What frequencies do you hear in these two cases?

(d) Which other notes might be excited by G_5? (G_5 also produces higher harmonics!) Verify your predictions.

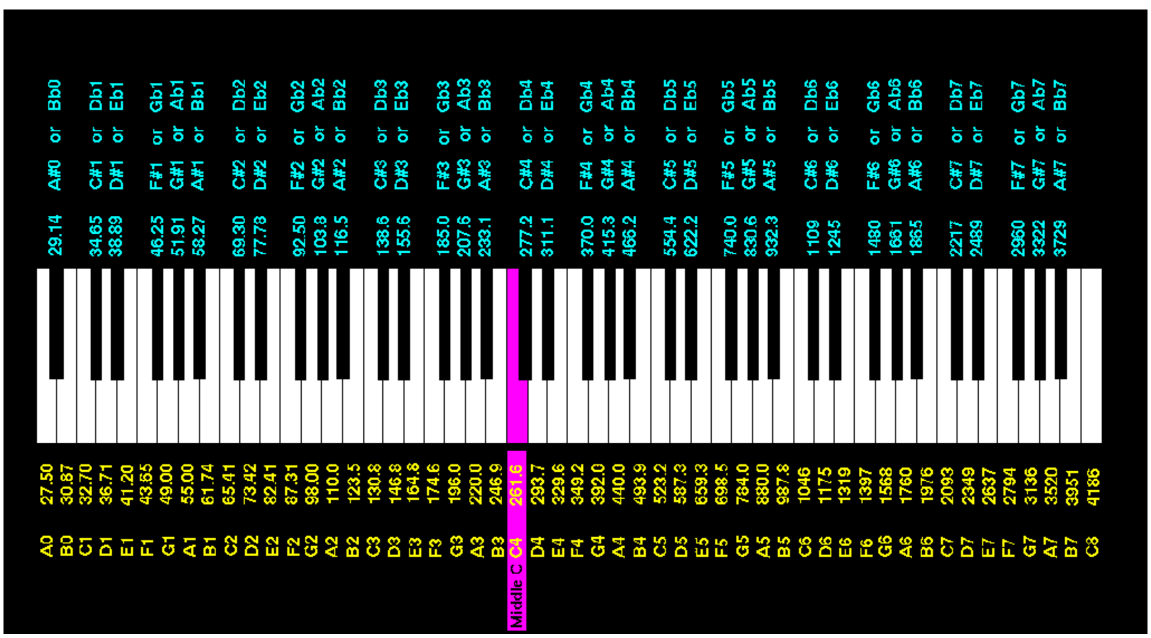
To demonstrate the presence of higher harmonics, we will make you "hear" the 6th harmonic of C_3. Hold the C_3 key down (keep your finger on it), strike G_5, hold it for a few seconds and let it go. Now listen carefully to the sound produced by your C_3 strings. This sound is the 6th harmonic of C_3.

Clearly if your piano is out of tune things may sound quite different. But even if it is in tune you may notice by listening carefully that the G_5 does not sound exactly like the 6th harmonic of the C_3. It seems that our piano strings do not behave as "ideally" as we earlier assumed.

(e) How would you explain that?

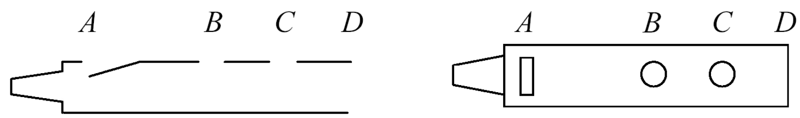
The lowest two notes on the piano are A_0 27.5 and $A\#_0$ 29.1. Their beat frequency is thus 1.6 Hz, which is easily detectable. Hit both notes together, gently. Once you think you hear beats, let one key up, but not the other.

(f) Do the beats go away?



Problem 5.2 – Holes in woodwind instruments

A simplified "flute" as shown in the figure is open at D. There is also a large opening at A (near the mouth piece) and there are two holes at B and C. [$AB = BD$, and $BC = CD$]. The distance AD ~37 cm. The speed of sound is ~ 340 m/sec. What frequency do you expect to hear when you blow and when you



cross section (side)   *top view*

(a) hold both holes at B and C closed?

(b) hold only hole C closed?

(c) hold only hole B closed?

(d) do not close either one of the holes B or C?

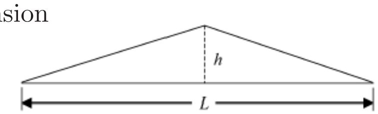
Keep in mind that wherever the air inside the "flute" is in "open" contact with the air outside, no pressure can build up (pressure nodes). Now read the pages 204 and 205 of Horns, Strings and Harmony by Benade and reconsider your answers.

If you play any woodwind instrument, we recommend that you read Chapter IX of Horns, Strings and Harmony by Benade. Very enjoyable!

Problem 5.3 – Plucked string

Do Problem 6-12 from French, A. P. *Vibrations and Waves*. New York, N.Y.: W. W. Norton and Company, January 1, 1971. ISBN: 0393099369.

A string of length L, which is clamped at both ends and has a tension T, is pulled aside a distance h at its center and released.



(a) What is the energy of the subsequent oscillations?

(b) How often will the shape shown in the figure reappear?

(Assume that the tension remains unchanged by the small increase of length caused by the transverse displacements.) [*Hint:* In part (a), consider the work done against the tension in giving the string its initial deformation.]

Problem 5.4 – Fourier analysis

(a) Find the Fourier series of the function shown in the figure of problem 5.3.

(b) If the release takes place at t=0. What will the string look like ($f(x,t)$) at time t?

(c) Make sketches of the string at $t = T_1/8$, $T_1/4$ and at $T_1/2$. T_1 is the period of the lowest frequency (first harmonic). With Matlab (though not required) you can do a great job!

Text removed due to copyright reasons. Please see:

Benade, Arthur H. *Horns, Strings, and Harmony*. NY: Dover Publications, 1992. ISBN: 0486273318.

Problem 5.5 – Fourier series

Do Problem 6-14 from French, A. P. *Vibrations and Waves*. New York, N.Y.: W. W. Norton and Company, January 1, 1971. ISBN: 0393099369.

Find the Fourier series for the following functions ($0 \leq x \leq L$):

(a) $y(x) = Ax(L - x)$.

(b) $y(x) = A\ sin(\pi x/L)$.

(c) $y(x) = \begin{cases} A\ sin(2\pi x/L) & (0 \leq x \leq L/2) \\ 0 & (L/2 \leq x \leq L) \end{cases}$

Problem 5.6 – Pianos can talk back

Revisit your piano (it does not have to be in tune!). Open the cover so that you can see the strings. Hold down the damper pedál. Shout "heyeyeyey" (hold it for a few seconds) into the region of the strings and sounding board. If you have a grand piano, that would be super! Shout "oooooooh". Try all vowels. The piano strings are responding to your sound. They sort of "Fourier analyze" the sound, and they produce your sound for several seconds.

(a) Explain how this remarkable process of "Fourier analysis" takes place. Why is it not necessary that the piano be in tune?

All components (with frequencies $\omega_1,\ 2\omega_1, \ldots n\omega_1$) in a real Fourier analysis are either in phase or out of phase. However, you won't succeed in making all piano strings that participate in the "analysis" of your voice vibrate in phase (or out of phase).

(b) Why not? Give a quantitative answer.

In spite of the complete absence of the phase relations we clearly hear the piano produce our sound.

(c) What does that tell you about the importance to your ears and brains of the relative phases of the Fourier components that make up the sound?

(d) How would you explain that?