Massachusetts Institute of Technology

Physics 8.03 Fall 2004

Problem Set 4

Due Friday, October 8, 2004 at 4 PM

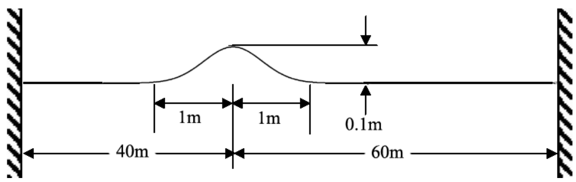
Reading Assignment

French pages 135-152, 201-230, 238-243, 253-264. Bekefi & Barrett pages 117-139.

Problem 4.1 – Travelling pulse

Do Problem 7-12 from French, A. P. *Vibrations and Waves*. New York, N.Y.: W. W. Norton and Company, January 1, 1971. ISBN: 0393099369.

The figure shows a pulse on a string of length 100 m with fixed ends. The pulse is travelling to the right without any change of shape, at a speed of 40 m/sec.



(a) Make a clear sketch showing how the transverse velocity of the string varies with distance along the string at the instant when the pulse is in the position shown.

(b) What is the maximum transverse velocity of the string (approximately)?

(c) If the total mass of the string is 2 kg, what is the tension T in it?

(d) Write an equation for $y(x,t)$ that numerically describes sinusoidal waves of wavelength 5 m and amplitude 0.2 m travelling in the negative x direction on a very long string made of the same material and under the same tension as above.

Problem 4.2 – Travelling pulse

Do Problem 7-13 from French, A. P. *Vibrations and Waves*. New York, N.Y.: W. W. Norton and Company, January 1, 1971. ISBN: 0393099369.

A pulse travelling along a stretched string is described by the following equation:

$$y(x,t) = \frac{b^3}{b^2 + (2x - ut)^2}$$

(a) Sketch the graph of y against x for $t = 0$.

(b) What are the speed of the pulse and its direction of travel?

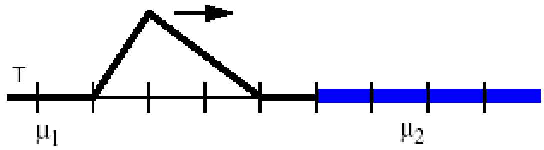
(c) The transverse velocity of a given point of the string is defined by

$$v_y = \frac{\partial y}{\partial t}$$

Calculate v_y as a function of x for the instant $t = 0$, and show by means of a sketch what this tells us about the motion of the pulse during a short time Δt.

Problem 4.3 – Pulse reflection at a boundary

Two strings with mass per unit length $\mu_1 = 0.1$ kg/m and $\mu_2 = 0.3$ kg/m, respectively, are jointed seamlessly. They are under tension $T = 20$ N. A travelling wave of a triangular shape shown in the figure is moving to the right along the lighter string. The tick marks set the scale of the pulse width.



(a) Find the reflection and transmission coefficients at the interface (including the signs).

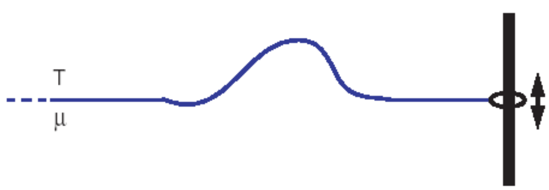
(b) Make a careful sketch of the *total* deformation of the string when the incident pulse has its peak exactly at the interface. Indicate how you arrived at your answer on your sketch.

(c) Make a careful sketch of the *total* deformation of the string when both the reflected and transmitted pulses have moved away from the interface.

(d) What is unphysical about the shape of this pulse? (Be quantitative)

Problem 4.4 – Boundary conditions on a string

A very long string of mass density μ and tension T is attached to a small hoop with negligible mass. The hoop slides on a greased vertical rod and experiences a vertical force $F_y = -b\frac{\partial y}{\partial t}$ when it moves.



(a) Apply Newton's law to the hoop to find the boundary condition at the end of the string. Express your result in terms of the partial derivatives of $y(x, t)$ at the location of the rod.

(b) Show that the boundary condition is satisfied by an incident pulse $f(x - vt)$ and a reflected pulse $g(x + vt)$. Find g in terms of f.

(c) Show that your result has the correct behavior in the limits $b \to 0$ (the string is free to slip) and $b \to \infty$ (the string is firmly clamped).

Problem 4.5 – Boundary conditions in a pipe

Pressure oscillations in a hollow pipe of length L are described by the wave equation

$$\frac{\partial^2 p}{\partial z^2} = \frac{\rho_0}{\kappa}\frac{\partial^2 p}{\partial t^2}$$

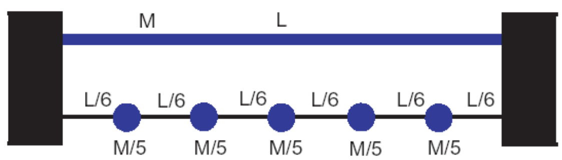
where p is the over-pressure (over and above the one atmosphere ambient pressure), ρ_0 is the density of the gas in the pipe, κ is the bulk modulus, and z is the longitudinal direction along the pipe. Assuming a solution of the form

$$p(z,t) = [A\ coskz + B\ sinkz]cos\omega t$$

find all the unknowns ($A,\ B,\ k$ and ω) for the case where the pipe is open at both ends and $p(z = L/2, t = 0) = p_0$.

Problem 4.6 – Normal modes of discrete vs. continuous systems

Referring to the diagram below, you are given a uniform string of length L and total mass M that is stretched to a tension T. You are also given a set of 5 beads, each of mass $M/5$, spaced at equal intervals on a massless string with tension T and total length L.



(a) Use boundary conditions to derive a general expression for the frequencies of the normal modes of oscillation of the string. Give the frequencies in terms of $n,\ T,\ L$ and M.

(b) Write down the frequencies of the five lowest normal modes of transverses oscillation of the string.

(c) Compare the numerical values of these normal mode frequencies with the normal mode frequencies of five beads on the massless string.
Hint: You do not have to solve the frequencies of the beads. You may use French, eqns. 5-25 and 5-26.

(d) Sketch the five lowest normal modes you found for the massive string. Sketch also the five normal modes of the massless-string-with-five-beads.

(e) In a sentence or two, discuss the differences, if any, in the normal modes of the two systems considered here.