Massachusetts Institute of Technology

Physics 8.03 Fall 2004

Problem Set 2

Due Friday, September 24, 2004 at 4 PM

Reading Assignment

French pages 77-112, Bekefi & Barrett pages 48-69.

Problem 2.1 – Take home experiment #1 – Influence of the mass on damping

The take-home experiments can be down-loaded from the 8.03 website.
Experiment #1: answer all six questions and give your reasoning and derivations.
All your measurements must include uncertainties (as we do in Lectures), and your final conclusions must take the uncertainties into account. The write-up mentions a pendulum length of 1-1.5 m. However, since our pendulum was about 65 cm long, we suggest you do the same. This will allow you to compare your results directly with ours (they will be posted Friday 9/24).

Problem 2.2 – Driven oscillator with damping

An object of mass 0.2 kg is hung from a spring whose spring constant is 80 N/m. The body is subject to a resistive force given by $-bv$, where v is its velocity (m/sec) and b =4 N m^{-1} sec.

(a) Set up the differential equation of motion for free oscillations of the system, and find the period of such oscillations.

(b) The object is subjected to a sinusoidal force given by $F(t) = F_0 \ sin\omega t$, where $F_0 = 2$ N and $\omega = 30$ sec^{-1}. In the steady state, what is the amplitude of the forced oscillation?

Instead of a driving force (see (b)), we now oscillate the end of the spring at the top end vertically with a harmonic displacement $X = X_0 \ sin(\omega t)$.

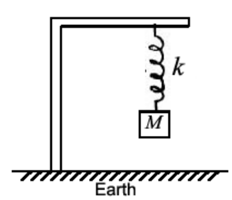
(c) Set up the differential equation of motion for this driven oscillator.

(d) What is the amplitude of the mass in steady state for $\omega = 0$, 30 and 300 sec^{-1}? $X_0 = 0.5$ cm in all cases.

Problem 2.3 – Seismograph

Do Problem 4-6 from French, *A. P. Vibrations and Waves*. New York, N.Y.: W. W. Norton and Company, January 1, 1971. ISBN: 0393099369.

Imagine a simple seismograph consisting of a mass M hung from a spring on a rigid frame attached to the earth (see figure). The spring force and the damping force depend on the displacement and velocity relative to the earth's surface, but the relevant acceleration (Newton's 2nd law) of M is relative to the fixed stars.



(a) Using y to denote the displacement of M relative to the earth and η to denote the displacement of the earth's surface relative to the fixed stars, show that the equation of motion is

$$\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + \omega_0^2 y = -\frac{d^2\eta}{dt^2}$$

(b) Solve for y (steady-state) if $\eta = C\ cos\omega t$.

(c) Sketch a graph of the amplitude A of the displacement y as a function of ω (supposing C the same for all ω).

(d) A typical long-period seismometer has a period of about 30 sec and a Q of about 2. As the result of a violent earthquake the earth's surface may oscillate with a period of about 20 min and with an amplitude such that the maximum acceleration is about 10^{-9} m/sec^2. How small a value of A must be observable if this is to be detected?

(e) Make sure you appreciate the big difference between this problem and problem 2.2. Compare the amplitudes of the two problems at very low and very high frequencies.

Problem 2.4 – Power dissipation

Do Problem 4-10 from French, A. P. *Vibrations and Waves*. New York, N.Y.: W. W. Norton and Company, January 1, 1971. ISBN: 0393099369.

The power input to maintain forced vibrations can be calculated by recognizing that this power is the mean rate of doing work against the resistive force $-bv$.

(a) Satisfy yourself that the instantaneous rate of doing work against this force is equal to bv^2.

(b) Using $x = A\ cos(\omega t - \delta)$, show that the mean rate of doing work is $b\omega^2 A^2/2$.

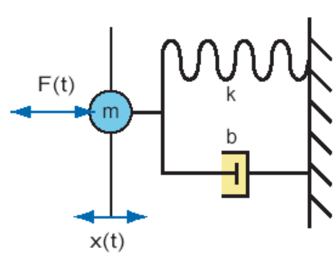
(c) Substitute the value of A at any arbitrary frequency and hence obtain the expression for $\bar{P}$ as given in Eq. (4-23).

Problem 2.5 – Transient behavior

Consider the simple damped spring-mass system shown in the first figure. The mass is driven by an external force given by

$$F(t) = F_0\ cos(\omega t + \phi)$$

The mass is at rest at its equilibrium position, $x = 0$, when the force is turned on instantaneously



at $t = 0$. The response of the mass to this driving force in shown in the second figure. Assuming that the mass is $m = 1$ kg, use the time series for $x(t)$ to get *estimates* (within 20%) for:

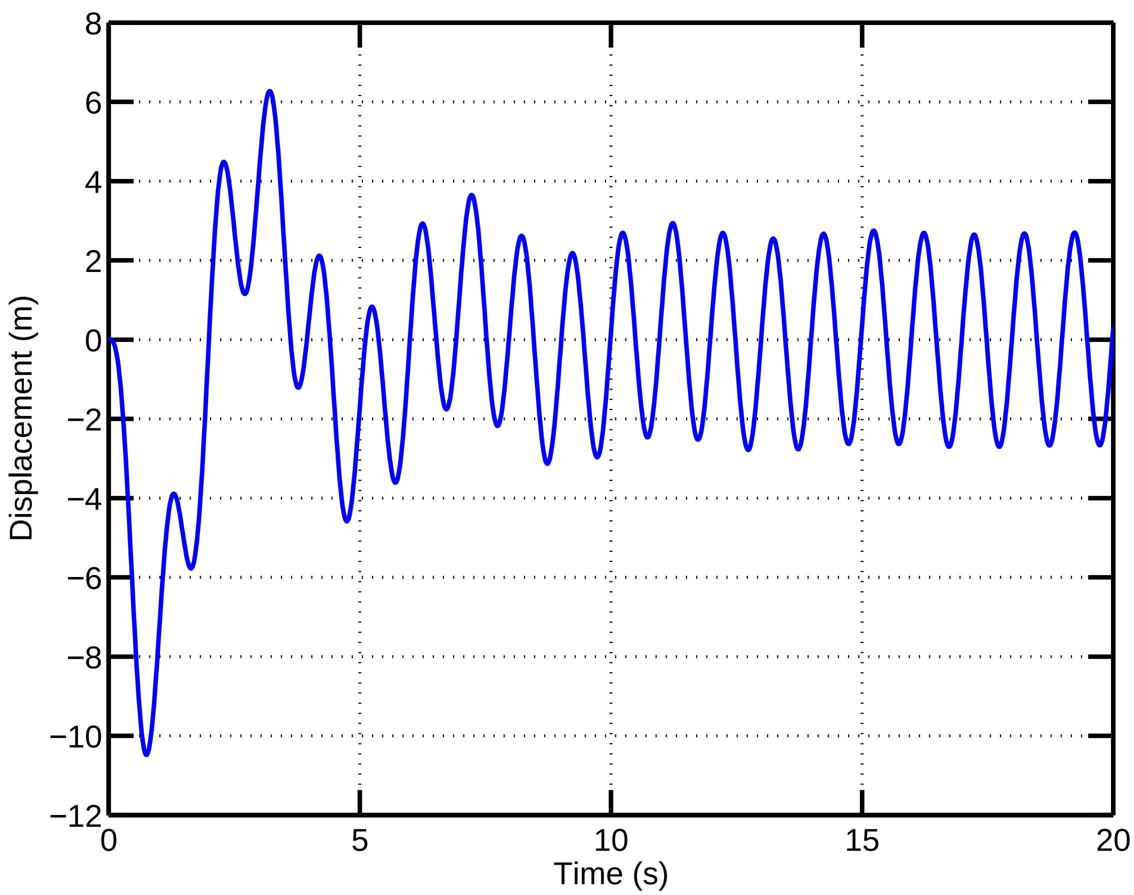
(a) The natural frequency of the undamped oscillator, $\omega_0/(2\pi)$ in Hz.
Hint: You may assume that γ is small, so that $\omega_1 \equiv \sqrt{\omega_0^2 - \gamma^2/4} \approx \omega_0$.

(b) The damping coefficient, b in N s/m.

(c) The frequency of the driving force, $\omega/(2\pi)$ in Hz.

(d) The amplitude of the driving force, F_0 in N.

(e) What is ϕ?



Problem 2.6 – Driven RLC circuit

A generator of emf $V(t) = V_0 \, cos\omega t$ is connected in series with a resistance R, an inductance L, and a capacitance C.

(a) Using Faraday's law, write down the differential equations for the current I in the circuit and for the charge, q, on the capacitor.

(b) Solve for $q(\omega, t)$.

(c) Solve for $I(\omega, t)$.

In what follows, $V_0 = 3$ V, $R = 50\ \Omega$, $L = 100$ mH, and $C = 0.01\ \mu$F.

(d) Plot the amplitude of the current (I_o) as a function of ω.

(e) At what value of ω is (I_o) a maximum?

(f) Plot q_o as a function of ω.

(g) At what frequency, ω, is the voltage across the capacitor a maximum?