Massachusetts Institute of Technology

Physics 8.03 Fall 2004

Problem Set 11

There is no due date for this problem set, but the topics covered here (including the Take-Home experiment) are all part of the Final Exam. I will post the solutions on Friday, December 10.

Reading Assignment

Bekefi & Barrett pages 559-584.

Problem 11.1 – Take-home experiment #9 – Transmission grating

The text mentions a mini-maglite to do some of the experiments. The maglite, however, was not put in your 8.03 kit this term. I suggest that you use a flashlight instead. Use some black paper with a small hole in it to reduce the size of the light. I also recommend that you use your color filters when doing some of the experiments with your grating even though that is not suggested in the text. Using the filters will give you some extra insight. That's why I added them to your optics kit/envelope.

The optics kit is yours to KEEP and to ENJOY! I always carry a grating and a linear polarizer on me. The polarizer is great to look at the blue sky, at rainbows and more, and the grating is priceless to look at various bright lights (mercury lamps!). Choose a light that is FAR AWAY (e.g., across the river) to minimize the angular dimension of the light; this will give you the best spectral resolution possible as demonstrated in lectures.
Please return the 8.03 experiment kits on Monday 12/6 3pm-6pm or on Tue 12/7 11am-1pm.

Problem 11.2 – Think big

The criterion for Fraunhofer diffraction is

$$z \geq \frac{2D^2}{\lambda} \text{ (Bekefi \& Barrett eq. 8.71)}$$

(a) What is the meaning of $z,\ D$, and λ?

The first zero in the case of single slit Fraunhofer diffraction is found when $\sin\phi = \frac{\lambda}{D}$ (Bekefi & Barrett page 584).

(b) Use this result to derive in a single line that, z should be larger than a few times $\frac{D^2}{\lambda}$. This is a very easy way to appreciate the Fraunhofer criterion.

Gedanken Experiment. Suppose we want to observe Fraunhofer's single slit diffraction from a star 10 light years away from us. As far as the above condition is concerned, we could make the slit about 12 meters wide without losing our Fraunhofer diffraction pattern that we want to observe on a screen. Take 500 nm as the wavelength.

(c) What should the approximate minimum distance be between the slit and our observation screen (photographic plate)?

(d) How wide would the central "bright" maximum approximately be on our screen? Try to predict first, then calculate!

Now leave the screen and slit in place but narrow down the slit width to 2 meters.

(e) First try to predict, then calculate the approximate width of the central "bright" maximum on the screen. Don't be too hasty with your prediction!

(f) What would be the approximate width of our central "bright" maximum due to diffraction if we made the "slit" 96 meters wide, without changing the distance between the gap (on Earth) and the screen (on the Moon)?

(g) List your reasons why the above "gedanken-experiment" would never give satisfying results.

Problem 11.3 (Bekefi & Barrett 8.7 amended) – Angular resolution

Two bright lights are 1 ft apart and 10 miles away. They are observed by a telescope, the lens of which has a diameter of 5 cm. A slit is placed in front of the lens and oriented so that **its width is parallel to the line that connects the two lights**. The slit width is variable, and it is narrowed until the two lights are just barely resolved. Find its width, assuming the effective wavelength to be 6000 $\AA$.

Problem 11.4 – Pinhole camera

Do problem 8.8 from Bekefi, and Barrett. *Electromagnetic Vibrations, Waves and Radiation.* Cambridge, MA: The MIT Press, September 15, 1977. ISBN: 0262520478.

A pinhole camera for visible light is made from a cubical box (length of one side = L) by drilling a small circular aperture (diameter = d) in one side and using the opposite inside wall as the screen where the film is placed. Approximately what value of d will provide the sharpest image on the film? (*Hint: Calculate the full size of the geometrical plus diffraction image of a distant source.*)

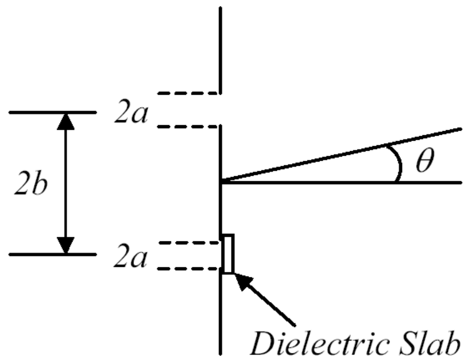
Problem 11.5 – Double slit interference

Do problem 8.9 from Bekefi, and Barrett. *Electromagnetic Vibrations, Waves and Radiation*. Cambridge, MA: The MIT Press, September 15, 1977. ISBN: 0262520478.

A plane electromagnetic wave of wavelength λ_0 is incident on two long, narrow slits, each having width $2a$ and separated by a distance $2b$, with $b \gg a$. One of the slits is covered by a thin dielectric plate of thickness d, and dielectric coefficient κ, with d chosen so that $(\sqrt{\kappa}-1)d/\lambda_0 = 5/2$.

The interference pattern due to the slits is observed in a plane a distance L from the slits, where L is large enough so that the far field approximations may be used, that is, the pattern depends only on the angle θ from the normal to the slits, as shown.

(a) Consider effects due to interference only. What is the condition for a maximum in the pattern? Sketch the interference pattern.



(b) Now include effects due to both interference and diffraction. How is the intensity distribution modified from that obtained in (a)? Let $b/a = 10$, sketch the resulting interference-diffraction pattern.
(Assume that all angles involved are small enough so that $\cos\theta \simeq 1$, and hence that the optical path through the dielectric is independent of angle.)