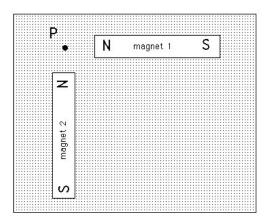# Magnetic Field
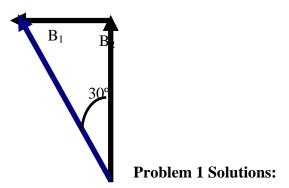## Challenge Problem Solutions

Problem 1:

Consider two bar magnets placed at right angles to each other, as pictured at left.

(a) If a small compass is placed at point P, what direction does the painted end of the compass needle point?

(b) If the compass needle instead pointed 15 degrees clockwise of where you predicted in (a), what could you qualitatively conclude about the relative strengths of the two magnets?

Problem 1 Solutions:

a)It points away from each magnetic North, which means toward the upper left corner (45 degrees if they are the same magnitude).

b)In order for it to point 15 degrees clockwise the second magnet must be stronger than the first. Since the total field is just a vector sum of the two we can see how much stronger.

$$\tan 30° = \frac{B_1}{B_2} = \frac{1}{\sqrt{3}} \Rightarrow \boxed{B_2 = \sqrt{3}B_1}$$

Problem 2:

(a) Can a constant magnetic field set into motion an electron which is initially at rest? Explain your answer.

(b) Is it possible for a constant magnetic field to alter the speed of a charged particle? What is the role of a magnetic field in a cyclotron?

(c) How can a current loop be used to determine the presence of a magnetic field in a given region of space?

(d) If a charged particle is moving in a straight line through some region of space, can you conclude that the magnetic field in that region is zero? Why or why not?

(e) List some similarities and differences between electric and magnetic forces.

Problem 2 Solutions:

(a) No. Changing the velocity of a particle requires an accelerating force. The magnetic force is proportional to the speed of the particle. If the particle is not moving, there can be no magnetic force on it.

(b) No, it is not possible. Because $\vec{\mathbf{F}}_B = q\left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$, the acceleration produced by a magnetic field on a particle of mass m is $\vec{\mathbf{a}}_B = \dfrac{q}{m}\left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$. For the acceleration to change the speed, a component of the acceleration must be in the direction of the velocity. The cross product tells us that the acceleration must be perpendicular to the velocity, and thus can only change the direction of the velocity.

The magnetic field in a cyclotron essentially keeps the charged particle in the electric field for a longer period of time, and thus experiencing a larger change in speed from the electric field, by forcing it in a spiral path. Without the magnetic field, the particle would have to move in a straight line through an electric field over a distance that is very large compared to the size of the cyclotron.

(c) If the current loop feels a torque, it must be caused by a magnetic field. If the current loop feels no torque, try a different orientation—the torque is zero if the field is along the axis of the loop.

(d) Not necessarily. If the magnetic field is parallel or anti-parallel to the velocity of the charged particle, then the particle will experience no magnetic force. There may also be an electric force acting on the particle such that $\vec{\mathbf{F}}_q = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}}_q \times \vec{\mathbf{B}}\right) = 0$.

(e)

Similarities:

1. Both can accelerate a charged particle moving through the field.

2. Both exert forces directly proportional to the charge of the particle feeling the force.
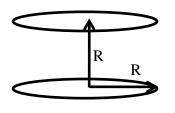
Differences:

1. The direction of the electric force is parallel or anti-parallel to the direction of the electric field, but the direction of the magnetic force is perpendicular to the magnetic field and to the velocity of the charged particle.

2. Electric forces can accelerate a charged particle from rest or stop a moving particle, but magnetic forces cannot.

Problem 3:

The magnitude of the component of a magnetic field along the axis of a coil with N turns to be given by:

$$B_{axial} = \frac{N \mu_0 I R^2}{2} \frac{1}{(z^2 + R^2)^{3/2}}$$

where z is measured from the center of the coil.

As pictured at left, a Helmholtz coil is created by placing two such coils (each of radius R and N turns) a distance R apart.

(a) If the current in the two coils is parallel (Helmholtz configuration), what is the magnitude of the magnetic field at the center of the apparatus (midway between the two coils)? How does this compare to the field strength at the center of the single coil configuration (e.g. what is the ratio)?

(b) In the anti-Helmholtz configuration the current in the two coils is anti-parallel. What is field strength at the center of the apparatus in this situation?

(c) Consider coils that have a radius $R = 7$ cm and $N = 168$ turns. Suppose $I = 0.6$ A runs in the single coil and 0.3 A runs in each in Helmholtz and anti-Helmholtz mode. What, approximately, are the largest on-axis fields we should expect in these three configurations? Where (approximately) are the fields the strongest?

Problem 3 Solution:

(a) We use superposition principle to determine the magnetic field due to the two coils. We are the same distance $z = R/2$ from each coil and since the currents are parallel they both create a field in the same direction (for example, if both currents are counter-clockwise they both create an upward magnetic field at the midpoint). The magnitude then is just twice that of a single coil:

$$B = 2 \times \frac{N \mu_0 I R^2}{2} \frac{1}{((R/2)^2 + R^2)^{3/2}} = \frac{N \mu_0 I}{R} \frac{1}{((1/2)^2 + 1)^{3/2}} = \frac{8 N \mu_0 I}{5^{3/2} R}$$

Comparing this to the field strength at the center of a single coil:

$$B_{sgl\ coil} = \frac{N \mu_0 I}{2R}$$

We find that the field of a Helmholtz coil is slightly larger:

$$\frac{B_{\text{Helmholtz}}}{B_{\text{Sgl Coil}}} = \left(\frac{8N\,\mu_0\,I}{5^{3/2}\,R}\right) \bigg/ \left(\frac{N\,\mu_0\,I}{2R}\right) = \frac{16}{5^{3/2}} \approx 1.4$$

(b) In this case the fields from the two coils are in opposite directions so they cancel each other out. That is, $B = 0$.

(c) For a single coil the maximum is at the center of the coil, for a Helmholtz at the center:

$$B_{\text{sgl coil}}^{\max} = \frac{(168)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(0.6\ \text{A})}{2\left(7\times10^{-2}\ \text{m}\right)} = 9.0\times10^{-4}\ \text{T} = 9.0\ \text{Gauss};$$

$$B_{\text{Helmholtz}}^{\max} = \frac{16}{5^{3/2}} \cdot 4.5\ \text{Gauss} = 6.5\ \text{Gauss}$$

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