Module 23:
LR Circuit

Module 23: Outline

LR Circuits

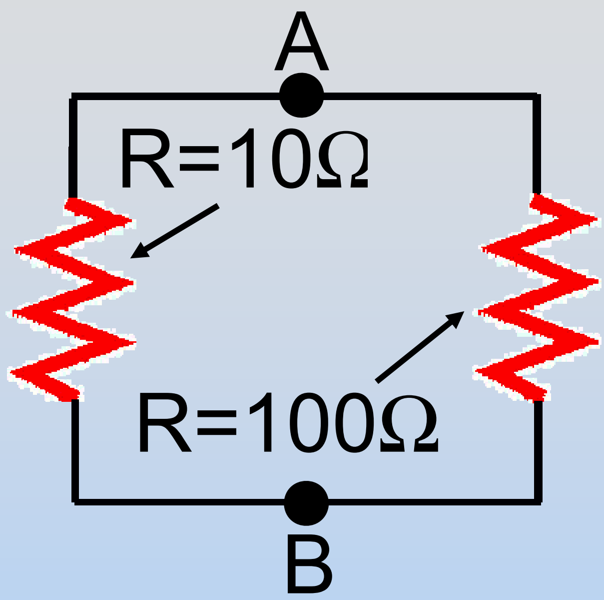
Expt. 8: Part 1: LR Circuits

Think Harder about Faraday

Concept Question Question: Faraday in Circuit

Concept Question: Faraday Circuit

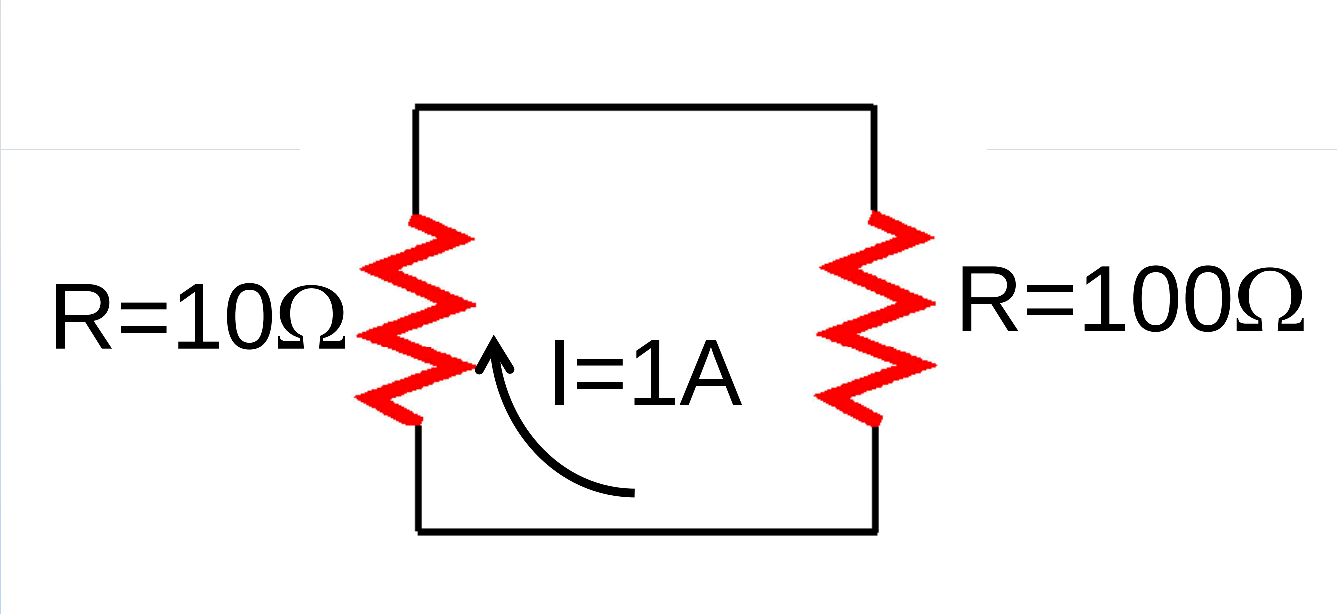
A magnetic field B penetrates this circuit outwards, and is increasing at a rate such that a current of 1 A is induced in the circuit (which direction?).



The potential difference VA-VB is:

1. **+10 V**
2. **-10 V**
3. **+100 V**
4. **-100 V**
5. **+110 V**
6. **-110 V**
7. **+90 V**
8. **-90 V**
9. **None of the above**

Non-Conservative Fields



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

E is no longer a conservative field –

Potential now meaningless

Kirchhoff's Modified 2nd Rule

$$\sum_i \Delta V_i = -\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = +\frac{d\Phi_B}{dt}$$

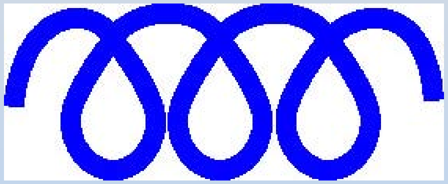
$$\Rightarrow \sum_i \Delta V_i - \frac{d\Phi_B}{dt} = 0$$

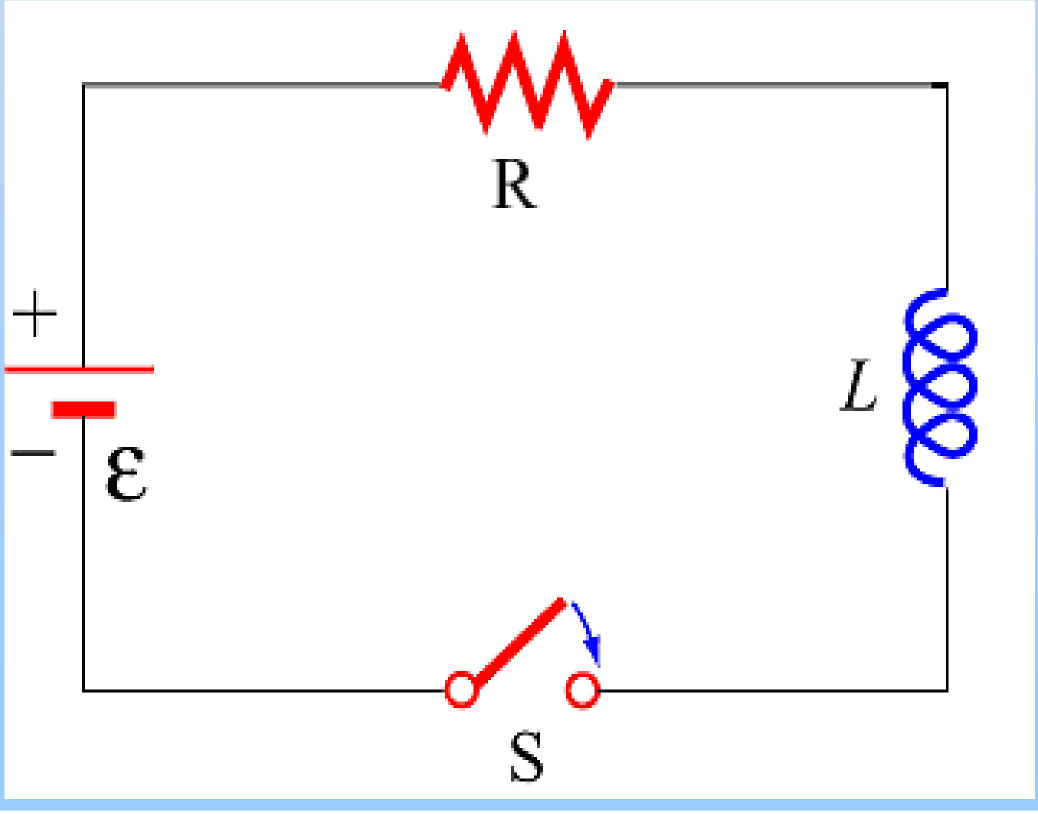
If all inductance is 'localized' in inductors then our problems go away – we just have:

$$\sum_i \Delta V_i - L\frac{dI}{dt} = 0$$

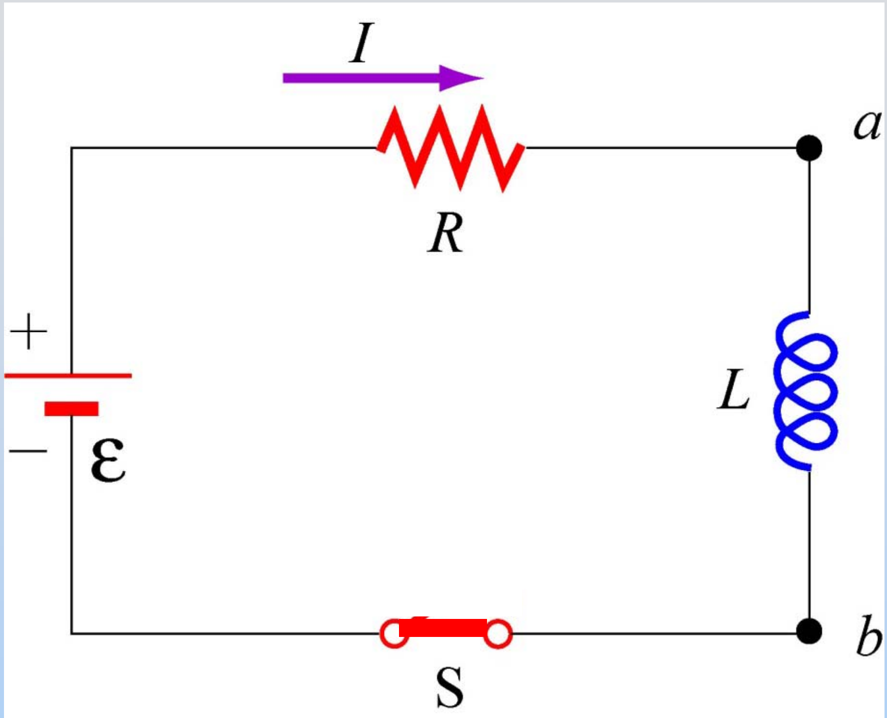
Inductors in Circuits

Inductor: Circuit element with self-inductance
Ideally it has zero resistance

Symbol: 



Ideal Inductor



BUT, EMF generated *by* an inductor is ***not*** a voltage drop *across* the inductor!

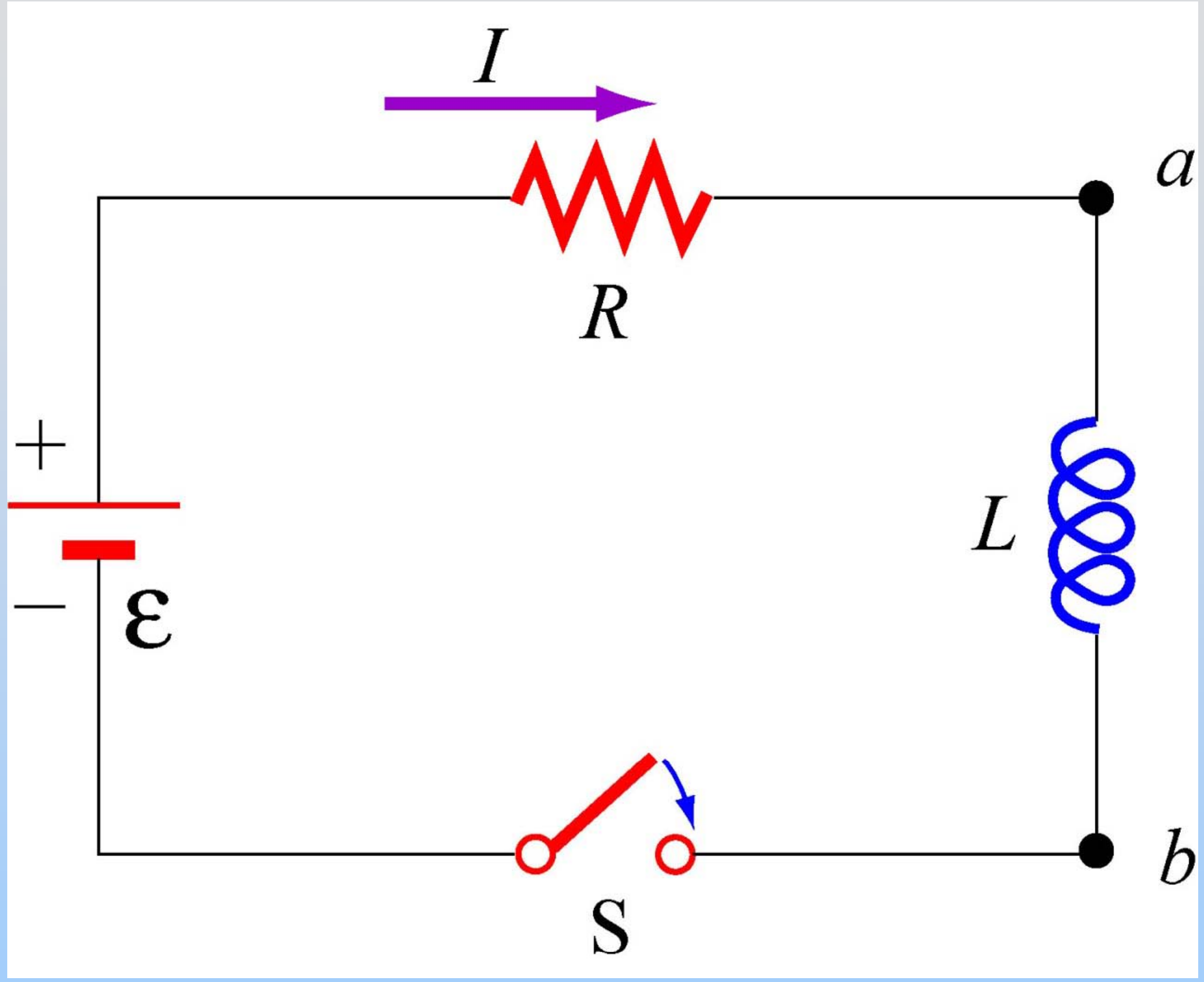
$$\varepsilon = -L\frac{dI}{dt}$$

$$\Delta V_{\text{inductor}} \equiv -\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

Because resistance is 0, E must be 0!

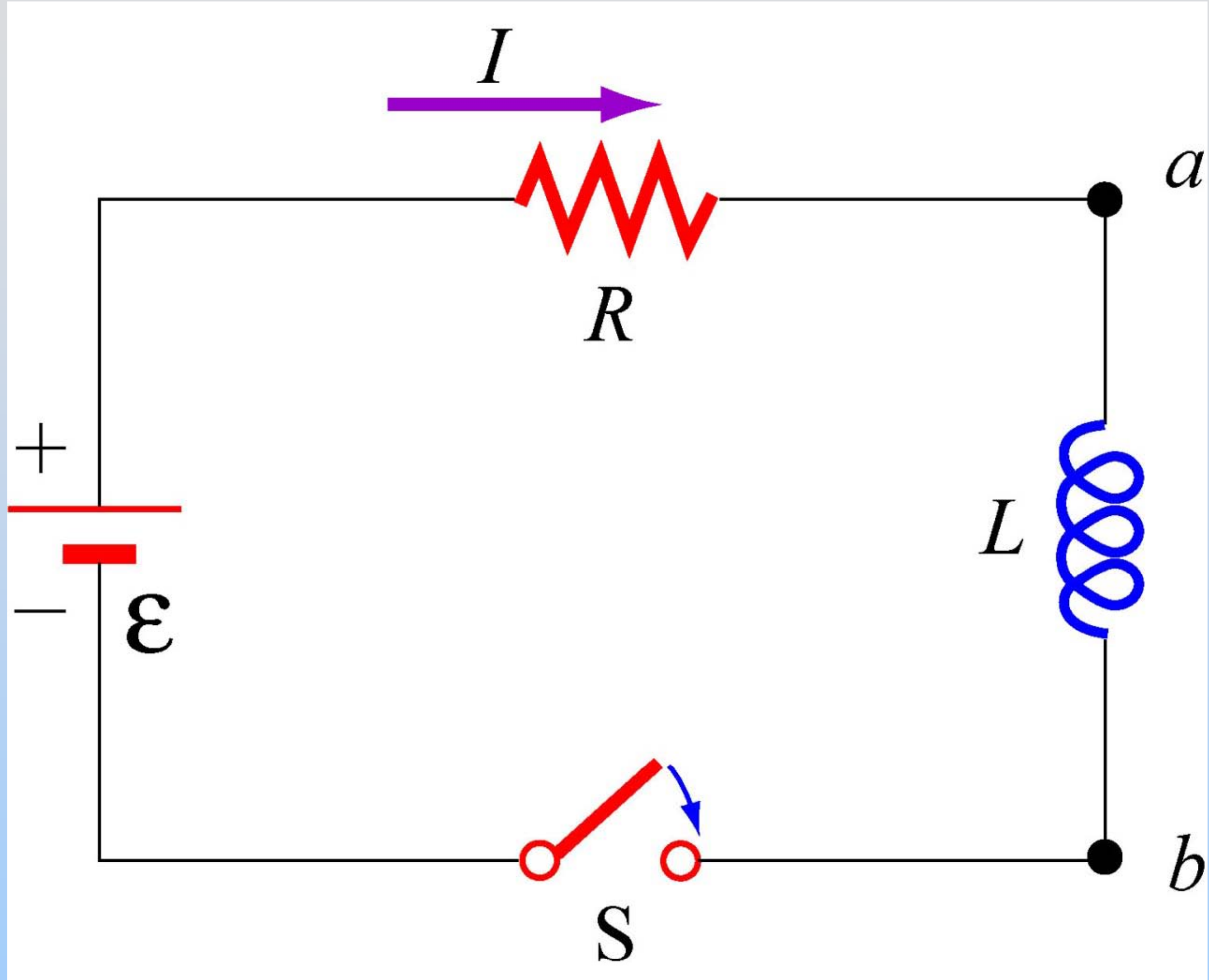
Circuits:
Applying Modified Kirchhoff's
(Really Just Faraday's Law)

LR Circuit



$$\sum_i V_i = \mathcal{E} - IR - L\frac{dI}{dt} = 0$$

LR Circuit



$$\varepsilon - IR - L\frac{dI}{dt} - 0 \quad = \quad \frac{dI}{dt} - -\frac{1}{L/R}\left(I - \frac{\varepsilon}{R}\right)$$

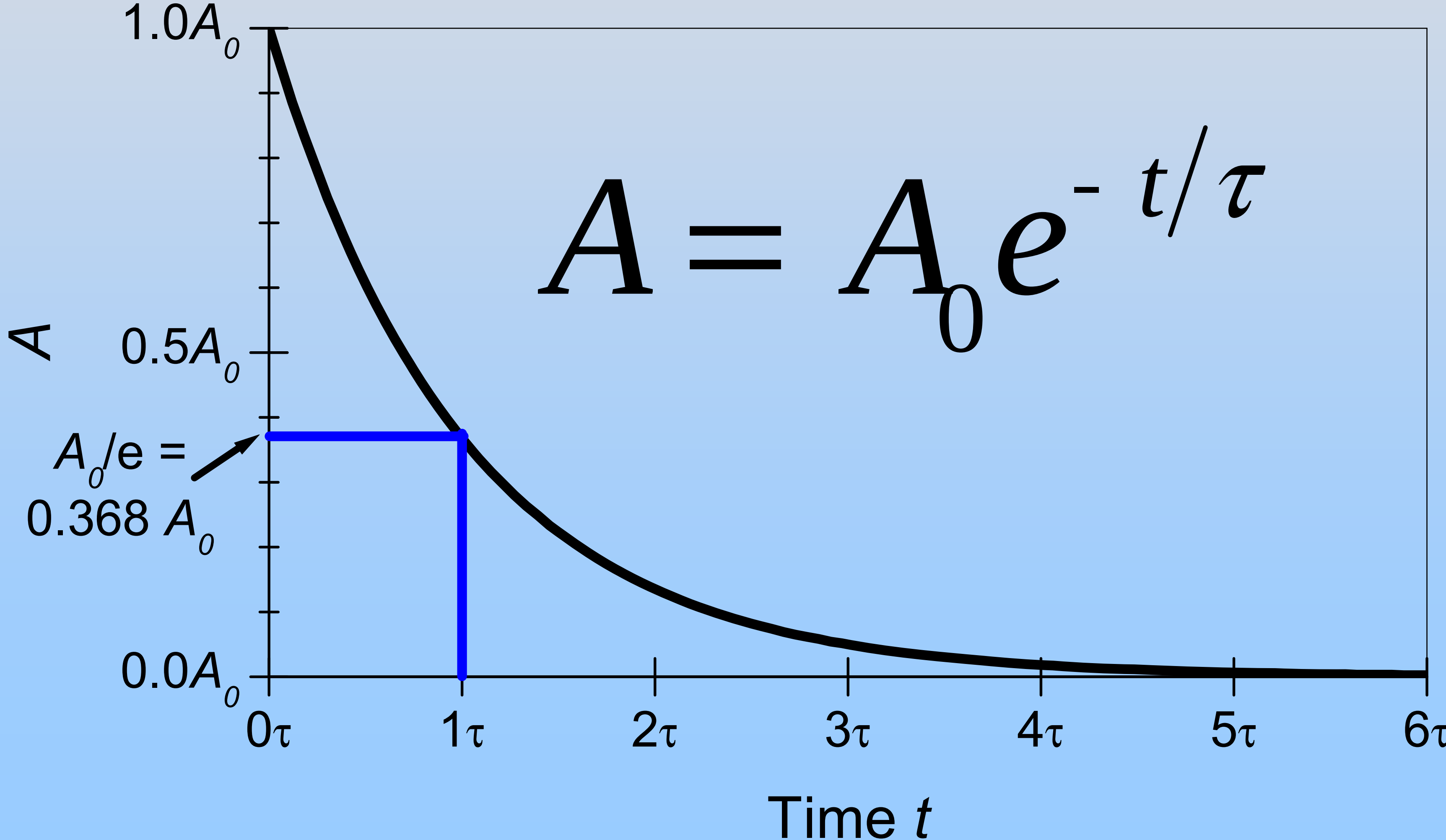
Review Some Math: Exponential Decay

Exponential Decay

Consider function *A* where: $$\frac{dA}{dt} = -\frac{1}{\tau}A$$

A decays exponentially:

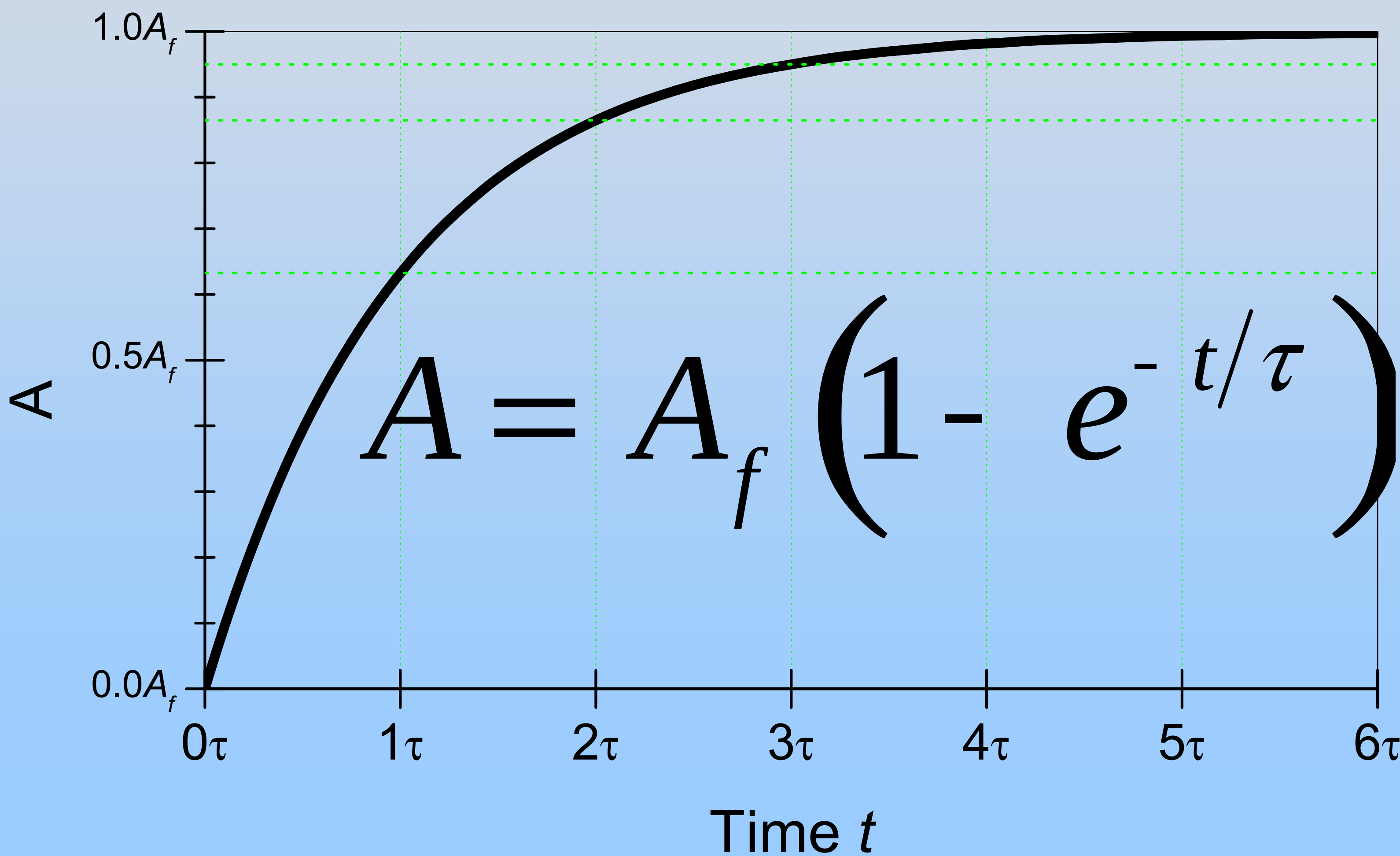
$$A = A_0 e^{-t/\tau}$$



Exponential Behavior

Slightly modify diff. eq.: $$\frac{dA}{dt} = -\frac{1}{\tau}\left(A - A_f\right)$$

A "decays" to A_f:



$$A = A_f\left(1 - e^{-t/\tau}\right)$$

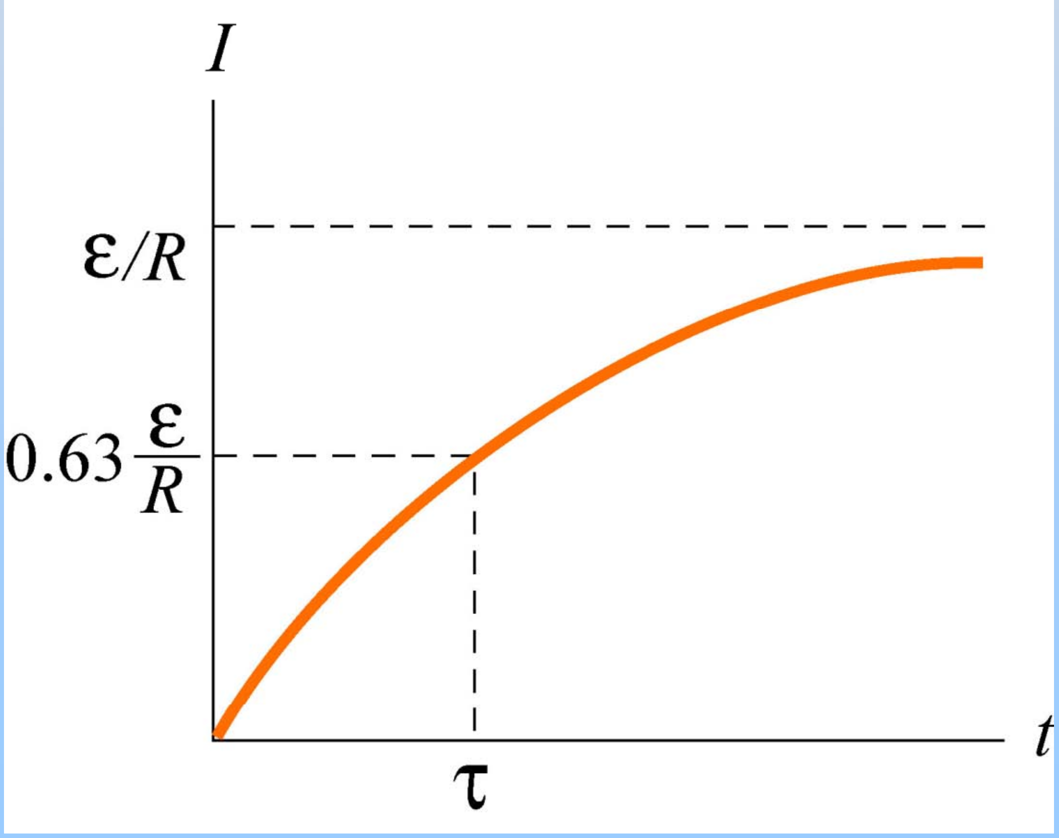
This is one of two differential equations we expect you to know how to solve (know the answer to).

The other is simple harmonic motion (more on that next module)

LR Circuit

$$\frac{dI}{dt} = -\frac{1}{L/R}\left(I - \frac{\varepsilon}{R} \right)$$

Solution to this equation when switch is closed at t = 0:

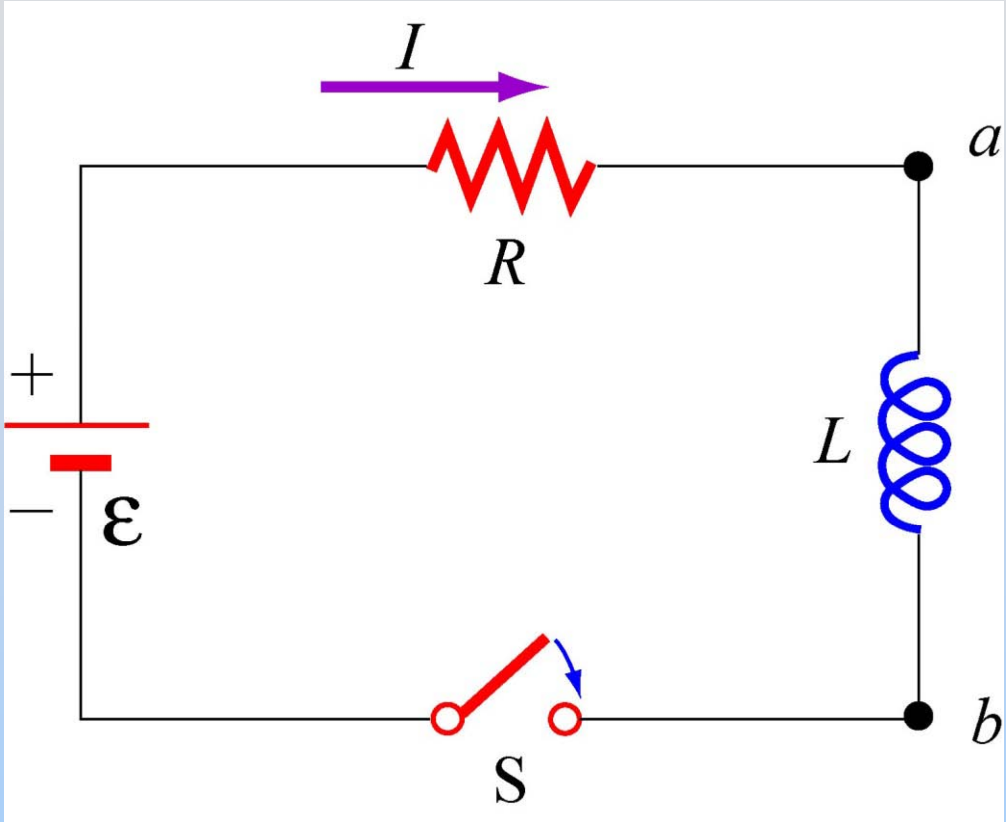


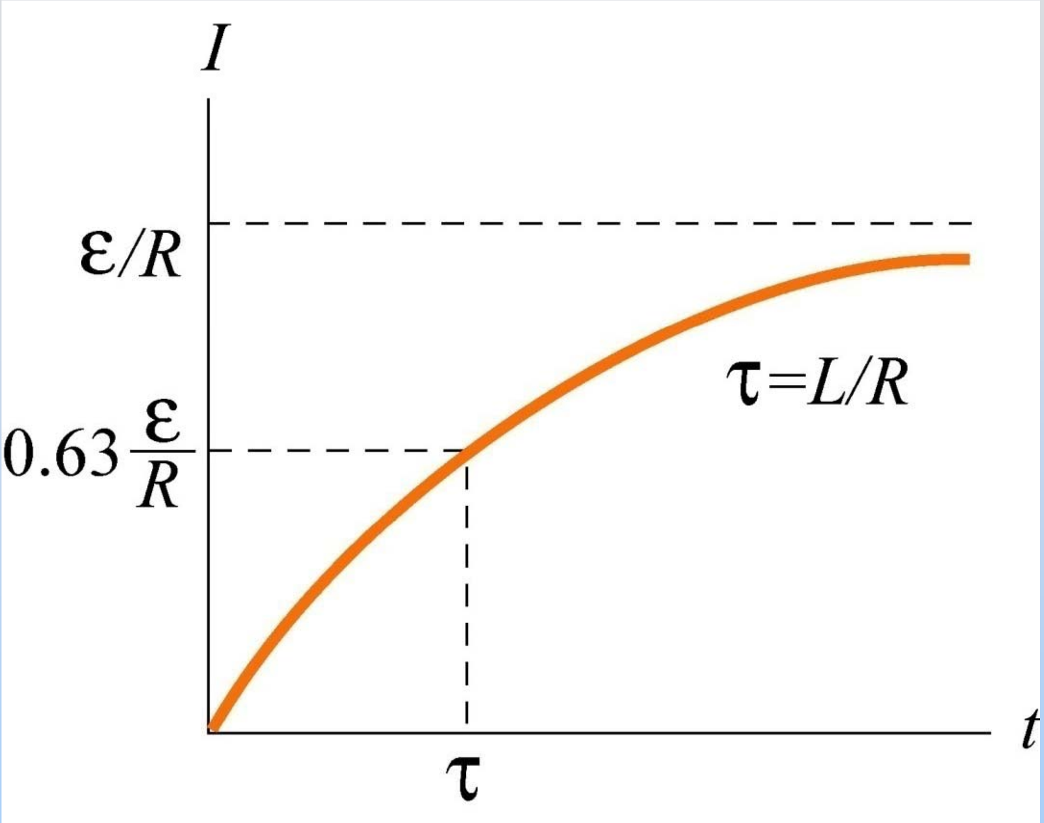
$$I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R} : \text{time constant}$$

(units: seconds)

LR Circuit

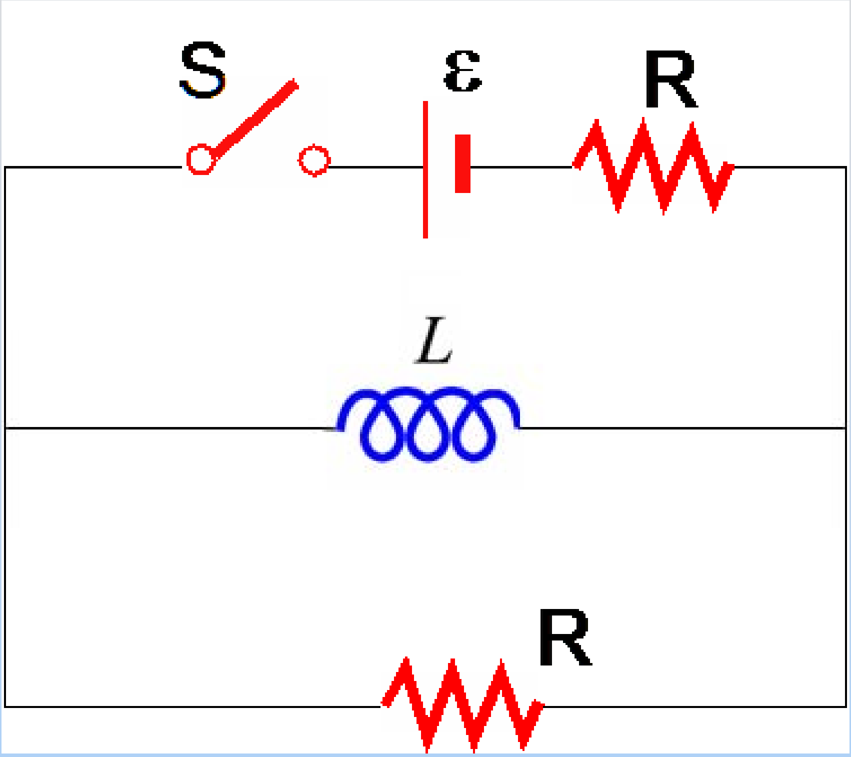




t=0⁺: Current is trying to change. Inductor works as hard as it needs to to stop it

t=∞: Current is steady. Inductor does nothing.

Problem: Circuits

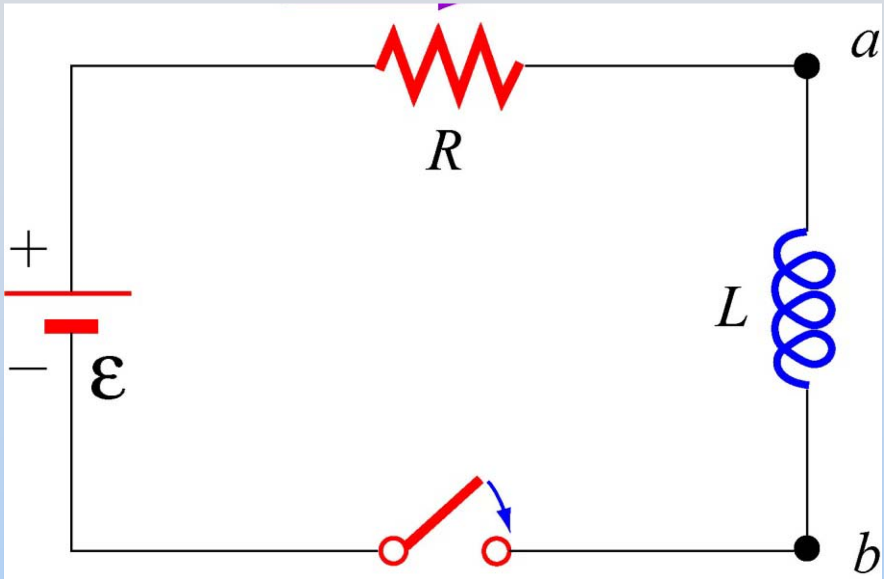


For the above circuit sketch the currents through the two bottom branches as a function of time (switch closes at $t = 0$, opens at $t = T$). State values at $t = 0^+$, T^-, T^+

Concept Question Question: Voltage Across Inductor

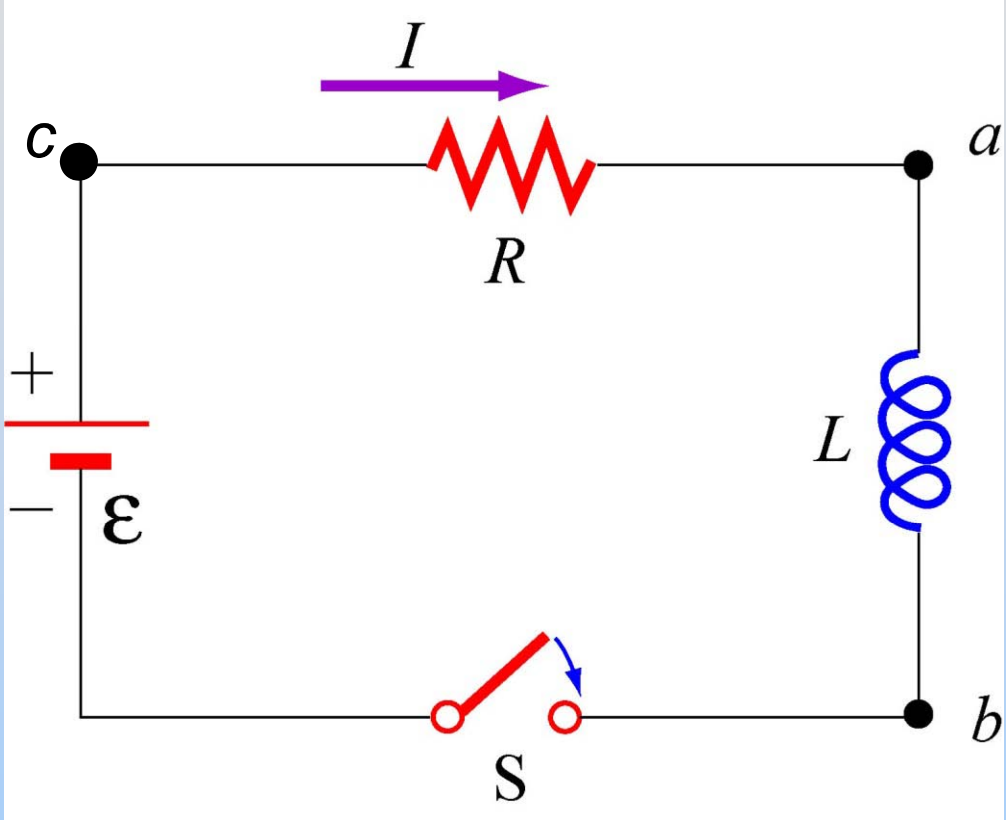
Concept Question: Voltage Across Inductor

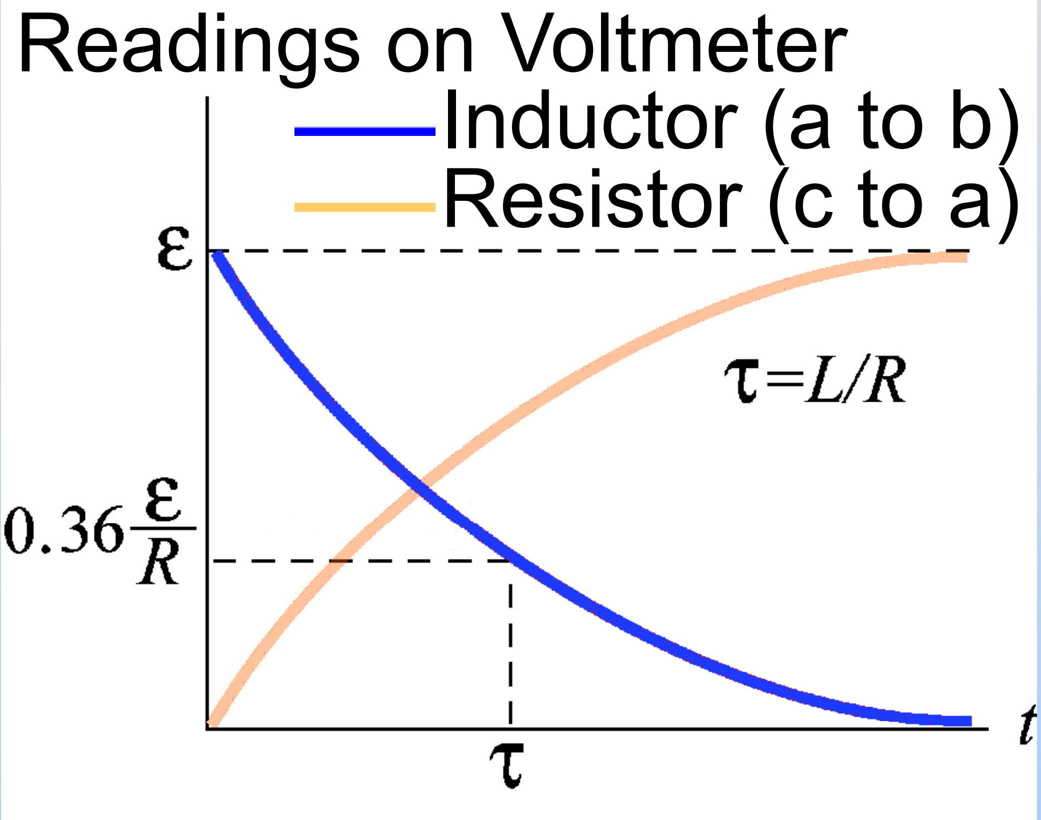
In the circuit at right the switch is closed at t = 0. A voltmeter hooked across the inductor will read:



1. $V_L = \varepsilon e^{-t/\tau}$
2. $V_L = \varepsilon(1 - e^{-t/\tau})$
3. $V_L = 0$
4. I don't know

LR Circuit



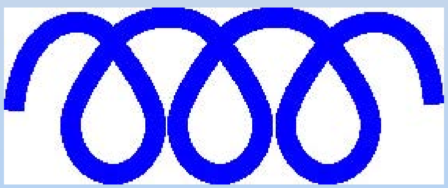


t=0$^+$: Current is trying to change. Inductor works as hard as it needs to to stop it

t=∞: Current is steady. Inductor does nothing.

Non-Ideal Inductors

Non-Ideal (Real) Inductor: Not only L but also some R



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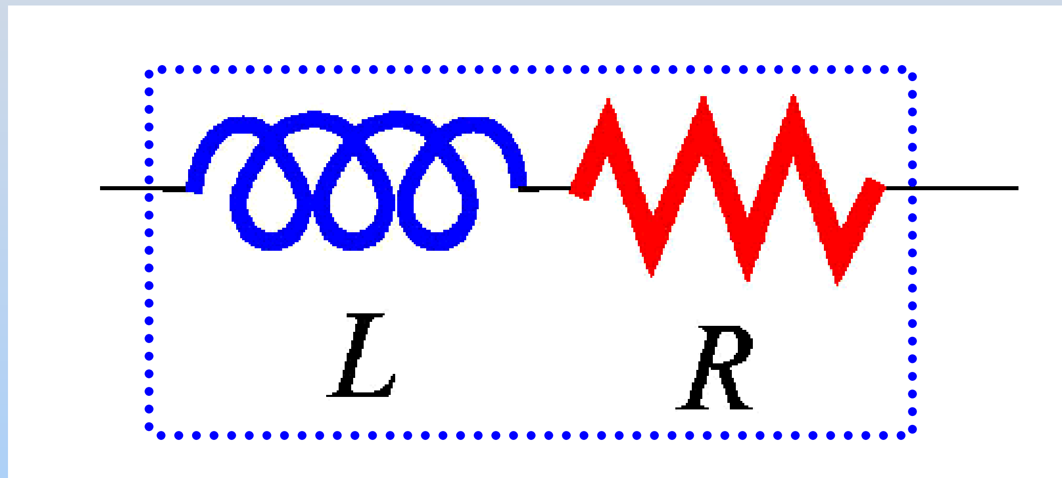


In direction of current: $\mathcal{E} = - L\frac{dI}{dt} - IR$

Experiment 8: Part 1
Inductance & LR Circuits

Concept Question Questions: LR Circuits

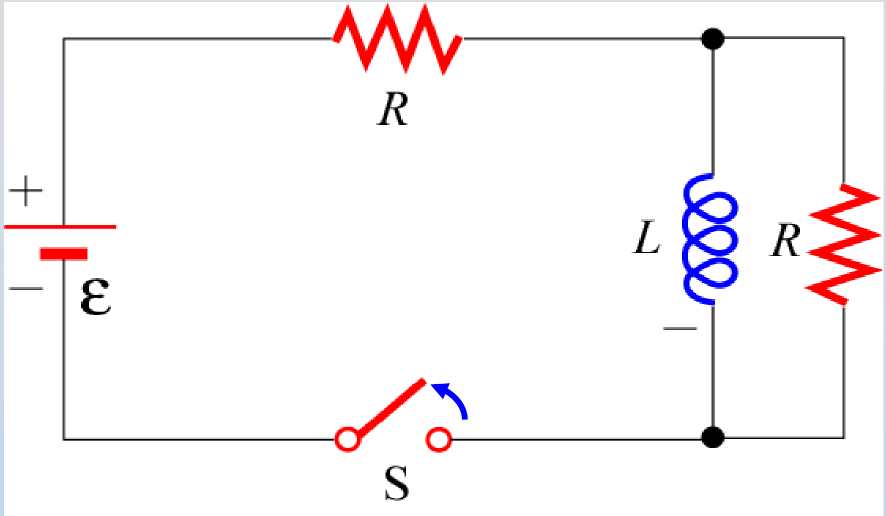
Concept Question: Inserting a Core

When you insert the iron core what happens?

1. B Increases so L does too
2. B Decreases so L does too
3. B Increases so L Decreases
4. B Decreases so L Increases
5. I don't know

Concept Q.: RL Circuit

In the circuit at right the switch S has been closed a very long time. At t = 0, the switch is opened. Taking downward current as positive, immediately after the switch is opened the current in the inductor is equal to



1. *ε /R*
2. *ε /2R*
3. *- ε /R*
4. *- ε /2R*
5. Zero
6. I don't know