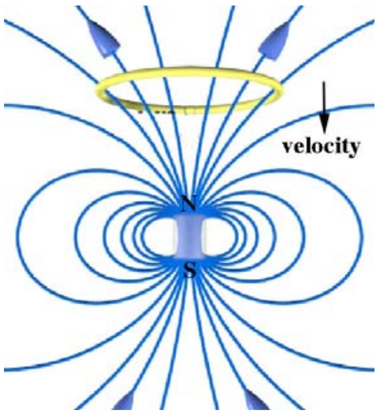
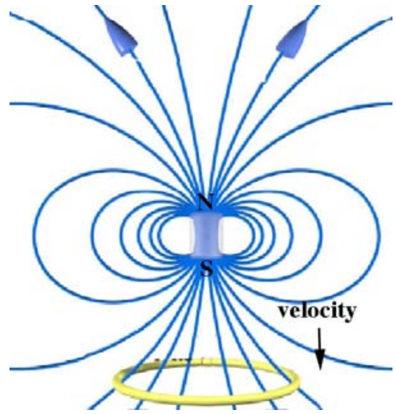
Faraday's Law
Challenge Problem Solutions

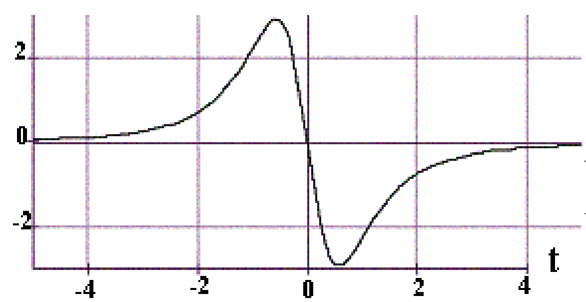
Problem 1:

A coil of wire is above a magnet whose north pole is pointing up. For current, counter-clockwise when viewed from above is positive. For flux, upwards is positive.

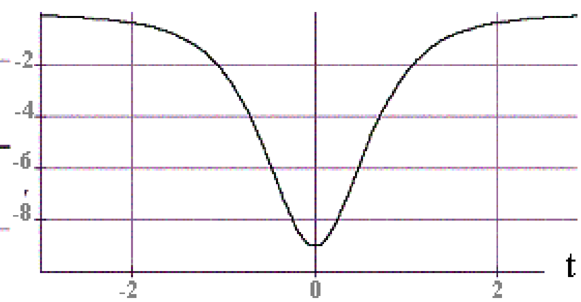




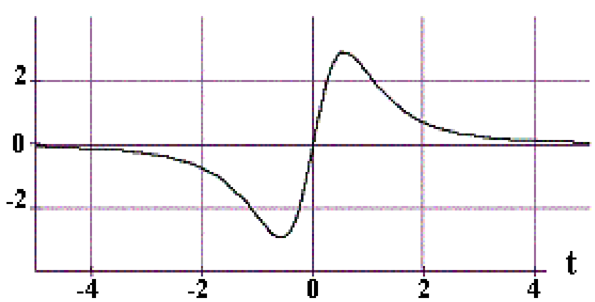
Suppose you moved the loop from well ***above*** the magnet to well ***below*** the magnet at a constant speed. Which graph most closely resembles the graph of ***current through the loop*** as a function of time?



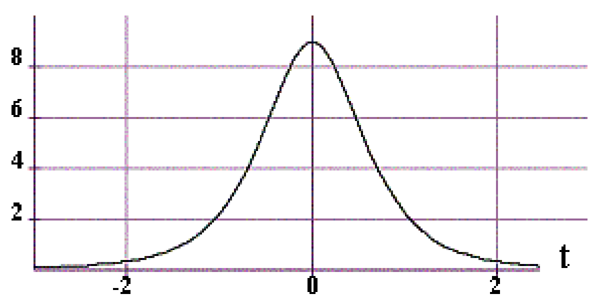
(a)



(b)



(c)



(d)

(e) None of the above.

Problem 1 Solution:

c. If you moved the loop from well *above* the magnet to well *below* the magnet at a constant speed, then as the loops approaches the magnet from below the flux through the loop is upward (positive) and increasing. Therefore an induced current flows through the loop in a clockwise direction as seen from above (negative) resulting in induced flux downward through the loop opposing the change. Once the loop passes the magnet, the flux through the loop is upward (positive) and decreasing. Therefore an induced current flows through the loop in a counterclockwise direction as seen from above (positive) resulting in induced flux upward through the loop opposing the change. Therefore graph (c) closely resembles the graph of *current through the loop* as a function of time.

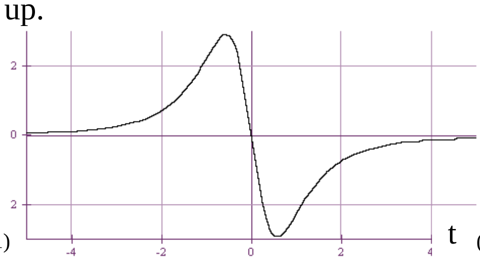
Problem 2:

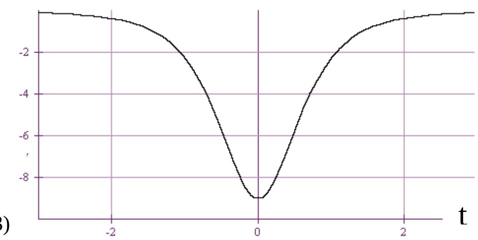
(a) Calculating Flux from Current and Faraday's Law. You move a coil from well above to well below a strong permanent magnet. You measure the current in the loop during this motion using a current sensor. You are able to graph the flux "measured" as a function of time.

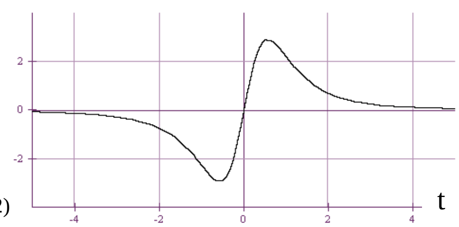
(i) Starting from Faraday's Law and Ohm's law, write an equation relating the current in the loop to the time derivative of the flux through the loop.

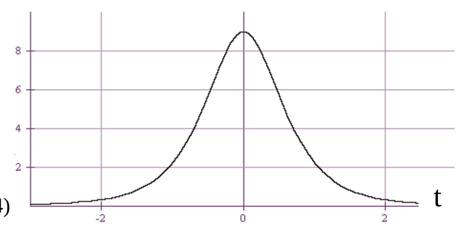
(ii) Now integrate that expression to get the time dependence of the flux through the loop $\Phi(t)$ as a function of current $I(t)$. What assumption must the software make before it can plot flux vs. time?

(b) Predictions: Coil Moving Past Magnetic Dipole
In moving the coil over the magnet, measurements of current and flux for each of several motions looked like one of the below plots. For current, counter-clockwise when viewed from above is positive. For flux, upwards is positive. The north pole of the magnet is pointing up.

(1)


(3)


(2)


(4)


Suppose you move the loop from well *above* the magnet to well *below* the magnet at a constant speed. Which graph most closely resembles the graph of:

(i) *magnetic flux through the loop* as a function of time?

(ii) *current through the loop* as a function of time?

Suppose you move the loop from well *below* the magnet to well *above* the magnet at a constant speed. Which graph most closely resembles the graph of:

(iii) *magnetic flux through the loop* as a function of time?

(iv) *current through the loop* as a function of time?

(c) Force on Coil Moving Past Magnetic Dipole

You can "feel" the force on a conducting loop as it moves past the magnet. For the following conditions, in what direction should the magnetic force point?

As you move the loop from well *above* the magnet to well *below* the magnet at a constant speed…

(i) … and the loop is *above* the magnet.

(ii) … and the loop is *below* the magnet

As you move the loop from well *below* the magnet to well *above* the magnet at a constant speed…

(iii) … and the loop is *below* the magnet.

(iv) … and the loop is *above* the magnet

(d) Feeling the Force

Now use an aluminum cylinder to "better feel" the force. To figure out why, answer the following.

(i) If we were to double the number of turns in the coil how would the force change?

(ii) Using the result of (a), how should we think about the Al tube? Why do we "better feel" the force?

In case you are interested, the wire is copper, and of roughly the same diameter as the thickness of the aluminum cylinder, although this information won't necessarily help you in answering the question.

Problem 2 Solutions:
(a) Calculating Flux from Current and Faraday's Law.

(i)

$$\varepsilon = -\frac{d\Phi}{dt} = IR$$

(ii)

$$d\Phi = -IR\,dt \quad \Rightarrow \quad \Phi(t) = -R\int_{t=0}^{t} I(t')\,dt'$$

The software must assume (as I did above) that the flux at time t=0 is zero.

(b) Predictions: Coil Moving Past Magnetic Dipole

(i) *magnetic flux through the loop* as a function of time? 4

(ii) *current through the loop* as a function of time? 2

Suppose you move the loop from well *below* the magnet to well *above* the magnet at a constant speed. Which graph most closely resembles the graph of:

(iii) *magnetic flux through the loop* as a function of time? 4

(iv) *current through the loop* as a function of time? 2

(c) Force on Coil Moving Past Magnetic Dipole

In all of these cases the force opposes the motion. For (a) & (b) it points upwards, for (c) and (d) downwards.

(d) Feeling the Force

(i)
If we were to double the number of turns we would double the total flux and hence EMF, but would also double the resistance so the current wouldn't change. But the force would double because the number of turns doubled.

(ii)
Going to the cylinder basically increases many times the number of coils (you can think about it as a bunch of thin wires stacked on top of each other). It also reduces the resistance and hence increases the current because the resistance is not through one very long wire but instead a bunch of short loops all in parallel with each other.

Problem 3:

(a) When a small magnet is moved toward a solenoid, an emf is induced in the coil. However, if the magnet is moved around inside a toroid, no measurable emf is induced. Explain.

(b) A piece of aluminum is dropped vertically downward between the poles of an electromagnet. Does the magnetic field affect the velocity of the aluminum? Explain.

(c) What happens to the generated current when the rotational speed of a generator coil is increased?

(d) If you pull a loop through a non-uniform magnetic field that is perpendicular to the plane of the loop which way does the induced force on the loop act?

Problem 3 Solutions:

(a)Moving a magnet inside the hole of the doughnut-shaped toroid will not change the magnetic flux through any turn of wire in the toroid, and thus not induce any current.

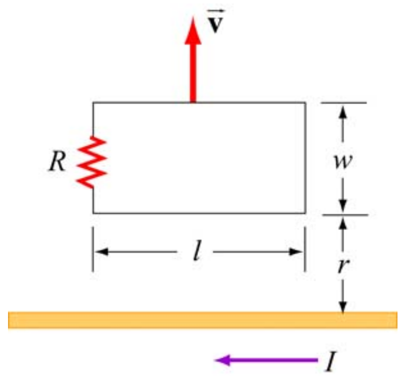
(b)Yes. The induced eddy currents on the surface of the aluminum will slow the descent of the aluminum. It may fall very slowly.

(c)The maximum induced emf will increase, increasing the terminal voltage of the generator resulting in a larger amplitude for the current.

(d)The direction of the induced force is opposite the direction of the pulling force.

Problem 4:

A rectangular loop of dimensions *l* and *w* moves with a constant velocity $\vec{\mathbf{v}}$ away from an infinitely long straight wire carrying a current *I* in the plane of the loop, as shown in the figure. The total resistance of the loop is *R*.



(a) Using Ampere's law, find the magnetic field at a distance *s* away from the straight current-carrying wire.

(b) What is the magnetic flux through the rectangular loop at the instant when the lower side with length l is at a distance *r* away from the straight current-carrying wire, as shown in the figure?

(c) At the instant the lower side is a distance *r* from the wire, find the induced emf and the corresponding induced current in the rectangular loop. Which direction does the induced current flow?

Problem 4 Solutions:

(a) Consider a circle of radius *s* centered on the current-carrying wire. Then around this Amperian loop, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi s) = \mu_0 I$
which gives

$$B = \frac{\mu_0 I}{2\pi s} \text{ (into the page)}$$

(b)

$$\Phi_B = \iint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int_r^{r+w} \left(\frac{\mu_0 I}{2\pi s}\right) l\, ds = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{r+w}{r}\right) \text{ (into the page)}$$

(c) The induce emf is

$$\varepsilon = -\frac{d}{dt}\Phi_B = -\frac{\mu_0 I l}{2\pi}\frac{r}{(r+w)}\left(\frac{-w}{r^2}\right)\frac{dr}{dt} = \frac{\mu_0 I l}{2\pi}\frac{vw}{r(r+w)}$$

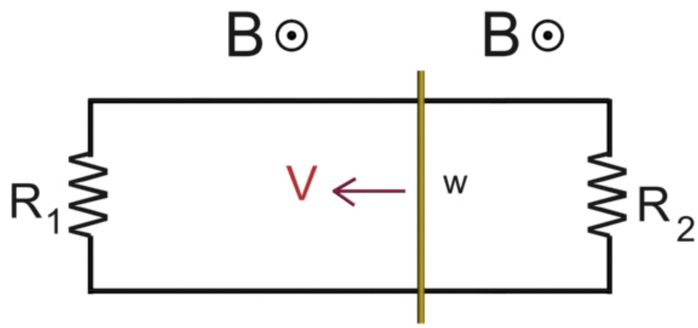
The induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{\mu_0 I l}{2\pi R}\frac{vw}{r(r+w)}$$

The flux into the page is decreasing as the loop moves away because the field is growing weaker. By Lenz's law, the induced current produces magnetic fields which tend to oppose the change in magnetic flux. Therefore, the current flows clockwise, which produces a self-flux that is positive into the page.

Problem 5:

A conducting rod with zero resistance and length *w* slides without friction on two parallel perfectly conducting wires. Resistors R_1 and R_2 are connected across the ends of the wires to form a circuit, as shown. A constant magnetic field **B** is directed out of the page. In computing magnetic flux through any surface, take the surface normal to be out of the page, parallel to **B**.



(a) The magnetic flux in the right loop of the circuit shown is (circle one)
   1) decreasing
   2) increasing.

What is the magnitude of the rate of change of the magnetic flux through the right loop?

(b) What is the current flowing through the resistor R_2 in the right hand loop of the circuit shown? Give its magnitude and indicate its direction on the figure.

(c) The magnetic flux in the left loop of the circuit shown is (circle one)
   1) decreasing
   2) increasing.

What is the magnitude of the rate of change of the magnetic flux through the right loop?

(d) What is the current flowing through the resistor R_1 in the left hand loop of the circuit shown? Gives its magnitude and indicate its direction on the figure.

(e) What is the magnitude and direction of the magnetic force exerted on this rod?

Problem 5 Solutions:

(a) The magnetic flux in the right loop of the circuit shown is (circle one)

   2) increasing.

What is the magnitude of the rate of change of the magnetic flux through the right loop?

$$\frac{d\Phi(t)}{dt} = \frac{d}{dt}BA = B\frac{d}{dt}A = BwV$$

(b) The flux out of the page is increasing so the current is clockwise to make a flux into the page. The magnitude we can get from Faraday:

$$I = \frac{|\varepsilon|}{R_2} = \frac{1}{R_2}\frac{d\Phi(t)}{dt} = \frac{BwV}{R_2}$$

(c) The magnetic flux in the left loop of the circuit shown is (circle one)

1) decreasing

What is the magnitude of the rate of change of the magnetic flux through the right loop?

$$\frac{d\Phi(t)}{dt} = \frac{d}{dt}BA = B\frac{d}{dt}A = -BwV$$

"Magnitude" is ambiguous – either a positive or negative number will do here. I use the negative sign to indicate that the flux is decreasing.

(d) The flux out of the page is decreasing so the current is counterclockwise to make a flux out of the page to make up for the loss. The magnitude we can get from Faraday:

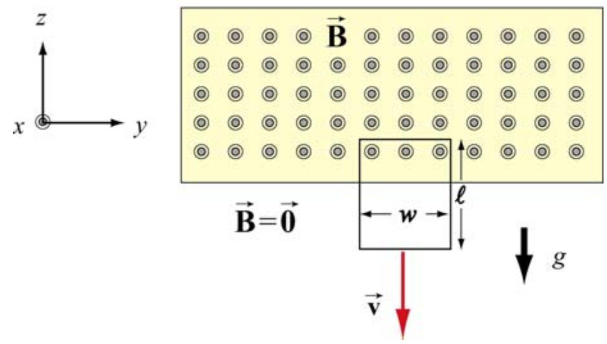
$$I = \frac{|\varepsilon|}{R_1} = \frac{1}{R_1}\frac{d\Phi(t)}{dt} = \frac{BwV}{R_1}$$

(e) The total current through the rod is the sum of the two currents (they both go up through the rod). Using the right hand rule on $\vec{\mathbf{F}} = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$ we see the force is to the **right**. You could also get this directly from Lenz. The magnitude of the force is:

$$F = \left|I\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right| = ILB = \left(BwV\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right)wB = \boxed{B^2w^2V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

Problem 6:

A rectangular loop of wire with mass *m*, width *w*, vertical length *l*, and resistance *R* falls out of a magnetic field under the influence of gravity, as shown in the figure below. The magnetic field is uniform and out of the paper ($\vec{\mathbf{B}} = B\hat{\mathbf{i}}$) within the area shown and zero outside of that area. At the time shown in the sketch, the loop is exiting the magnetic field at speed $\vec{\mathbf{v}} = -v\hat{\mathbf{k}}$.



(a) What is the direction of the current flowing in the circuit at the time shown, clockwise or counterclockwise? Why did you pick this direction?

(b) Using Faraday's law, find an expression for the magnitude of the emf in this circuit in terms of the quantities given. What is the magnitude of the current flowing in the circuit at the time shown?

(c) Besides gravity, what other force acts on the loop in the $\pm\hat{\mathbf{k}}$ direction? Give its magnitude and direction in terms of the quantities given.

(d) Assume that the loop has reached a "terminal velocity" and is no longer accelerating. What is the magnitude of that terminal velocity in terms of given quantities?

(e) Show that at terminal velocity, the rate at which gravity is doing work on the loop is equal to the rate at which energy is being dissipated in the loop through Joule heating.

Problem 6 Solutions:

(a) As the loop falls down, the magnetic flux is pointing out of the page and decreasing. Therefore an induced current flows in the counterclockwise direction. The effect of this induced current is to produce magnetic flux out of page through the surface enclosed by the loop, and thus opposing the change of the external magnetic flux.

(b) For the loop, we choose out of the page ($+\hat{\mathbf{i}}$ -direction) as the positive direction for the unit normal to the area of the loop. This means that a current flowing in the counterclockwise direction (looking at the page) has positive sign.

Choose the plane $z = 0$ at the bottom of the area where the magnetic field is non-zero. Then at time t, the top of the loop is located at $z(t)$. The area of the loop at time t is then

$$A(t) = z(t)w\,.$$

where w is the width of the loop. The magnetic flux through the loop is then given by

$$\Phi_{magnetic} = \iint \vec{\mathbf{B}} \cdot \hat{\mathbf{n}}\, da = \iint B_x\, \hat{\mathbf{i}} \cdot \hat{\mathbf{i}}\, da = \iint B_x\, da = B_x A(t) = B_x z(t) w\,.$$

The electromotive force is then

$$\varepsilon = -\frac{d}{dt}\Phi_{magnetic} = -B_x \frac{dz}{dt} w = -B_x v_z w > 0\,.$$

Note that the *z*-component of the velocity of the loop is negative, $v_z < 0$, so the electromotive force is positive.

The current that flows in the loop is therefore

$$I_{ind} = \frac{\varepsilon}{R} = -\frac{B_x v_z w}{R} > 0\,.$$

Note that a positive current corresponds to a counterclockwise flow of charge agreeing with our Lenz's Law analysis in part (a).

(c) There is an induced magnetic force acting on the upper leg of the loop given by

$$\vec{\mathbf{F}}_{ind} = I\vec{\mathbf{w}} \times \vec{\mathbf{B}} = \frac{B_x v_z w}{R} w\, \hat{\mathbf{j}} \times B_x\, \hat{\mathbf{i}} = -\frac{B_x{}^2 v_z w^2}{R}\hat{\mathbf{k}} > 0\,.$$

Note that this force is in the positive $\hat{\mathbf{k}}$ -direction since $v_z < 0$.

(d) If terminal velocity (denote the z-component by $(v_z)_{term}$) is reached, some portion of the loop must still be in the magnetic field. Otherwise there will no longer be an induced magnetic force and the loop will accelerate uniformly downward due to the gravitational force. Terminal velocity is reached when the total force on the loop is zero, therefore

$$\vec{\mathbf{F}}_{ind,term} - mg\;\hat{\mathbf{k}} = \vec{\mathbf{0}}$$

The substitute our result for $(v_z)_{term}$ in the expression for the induced force to yield

$$-\frac{B_x{}^2 (v_z)_{term} w^2}{R}\hat{\mathbf{k}} - mg\ \hat{\mathbf{k}} = 0 .$$

We now solve this equation for the z-component of the terminal velocity:

$$(v_z)_{term} = -\frac{mgR}{B_x{}^2 w^2} < 0 .$$

Substitute the above results for $(v_z)_{term}$ into our expression for the induced current to find the induced current at terminal velocity,

$$I_{ind,term} = -\frac{B_x w}{R}(v_z)_{term} = \left(-\frac{B_x w}{R}\right)\left(-\frac{mgR}{B_x{}^2 w^2}\right) = \frac{mg}{B_x w}$$

(e) When the loop is moving at terminal velocity the power exerted by gravitational force is given by

$$P_{grav,term} = \vec{F}_{grav} \cdot \vec{v}_{term} = -mg\ \hat{\mathbf{k}} \cdot \left(-\frac{mgR}{B_x{}^2 w^2}\hat{\mathbf{k}}\right) = \frac{m^2 g^2 R}{B_x{}^2 w^2} .$$

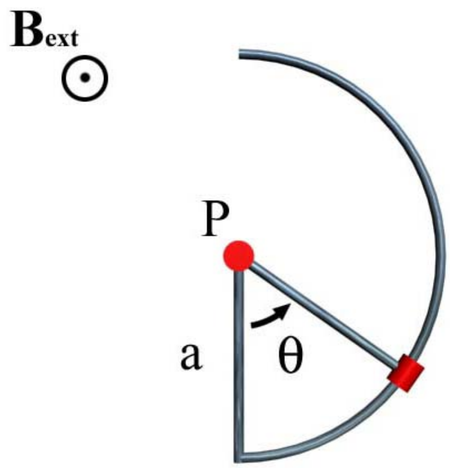
The power associated with the Joule heating at terminal velocity is given by

$$P_{joule,term} = (I^2)_{ind,term} R = \frac{m^2 g^2}{B_x{}^2 w^2} R .$$

Thus comparing these two last equations shows that

$$P_{grav,term} = P_{Joule,term} .$$

Problem 7:



A "pie-shaped" circuit is made from a straight vertical conducting rod of length *a* welded to a conducting rod bent into the shape of a semi-circle with radius *a* (see sketch). The circuit is completed by a conducting rod of length *a* pivoted at the center of the semi-circle, *Point P*, and free to rotate about that point. This moving rod makes electrical contact with the vertical rod at one end and the semi-circular rod at the other end. The angle θ is the angle between the vertical rod and the moving rod, as shown. The circuit sits in a constant magnetic field $\mathbf{B}_{ext}$ pointing out of the page.

(a) If the angle θ is increasing with time, what is the direction of the resultant current flow around the "pie-shaped" circuit? What is the direction of the current flow at the instant shown on the above diagram? To get credit for the right answer, you must justify your answer.

For the next two parts, assume that the angle θ is increasing at a constant rate, $d\theta(t)/dt = \omega$.

(b) What is the magnitude of the rate of change of the magnetic flux through the "pie-shaped" circuit due to $\mathbf{B}_{ext}$ only (do **not** include the magnetic field associated with any induced current in the circuit)?

(c) If the "pie-shaped" circuit has a constant resistance *R*, what is the magnitude and direction of the magnetic force due to the external field on the moving rod in terms of the quantities given. What is the direction of the force at the instant shown on the above diagram?

Problem 7 Solutions:

(a) The flux out of the page is increasing, so we want to generate a field into the page (Lenz' Law). This requires a clockwise current (see arrows beside pie shaped wedge).

(b)

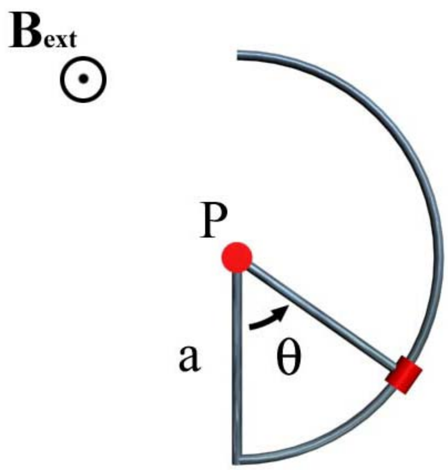
$$\frac{d}{dt}\Phi_B = \frac{d}{dt}\left(B_{\text{ext}}A\right) = B_{\text{ext}}\frac{d}{dt}\left(\pi a^2 \cdot \frac{\theta}{2\pi}\right) = \frac{B_{\text{ext}}a^2}{2}\frac{d\theta}{dt} = \frac{B_{\text{ext}}a^2}{2}\omega$$

(c) The magnetic force is determined by the current, which is determined by the EMF, which is determined by Faraday's Law:

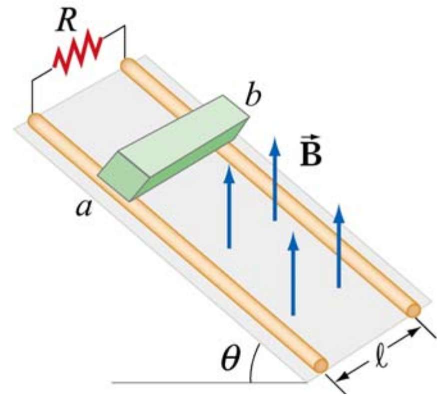
$$\varepsilon = \frac{d}{dt}\Phi_B = \frac{B_{\text{ext}}a^2}{2}\omega \quad \Rightarrow \qquad I = \frac{\varepsilon}{R} = \frac{B_{\text{ext}}a^2\omega}{2R}$$

$$\Rightarrow F = IaB_{\text{ext}} = \frac{B_{\text{ext}}^2 a^3\omega}{2R}$$

The force opposes the motion, which means it is currently down and to the left (the cross product of a radially outward current with a B field out of the page).



Problem 8:



A conducting bar of mass *m* slides down two frictionless conducting rails which make an angle θ with the horizontal, separated by a distance ℓ and connected at the top by a resistor *R*, as shown in the figure. In addition, a uniform magnetic field $\vec{\mathbf{B}}$ is applied vertically upward. The bar is released from rest and slides down. At time *t* the bar is moving along the rails at speed *v*(*t*).

(a) Find the induced current in the bar at time *t*. Which way does the current flow, from *a* to *b* or *b* to *a*?

(b) Find the terminal speed v_T of the bar.

After the terminal speed has been reached,

(c) What is the induced current in the bar?

(d) What is the rate at which electrical energy is being dissipated through the resistor?

(e) What is the rate of work done by gravity on the bar? The rate at which work is done is $\vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$. How does this compare to your answer in (d)? Why?

Problem 8 Solutions:

(a) The flux between the resistor and bar is given by

$$\Phi_B = B\ell x(t)\cos\theta$$

where *x*(*t*) is the distance of the bar from the top of the rails.

Then,

$$\varepsilon = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}B\ell x(t)\cos\theta = -B\ell v(t)\cos\theta$$

Because the resistance of the circuit is R, the magnitude of the induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{B\ell v(t)\cos\theta}{R}$$

By Lenz's law, the induced current produces magnetic fields which tend to oppose the change in magnetic flux. Therefore, the current flows clockwise, from *b* to *a* across the bar.

(b) At terminal velocity, the net force along the rail is zero, that is gravity is balanced by the magnetic force:

$$mg\sin\theta = IB\ell\cos\theta = \left(\frac{B\ell v_t(t)\cos\theta}{R}\right)B\ell\cos\theta$$

or

$$v_t(t) = \frac{Rmg\sin\theta}{(B\ell\cos\theta)^2}$$

(c)

$$I = \frac{B\ell v_t(t)\cos\theta}{R} = \frac{B\ell\cos\theta}{R}\left(\frac{Rmg\sin\theta}{(B\ell\cos\theta)^2}\right) = \frac{mg\sin\theta}{B\ell\cos\theta} = \frac{mg}{B\ell}\tan\theta$$

(d) The power dissipated in the resistor is

$$P = I^2R = \left(\frac{mg}{B\ell}\tan\theta\right)^2 R$$

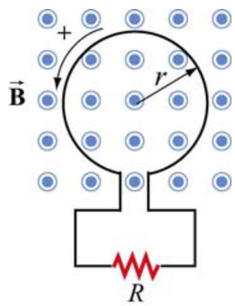
(e)

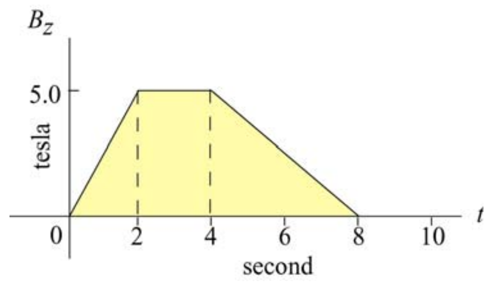
$$\vec{F}\cdot\vec{V} = (mg\sin\theta)v_t(t) = mg\sin\theta\left(\frac{Rmg\sin\theta}{(B\ell\cos\theta)^2}\right) = \left(\frac{mg}{B\ell}\tan\theta\right)^2 R = P$$

That is, they are equal. All of the work done by gravity is dissipated in the resistor, which is why the rod isn't accelerating past its terminal velocity.

Problem 9:

A uniform magnetic field $\vec{\mathbf{B}}$ is perpendicular to a one-turn circular loop of wire of negligible resistance, as shown in the figure below. The field changes with time as shown (the *z* direction is out of the page). The loop is of radius $r = 50\text{ cm}$ and is connected in series with a resistor of resistance $R = 20\ \Omega$. The "+" direction around the circuit is indicated in the figure.





(a) What is the expression for EMF in this circuit in terms of $B_z(t)$ for this arrangement?

(b) Plot the EMF in the circuit as a function of time. Label the axes quantitatively (numbers and units). Watch the signs. Note that we have labeled the positive direction of the emf in the left sketch consistent with the assumption that positive $\vec{\mathbf{B}}$ is out of the paper.

(c) Plot the current I through the resistor *R*. Label the axes quantitatively (numbers and units). Indicate with arrows on the sketch the *direction* of the current through *R* during each time interval.

(d) Plot the power dissipated in the resistor as a function of time.

Problem 9 Solutions:

(a) When we choose a "+" direction around the circuit shown in the figure above, then we are also specifying that magnetic flux out of the page is positive. (The unit vector $\hat{\mathbf{n}} = +\hat{\mathbf{k}}$ points out of the page). Thus the dot product becomes

$$\vec{\mathbf{B}} \cdot \hat{\mathbf{n}} = \vec{\mathbf{B}} \cdot \hat{\mathbf{k}} = B_z . \tag{0.1}$$

From the graph, the z-component of the magnetic field B_z is given by

$$B_z = \begin{cases} (2.5\text{ T}\cdot\text{s}^{-1})t\ ;\ 0 < t < 2\text{ s} \\ 5.0\text{ T}\ ;\ 2\text{ s} < t < 4\text{ s} \\ 10\text{ T} - (1.25\text{ T}\cdot\text{s}^{-1})t\ ;\ 4\text{ s} < t < 8\text{ s} \\ 0\ ;\ t > 8\text{ s} \end{cases} . \tag{0.2}$$

The derivative of the magnetic field is then

$$\frac{dB_z}{dt} = \begin{cases} 2.5\ \mathrm{T\cdot s^{-1}}\ ;\ 0 < t < 2\ \mathrm{s} \\ 0\ ;\ 2\ \mathrm{s} < t < 4\ \mathrm{s} \\ -1.25\ \mathrm{T\cdot s^{-1}}\ ;\ 4\ \mathrm{s} < t < 8\ \mathrm{s} \\ 0\ ;\ t > 8\ \mathrm{s} \end{cases}. \qquad (0.3)$$

The magnetic flux is therefore

$$\Phi_{magnetic} = \iint \vec{\mathbf{B}} \cdot \hat{\mathbf{n}}\, d\vec{\mathbf{A}} = \iint B_z dA = B_z \pi r^2\ .(0.4)$$

The electromotive force is

$$\mathcal{E} = -\frac{d}{dt}\Phi_{magnetic} = -\frac{dB_z}{dt}\pi r^2. \qquad (0.5)$$

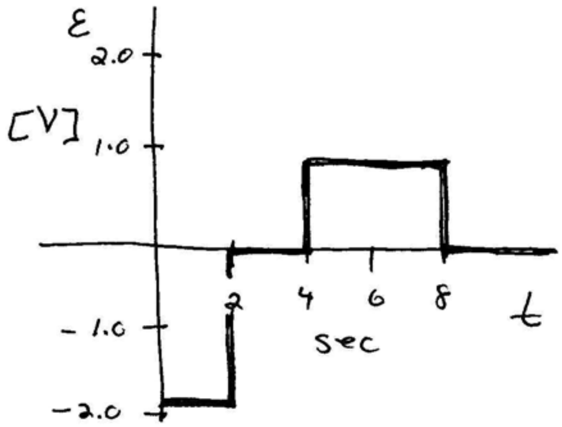
So we calculate the electromotive force by substituting Eq. (0.3) into Eq. (0.5) yielding

$$\mathcal{E} = \begin{cases} -(2.5\ \mathrm{T\cdot s^{-1}})\pi r^2\ ;\ 0 < t < 2\ \mathrm{s} \\ 0\ ;\ 2\ \mathrm{s} < t < 4\ \mathrm{s} \\ (1.25\ \mathrm{T\cdot s^{-1}})\pi r^2\ ;\ 4\ \mathrm{s} < t < 8\ \mathrm{s} \\ 0\ ;\ t > 8\ \mathrm{s} \end{cases}. \qquad (0.6)$$

Using $r = 0.5\ \mathrm{m}$, the electromotive force is then

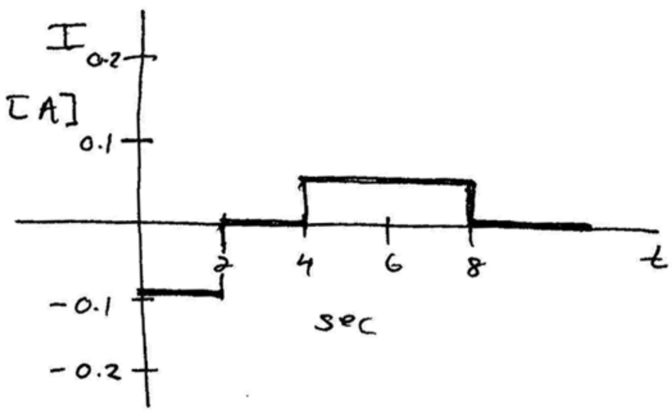
$$\mathcal{E} = \begin{cases} -1.96\ \mathrm{V}\ ;\ 0 < t < 2\ \mathrm{s} \\ 0\ ;\ 2\ \mathrm{s} < t < 4\ \mathrm{s} \\ 0.98\ \mathrm{V}\ ;\ 4\ \mathrm{s} < t < 8\ \mathrm{s} \\ 0\ ;\ t > 8\ \mathrm{s} \end{cases} \qquad (0.7)$$

(b)



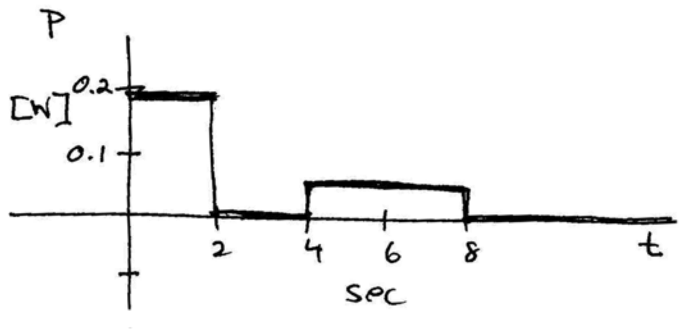
(c) The current through the resistor ($R = 20\ \Omega$) is given by

$$I = \frac{\mathcal{E}}{R} = \begin{cases} -9.8\times10^{-2}\ \text{A}\ ;\ 0 < t < 2\ \text{s} \\ 0\ ;\ 2\ \text{s} < t < 4\ \text{s} \\ 4.9\times10^{-2}\ \text{A}\ ;\ 4\ \text{s} < t < 8\ \text{s} \\ 0\ ;\ t > 8\ \text{s} \end{cases}. \qquad (0.8)$$



(d) The power dissipated in the resistor is given by

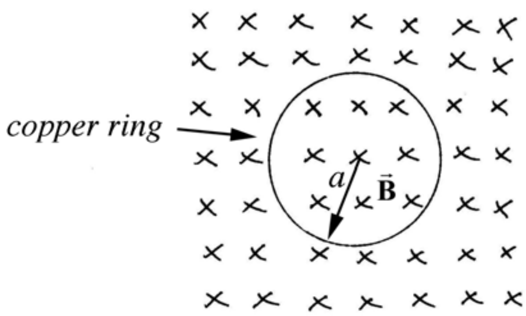
$$P = I^2 R = \begin{cases} 1.9\times10^{-1}\ \text{W}\ ;\ 0 < t < 2\ \text{s} \\ 0\ ;\ 2\ \text{s} < t < 4\ \text{s} \\ 4.8\times10^{-2}\ \text{W}\ ;\ 4\ \text{s} < t < 8\ \text{s} \\ 0\ ;\ t > 8\ \text{s} \end{cases}. \qquad (0.9)$$

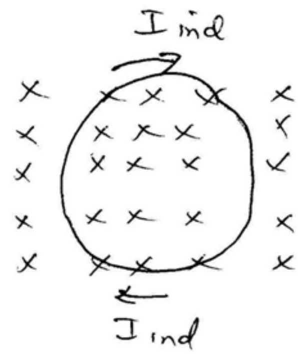


P
[W]
0.2
0.1
2
4
6
8
t
sec

Problem 10:

Consider a copper ring of radius a and resistance R. The loop is in a constant magnetic field $\vec{\mathbf{B}}$ of magnitude B_0 perpendicular to the plane of the ring (pointing into the page, as shown in the diagram).





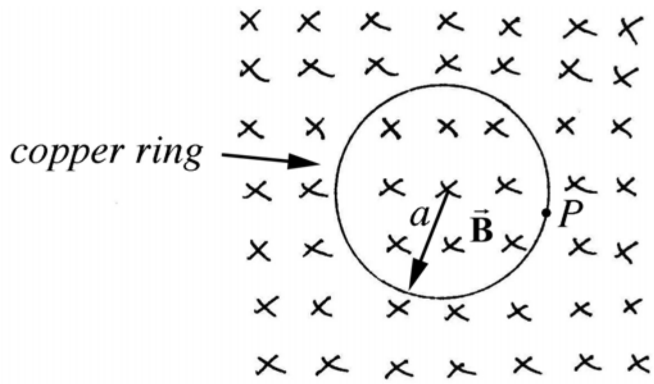
(a) What is the magnetic flux Φ through the ring? Express your answer in terms of B_0, a, R, and μ_0 as needed.

Now, the magnitude of the magnetic field is decreased during a time interval from $t = 0$ to $t = T$ according to

$$B(t) = B_0\left(1 - \frac{t}{T}\right), \text{ for } 0 < t \le T$$

(b) What are the magnitude and direction (draw the direction on the figure above) of the current I in the ring? Express your answer in terms of B_0, T, a, R, t, and μ_0 as needed.

(c) What is the total charge Q that has moved past a fixed point P in the ring during the time interval that the magnetic field is changing? Express your answer in terms of B_0, T, a, R, t, and μ_0 as needed.



Problem 10 Solutions:

(a) $\Phi = B_0 \pi a^2$

(b) The external flux is into the page and decreasing so the induced current is in the clockwise direction producing flux into the page through the ring opposing the change. The magnitude of the induced current is non-zero during the interval $0 < t \le T$ and is equal to

$$I = \frac{1}{R}\left|\frac{d\Phi}{dt}\right| = \frac{1}{R}\left|\frac{d}{dt}\left(B_0\left(1-\frac{t}{T}\right)\pi a^2\right)\right| = \frac{1}{R}\left|\frac{d}{dt}\left(B_0\left(1-\frac{t}{T}\right)\pi a^2\right)\right| = \frac{B_0 \pi a^2}{TR}, \textbf{ for } 0 < t \le T$$

(c) The total charge moving past a fixed point P in the ring is the integral

$$Q = \int_0^T I dt = \int_0^T \frac{B_0 \pi a^2}{TR} dt = \frac{B_0 \pi a^2}{R}.$$