Module 15: DC Circuits with Capacitors

Modules 15: Outline

Capacitors in Series and Parallel

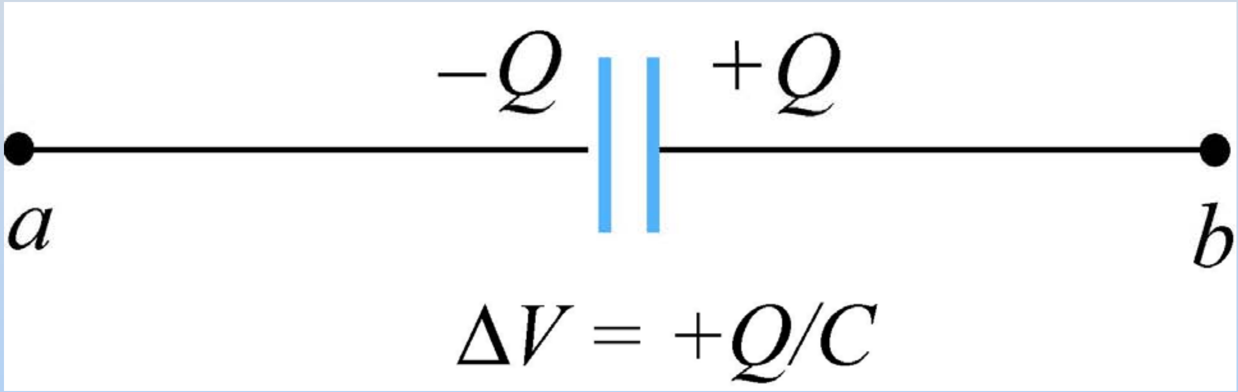
RC Circuits

Expt 4: RC Circuits

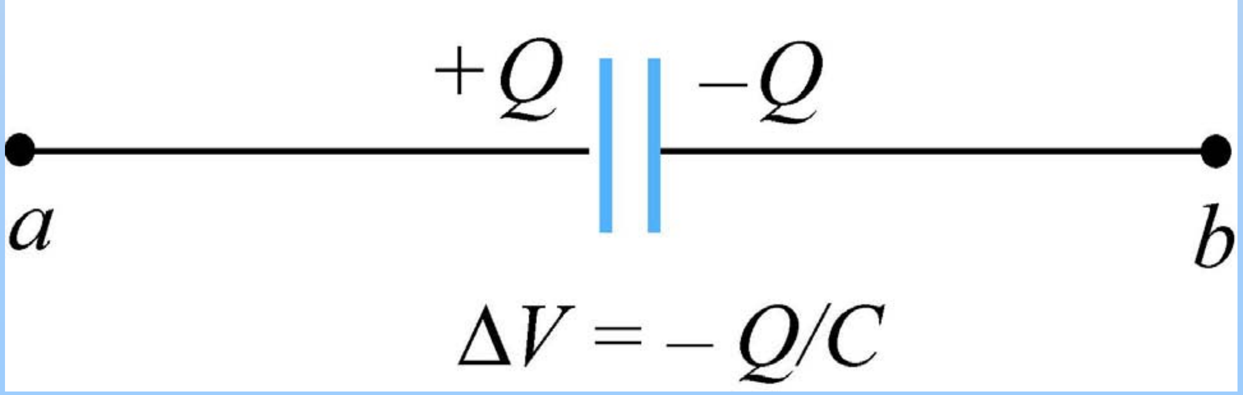
DC Circuits with Capacitors

Sign Conventions - Capacitor

Moving across a capacitor from the negatively to positively charged plate **increases** your potential



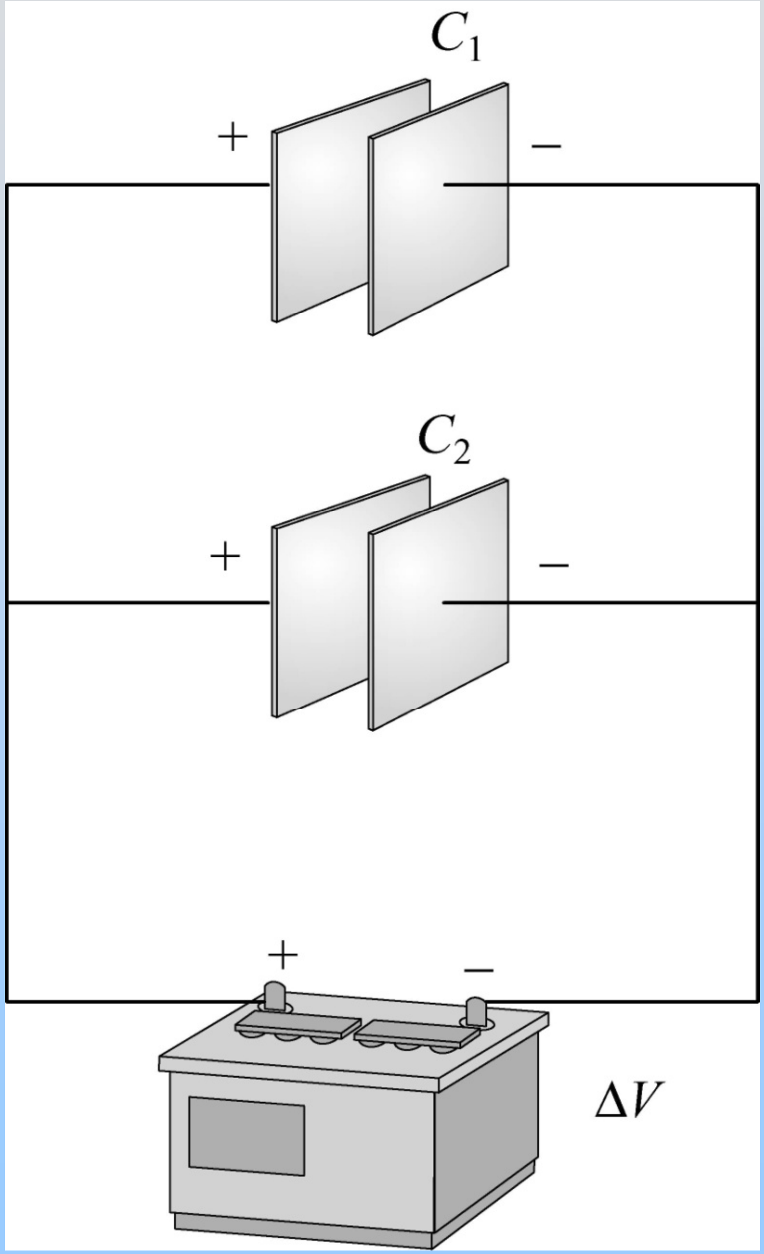
$$\Delta V = V_b - V_a$$



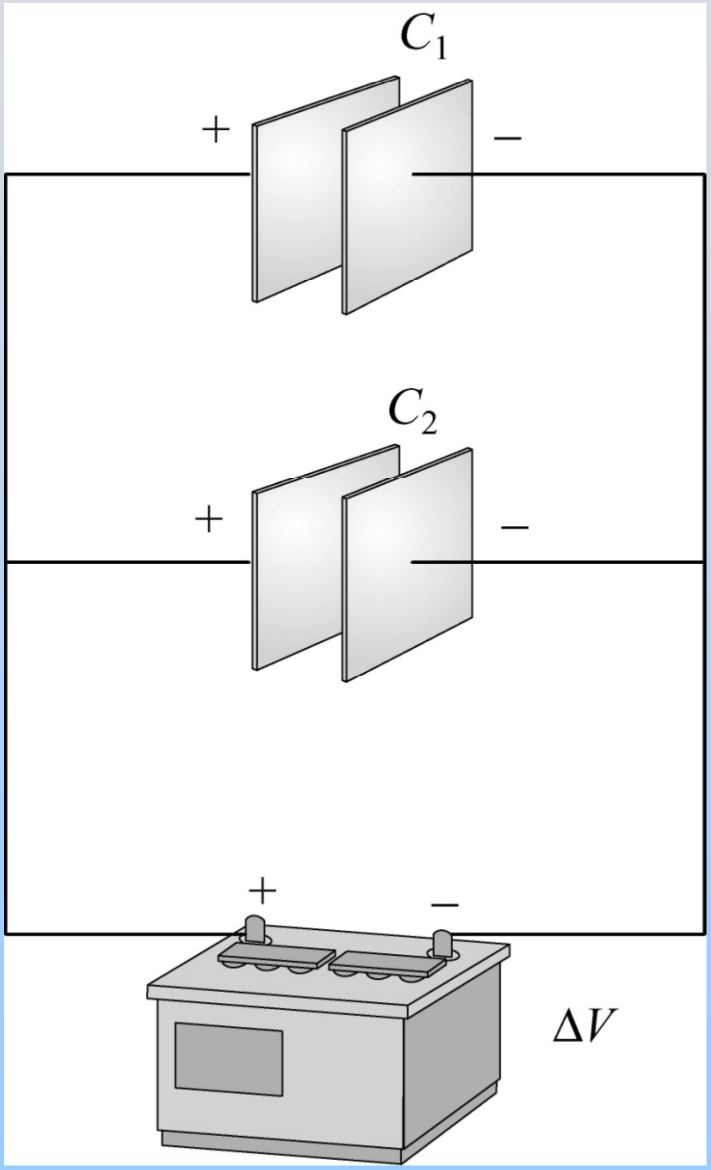
Think:

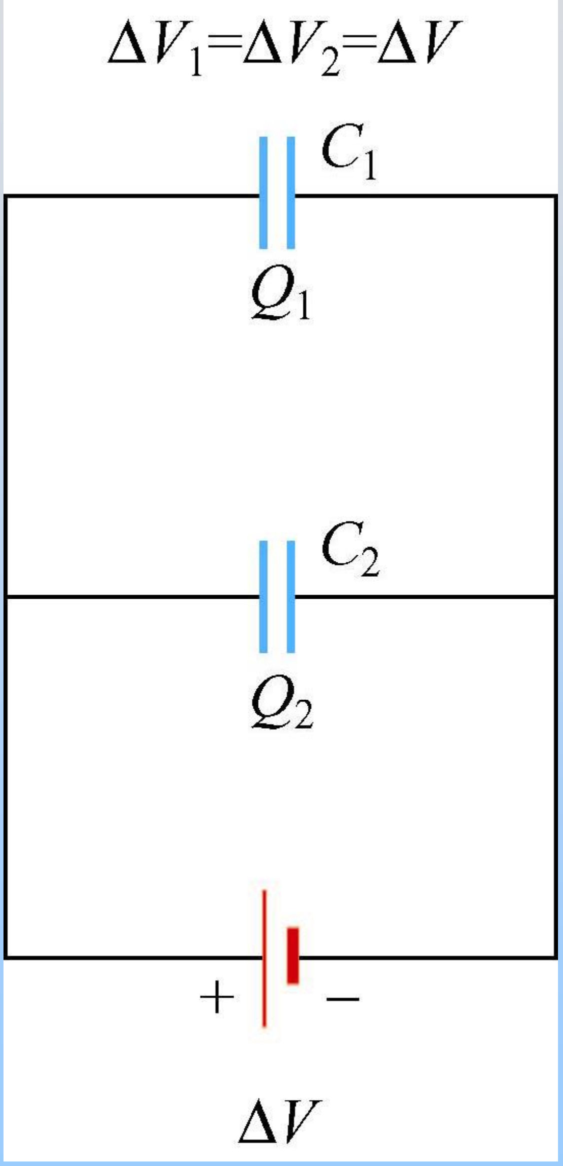
Ski Lodge

Capacitors in Parallel



Capacitors in Parallel

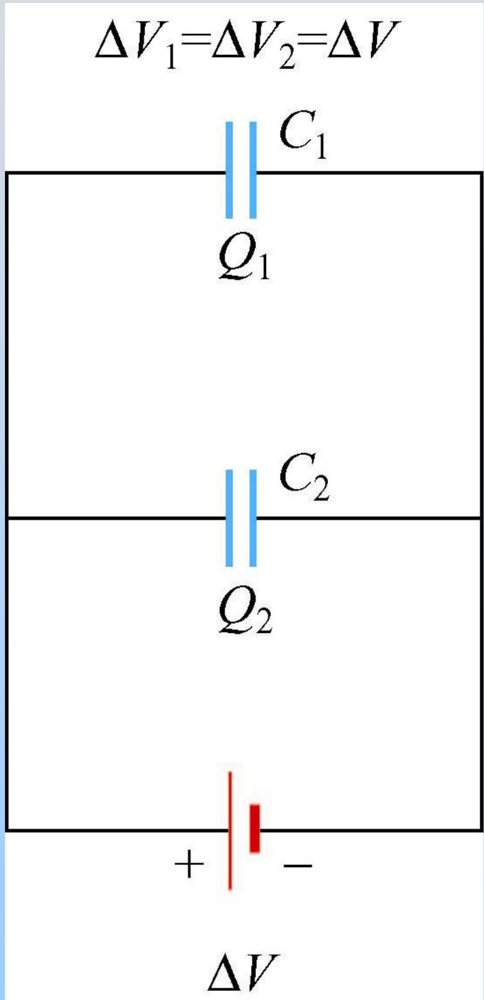




Same potential!

$$C_1 = \frac{Q_1}{\Delta V},\ C_2 = \frac{Q_2}{\Delta V}$$

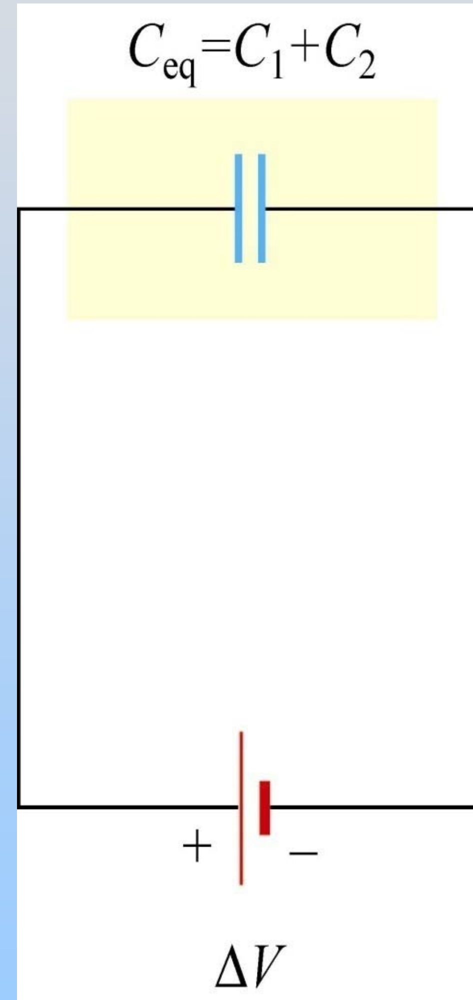
Equivalent Capacitance



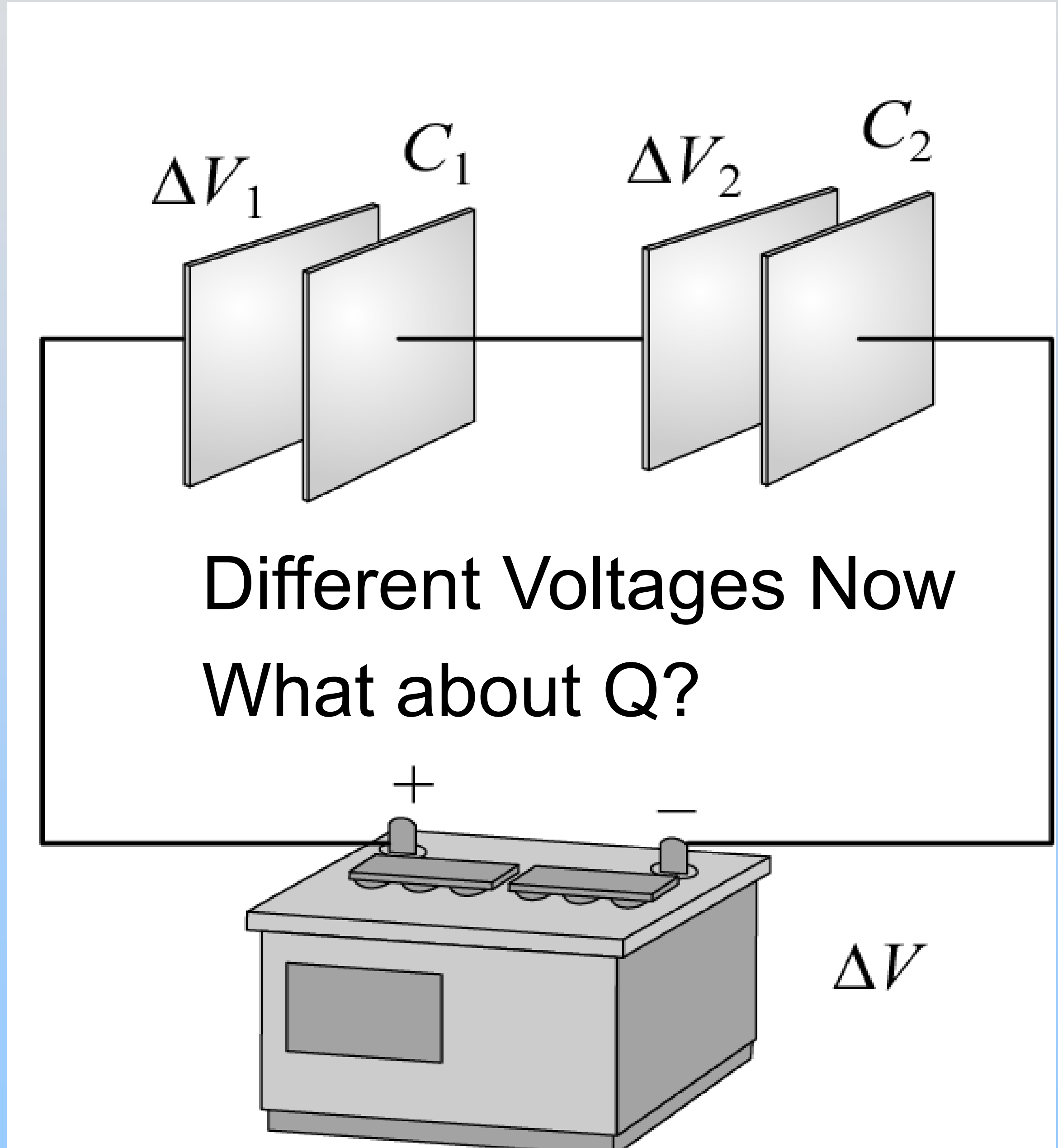
?

$$Q = Q_1 + Q_2 = C_1 \Delta V + C_2 \Delta V$$
$$= \left(C_1 + C_2\right) \Delta V$$

$$C_{eq} = \frac{Q}{\Delta V} = C_1 + C_2$$



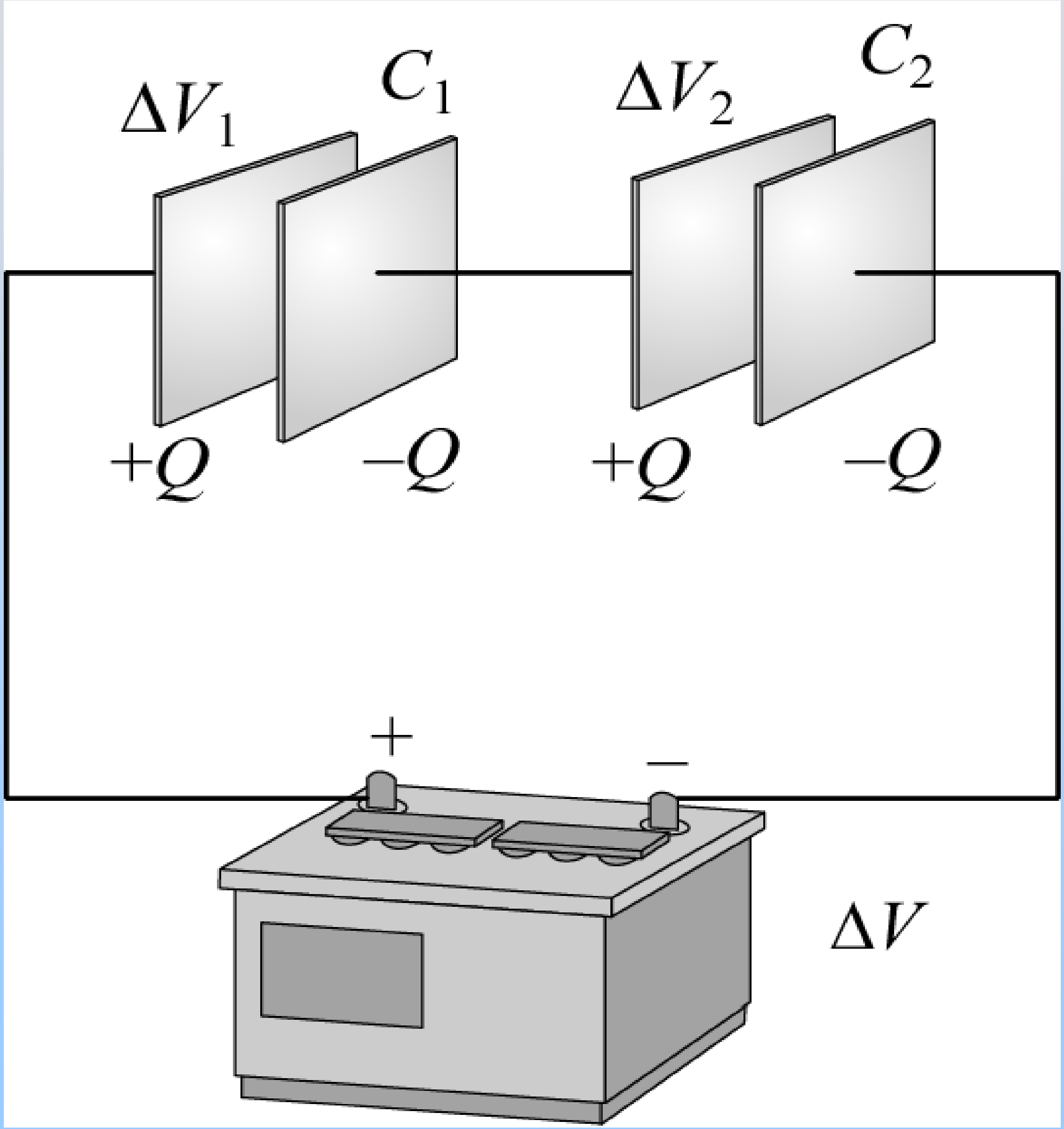
Capacitors in Series



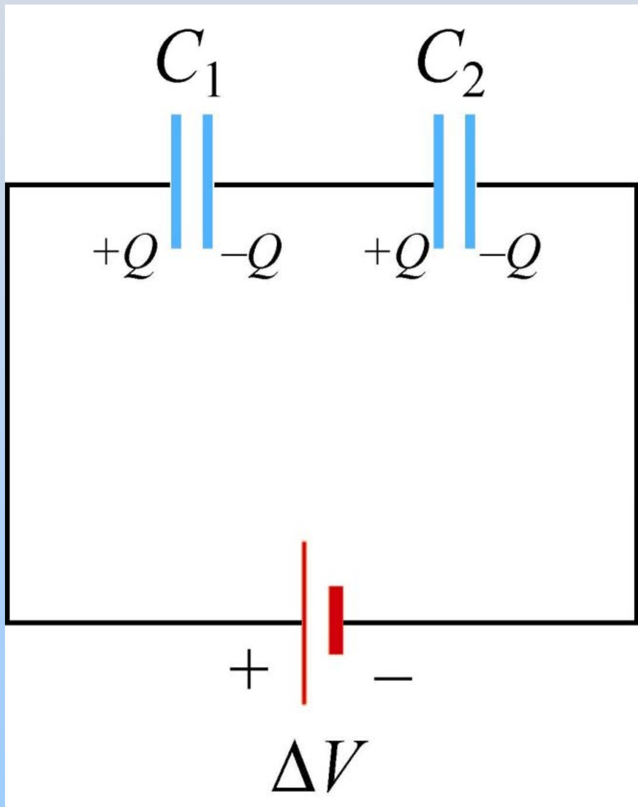
Different Voltages Now

What about Q?

Capacitors in Series



Equivalent Capacitance



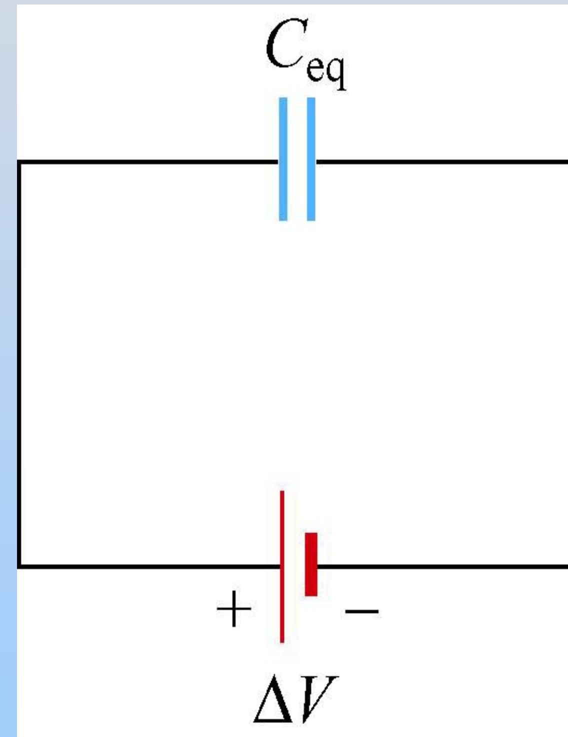
$$\Delta V_1 = \frac{Q}{C_1},\ \ \Delta V_2 = \frac{Q}{C_2}$$

$$\Delta V = \Delta V_1 + \Delta V_2$$

(voltage adds in series)

$$\Delta V = \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

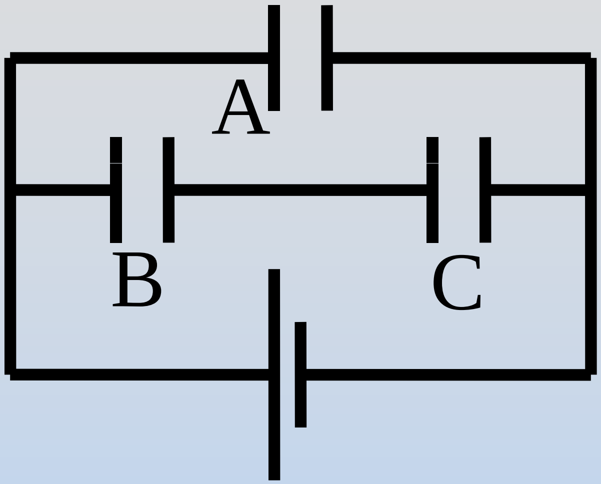
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$



Concept Question Question: Capacitors in Series and Parallel

Concept Question: Capacitors

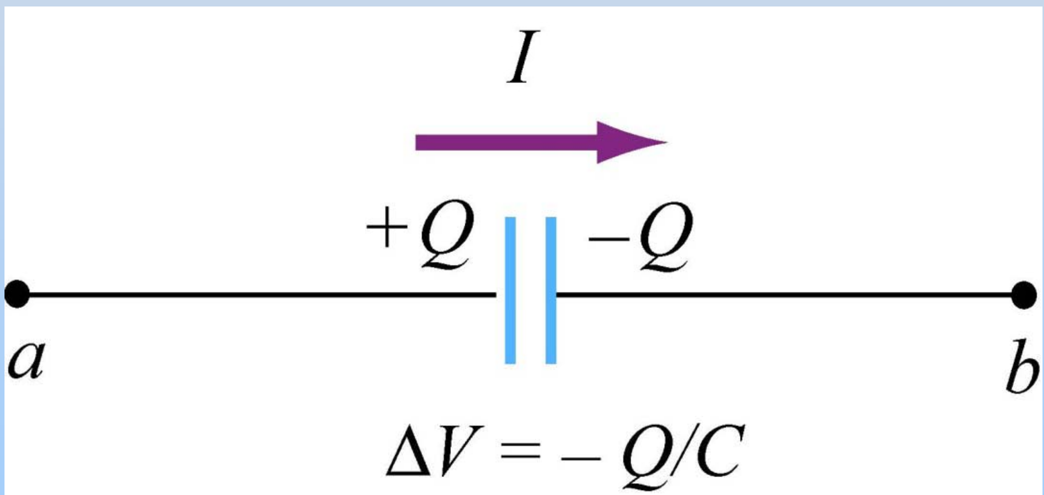
Three identical capacitors are connected to a battery. The battery is then disconnected. How do the charge on A, B & C compare before and after the battery is removed?



BEFORE;	AFTER
1. $Q_A = Q_B = Q_C$;	No Change
2. $Q_A = Q_B = Q_C$;	$Q_A > Q_B = Q_C$
3. $Q_A = Q_B = Q_C$;	$Q_A < Q_B = Q_C$
4. $Q_A > Q_B = Q_C$;	No Change
5. $Q_A > Q_B = Q_C$;	$Q_A = Q_B = Q_C$
6. $Q_A < Q_B = Q_C$;	No Change
7. $Q_A < Q_B = Q_C$;	$Q_A = Q_B = Q_C$

Power - Capacitor

Moving across a capacitor from the positive to negative plate **decreases** your potential. If current flows in that direction the capacitor **absorbs** power (stores charge)



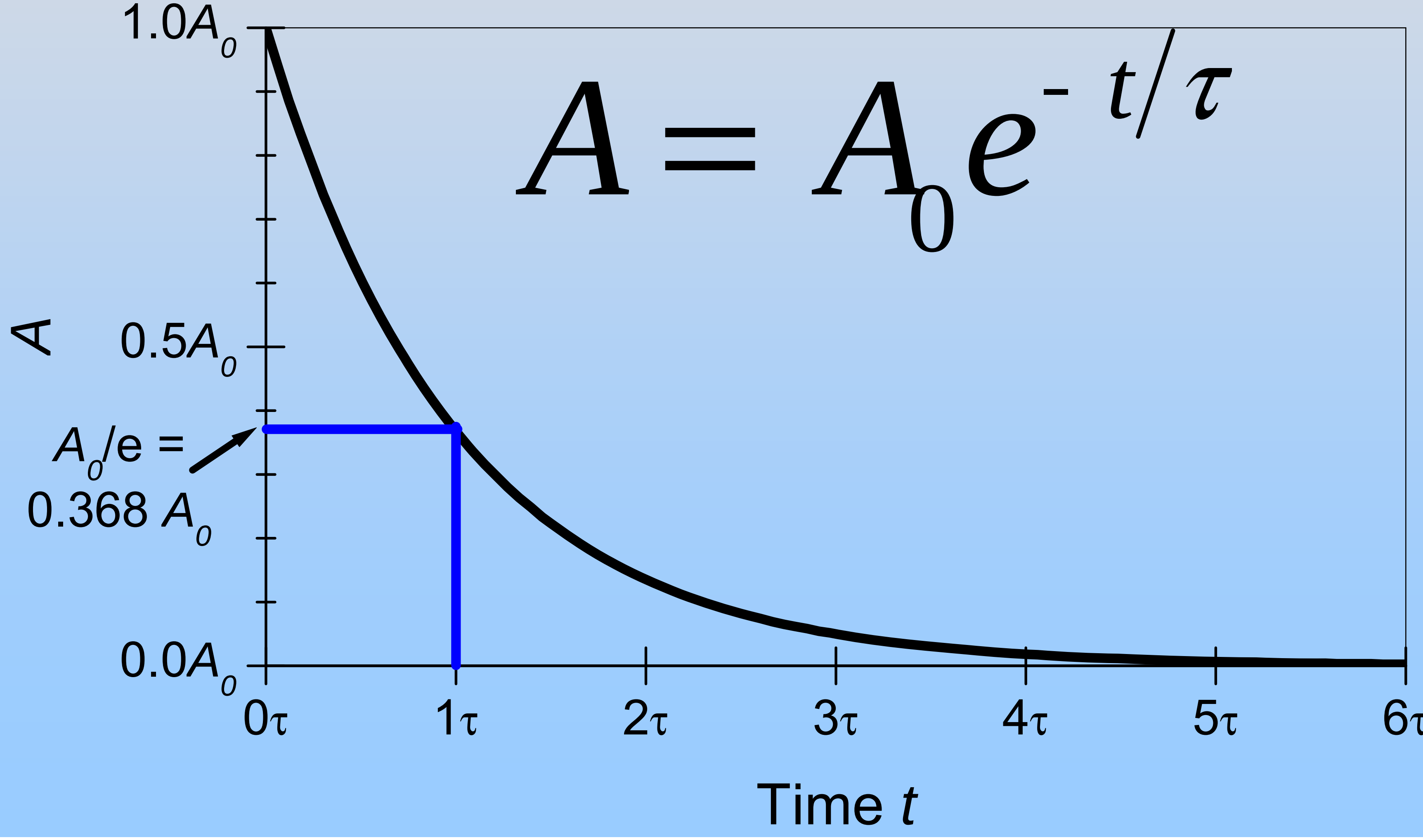
$$P_{\text{absorbed}} = I\,\Delta V = \frac{dQ}{dt}\frac{Q}{C} = \frac{d}{dt}\frac{Q^2}{2C} = \frac{dU}{dt}$$

Exponential Decay

Consider function *A* where: $$\frac{dA}{dt} = -\frac{1}{\tau}A$$

A decays exponentially:

$$A = A_0 e^{-t/\tau}$$

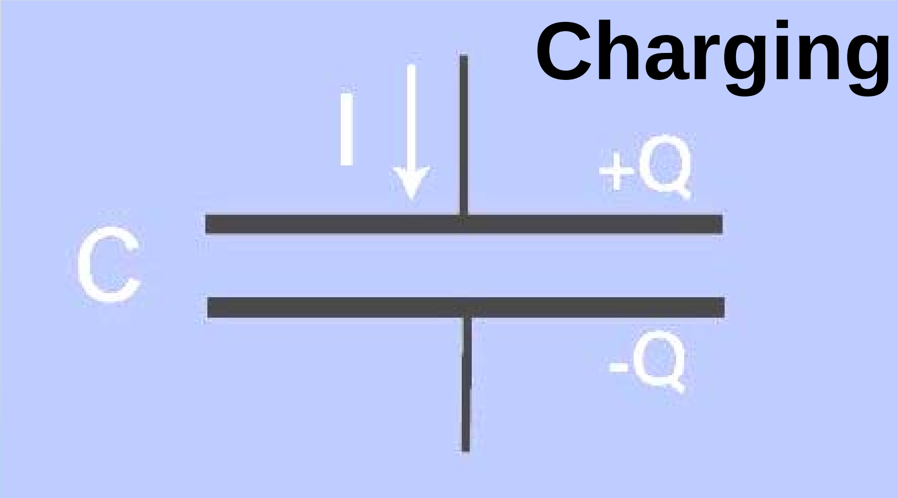


RC Circuits

(Dis)Charging a Capacitor

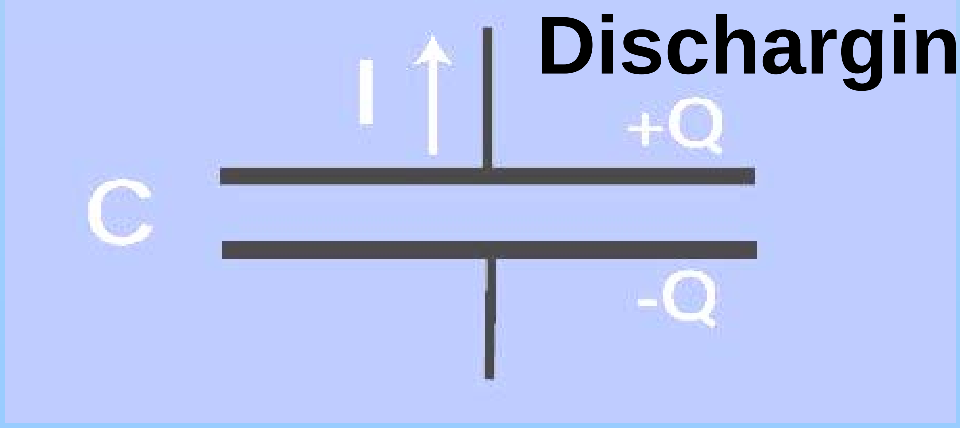
1. When the direction of current flow is toward the positive plate of a capacitor, then

$$I = +\frac{dQ}{dt}$$

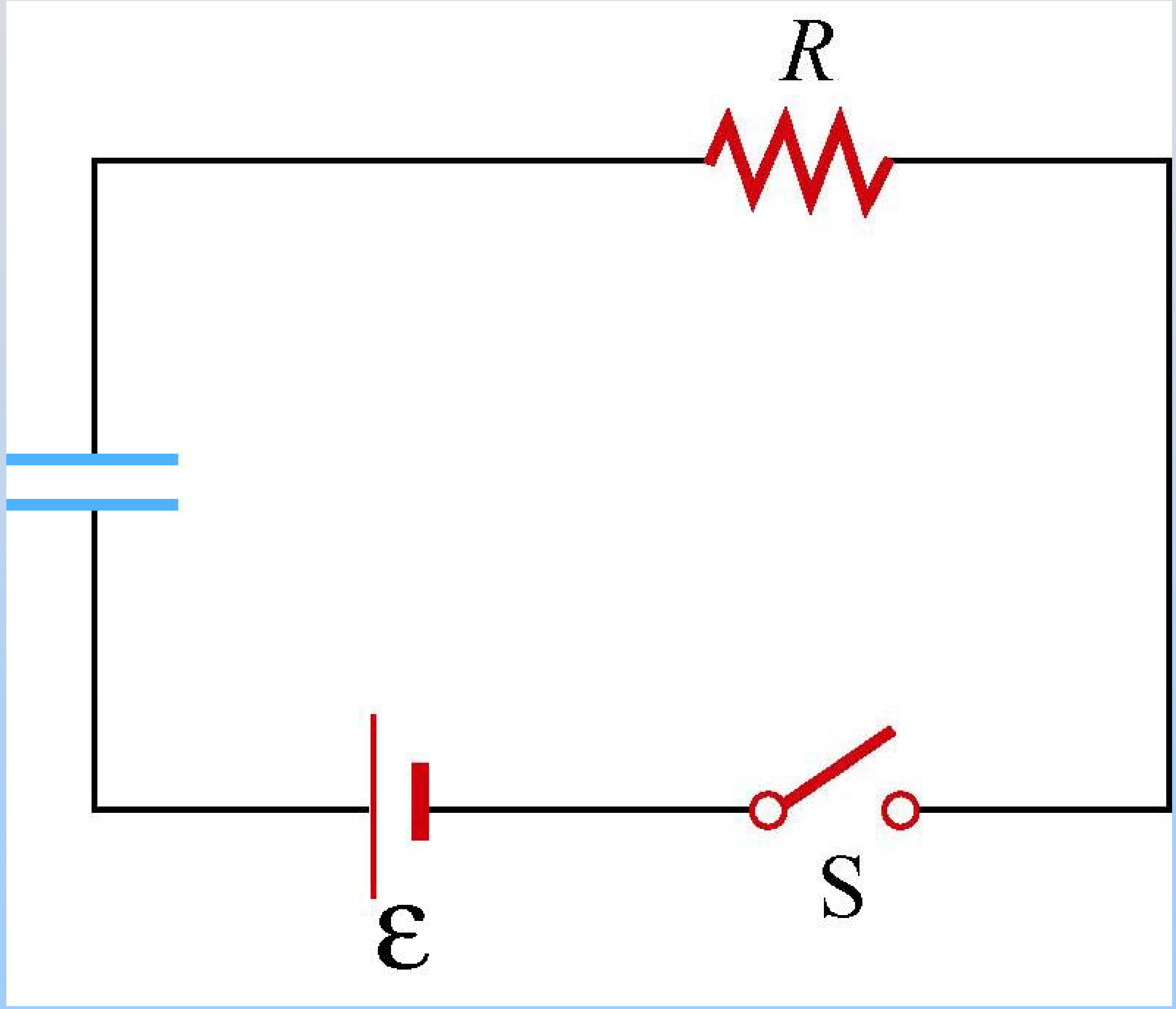


2. When the direction of current flow is away from the positive plate of a capacitor, then

$$I = -\frac{dQ}{dt}$$

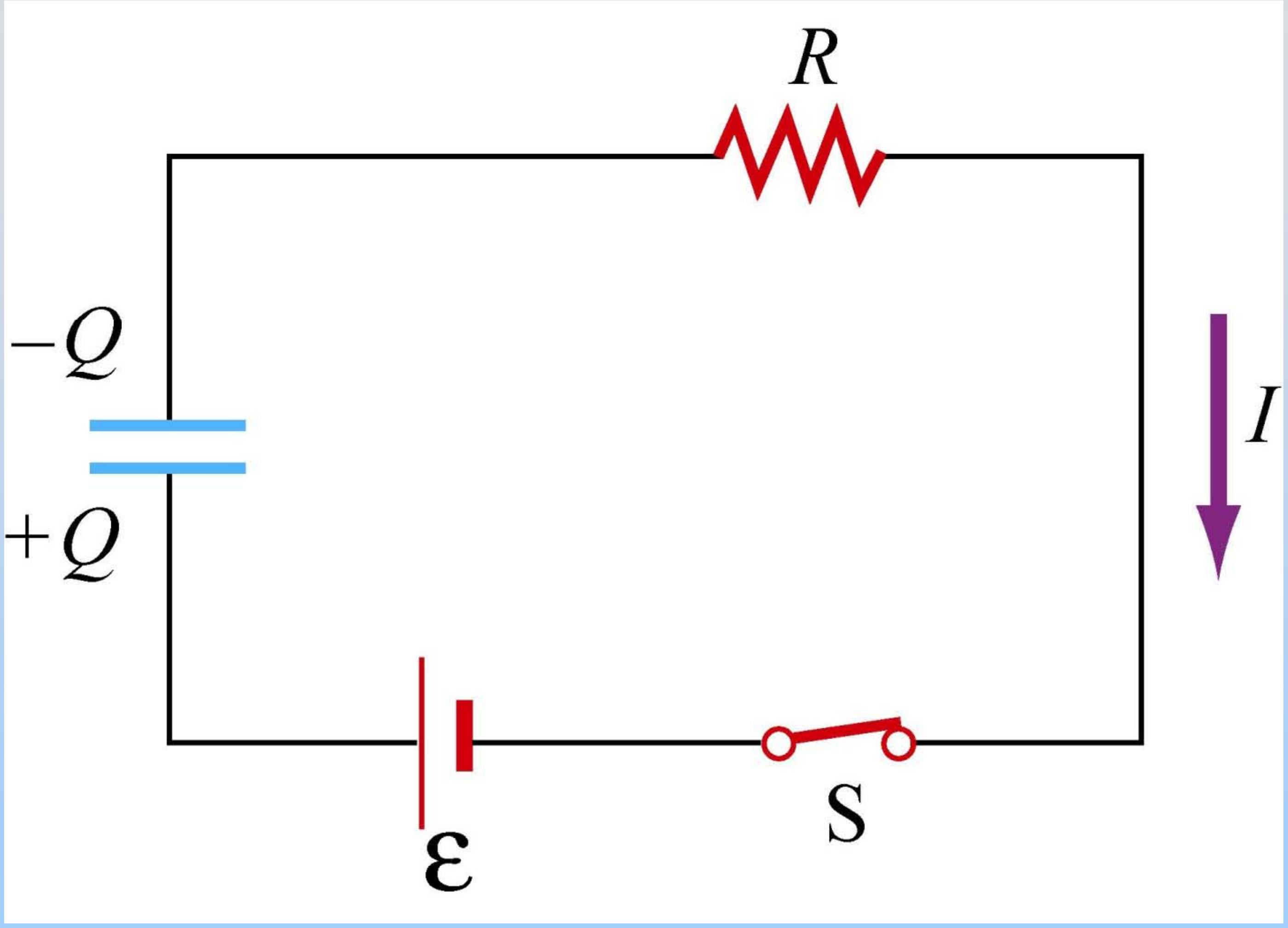


Charging A Capacitor



What happens when we close switch S?

Charging A Capacitor

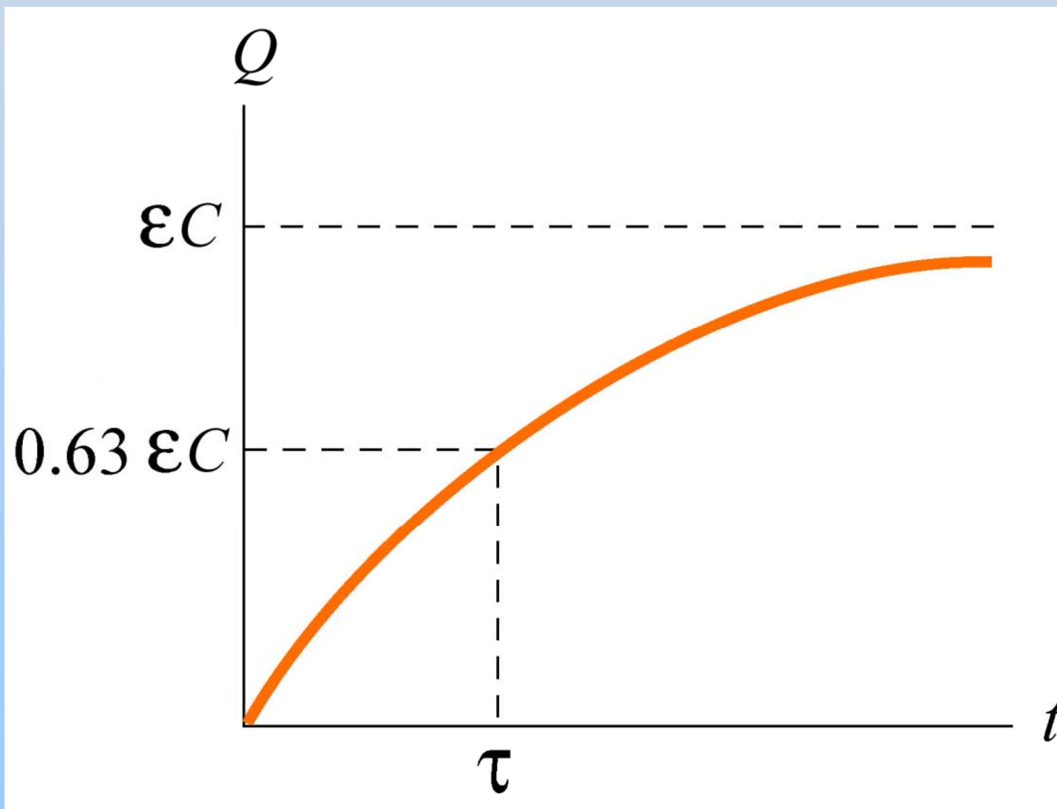


$$\sum_i \Delta V_i = \varepsilon - \frac{Q}{C} - \frac{dQ}{dt} R = 0$$

RC Circuit

$$\frac{dQ}{dt} = -\frac{1}{RC}\left(Q - C\varepsilon\right)$$

Solution to this equation when switch is closed at t = 0:



$$Q(t) - C\mathcal{E}\left(1 - e^{-t/\tau}\right)$$

$$\tau = RC : \text{time constant}$$

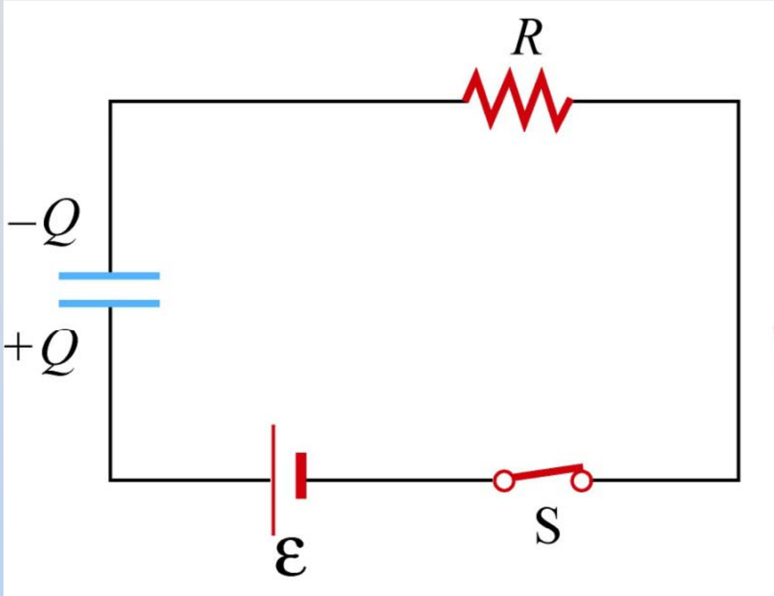
(units: seconds)

Solve Diferential Equation for Charging RC Circuits

Concept Question Question: Current in RC Circuit

Concept Question: RC Circuit

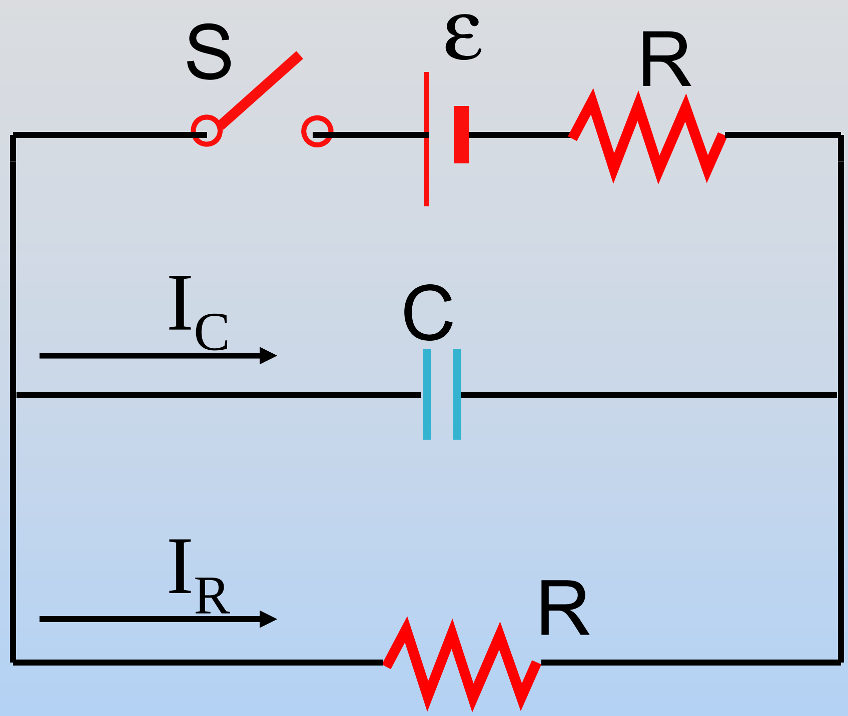
An uncharged capacitor is connected to a battery, resistor and switch. The switch is initially open but at t = 0 it is closed. A very long time after the switch is closed, the current in the circuit is



1. Nearly zero
2. At a maximum and decreasing
3. Nearly constant but non-zero
4. I don’t know

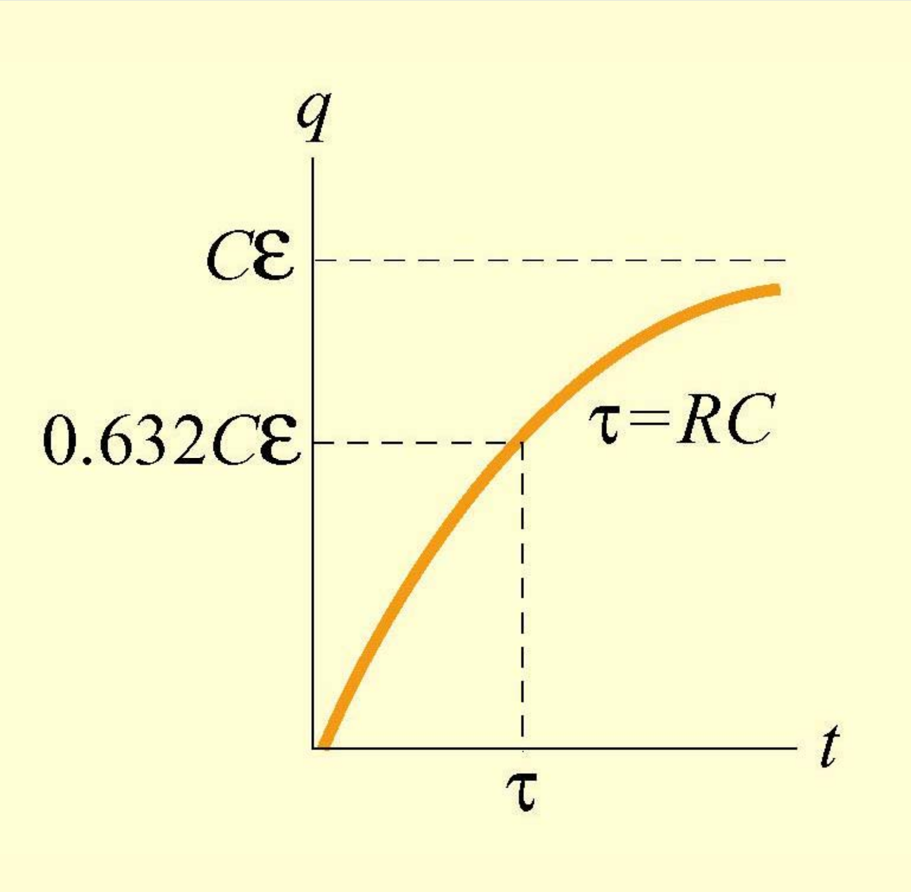
Concept Question: RC Circuit

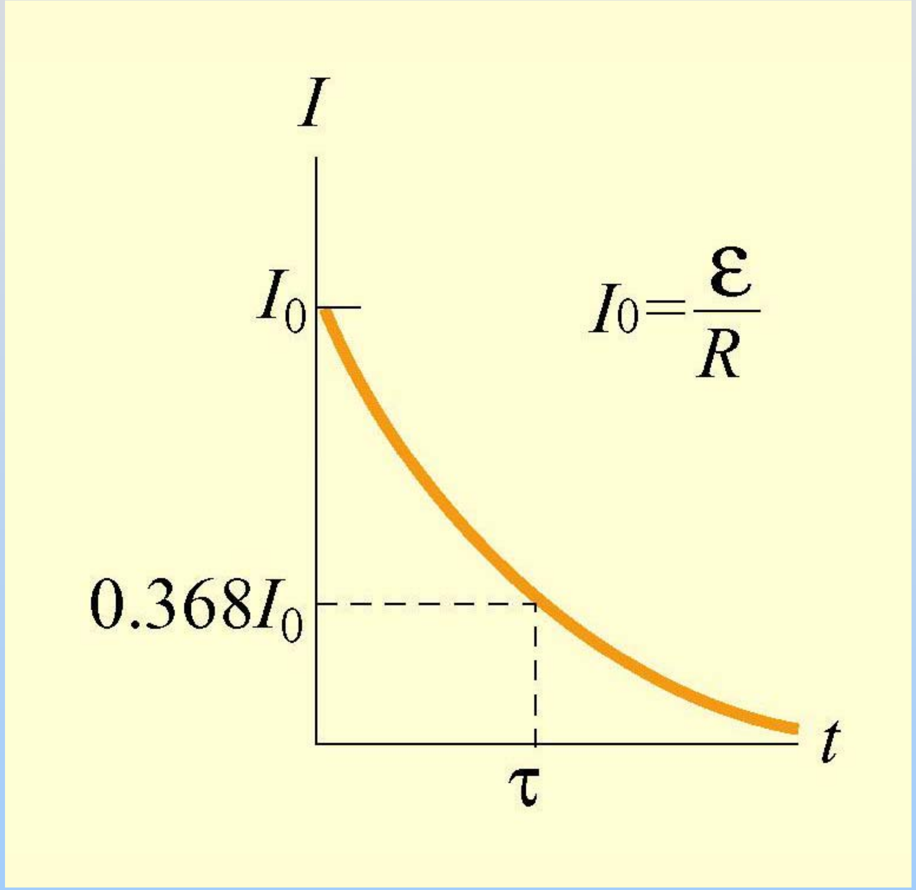
Consider the circuit at right, with an initially uncharged capacitor and two identical resistors. At the instant the switch is closed:



1. $I_R = I_C = 0$
2. $I_R = \varepsilon/2R$; $I_C = 0$
3. $I_R = 0$; $I_C = \varepsilon/R$
4. $I_R - \varepsilon/2R$; $I_C - \varepsilon/R$
5. I don’t know

Charging A Capacitor

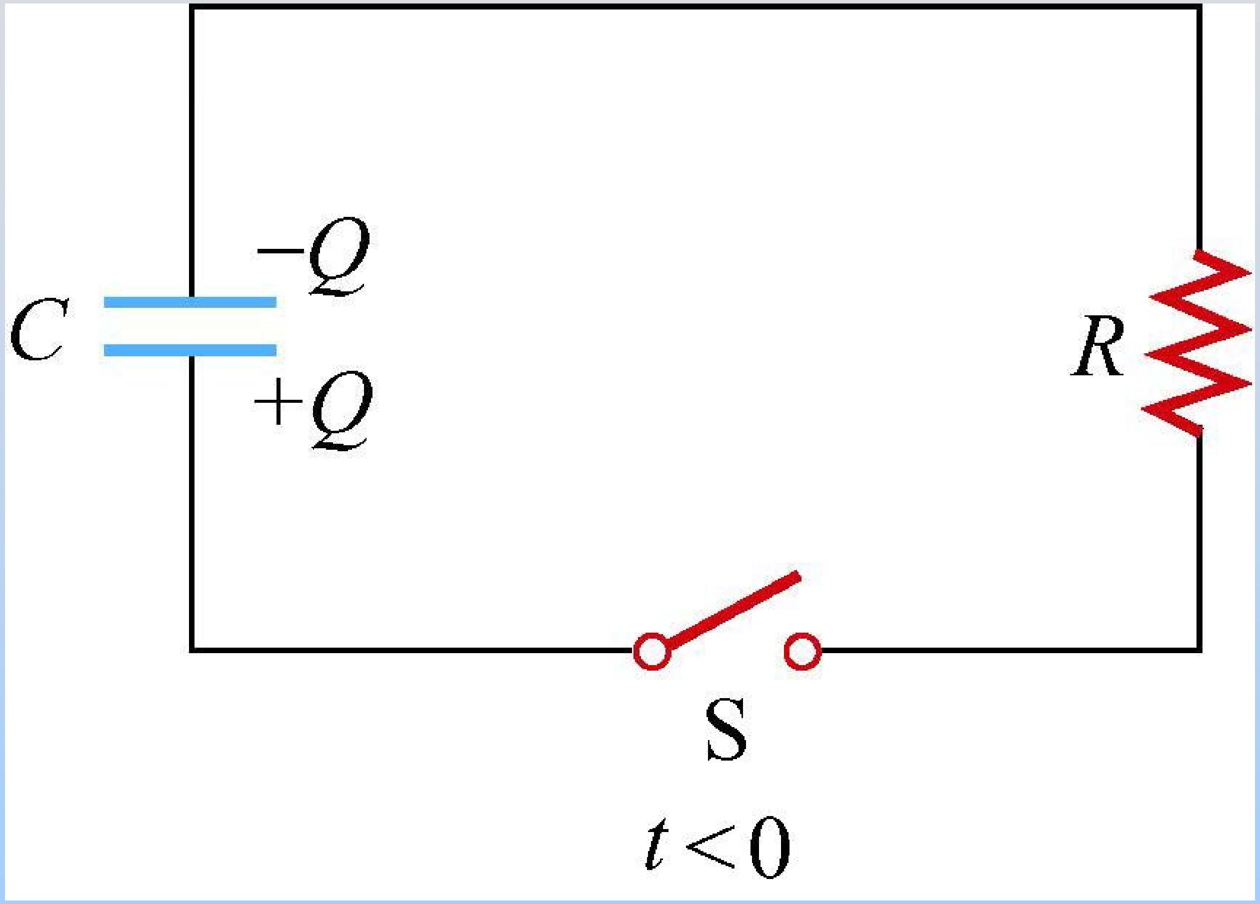




$$Q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

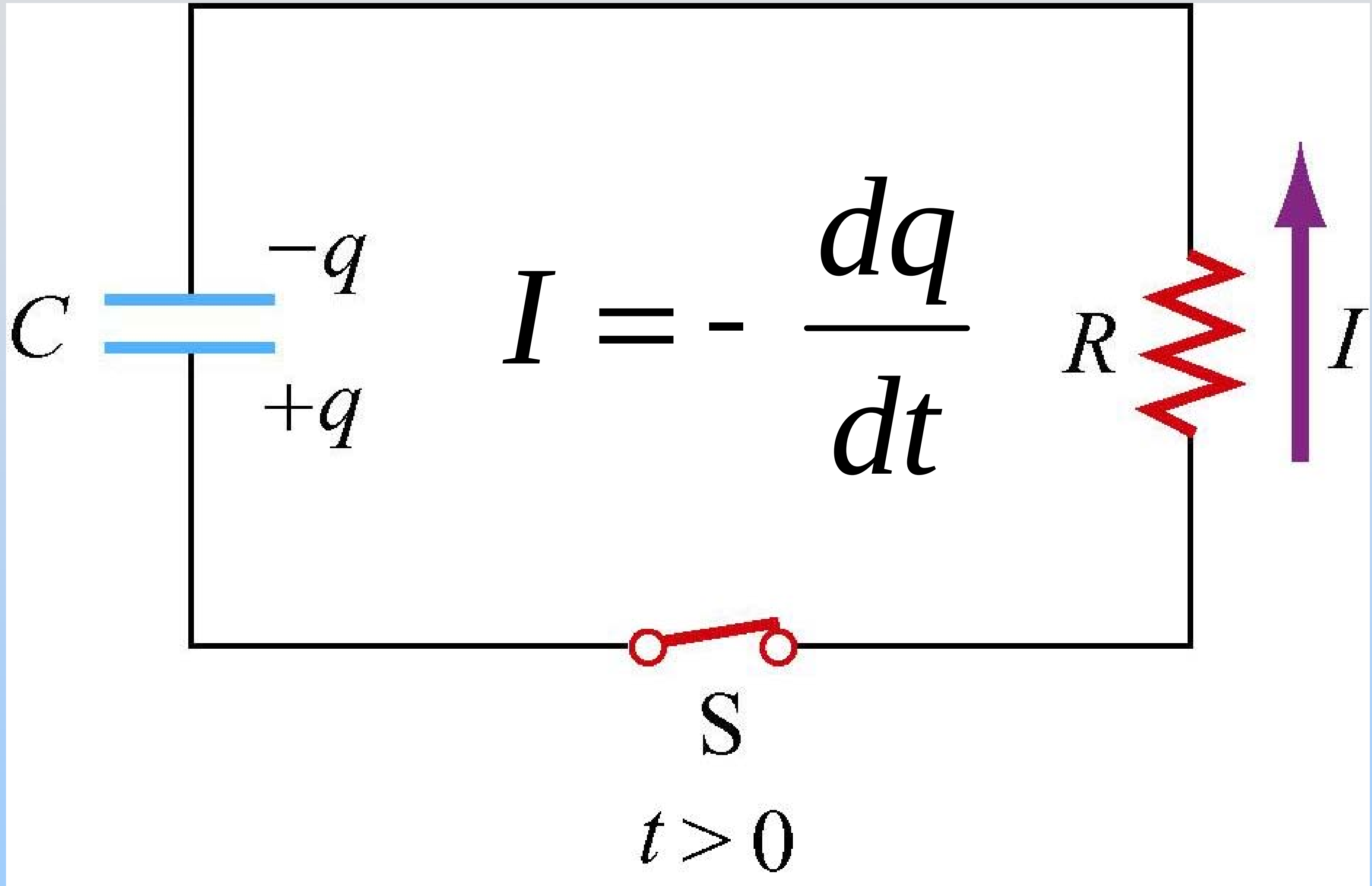
$$I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC}$$

Discharging A Capacitor



What happens when we close switch S?

Discharging A Capacitor

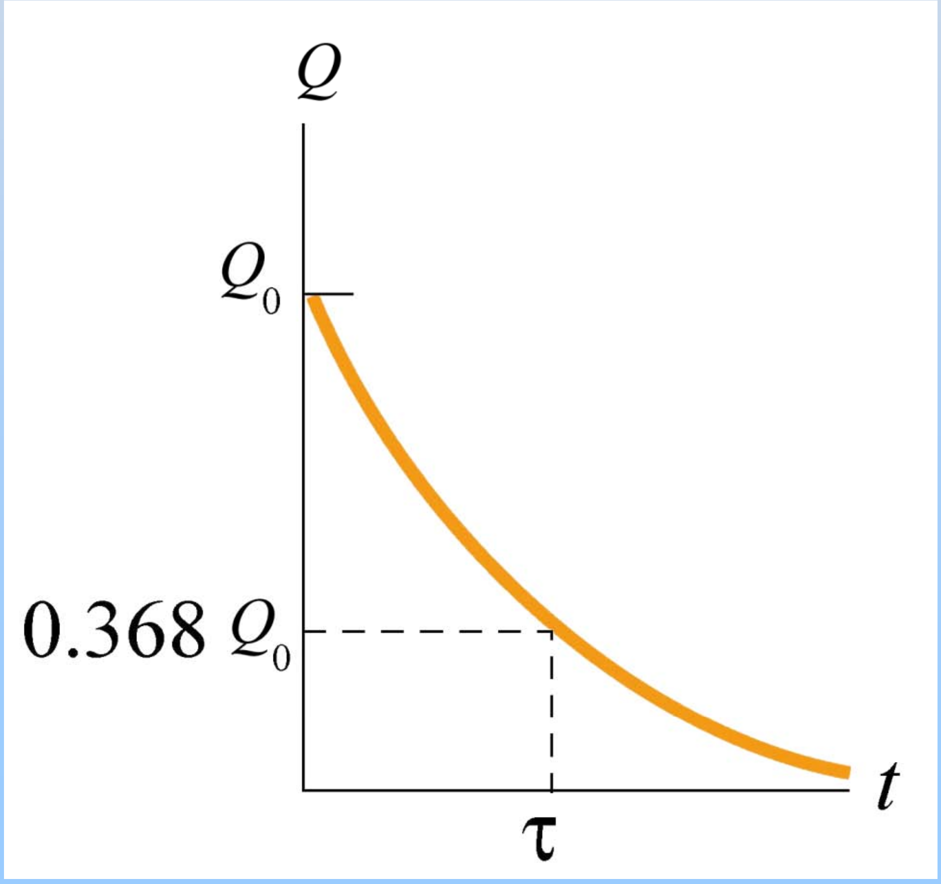


$$\sum_i \Delta V_i = \frac{q}{C} - IR = 0 \qquad \sum_i \Delta V_i = \frac{q}{C} + \frac{dq}{dt} R = 0$$

RC Circuit: Discharging

$$\frac{dQ}{dt} = -\frac{1}{RC}Q$$

Solution to this equation when switch is closed at t = 0:

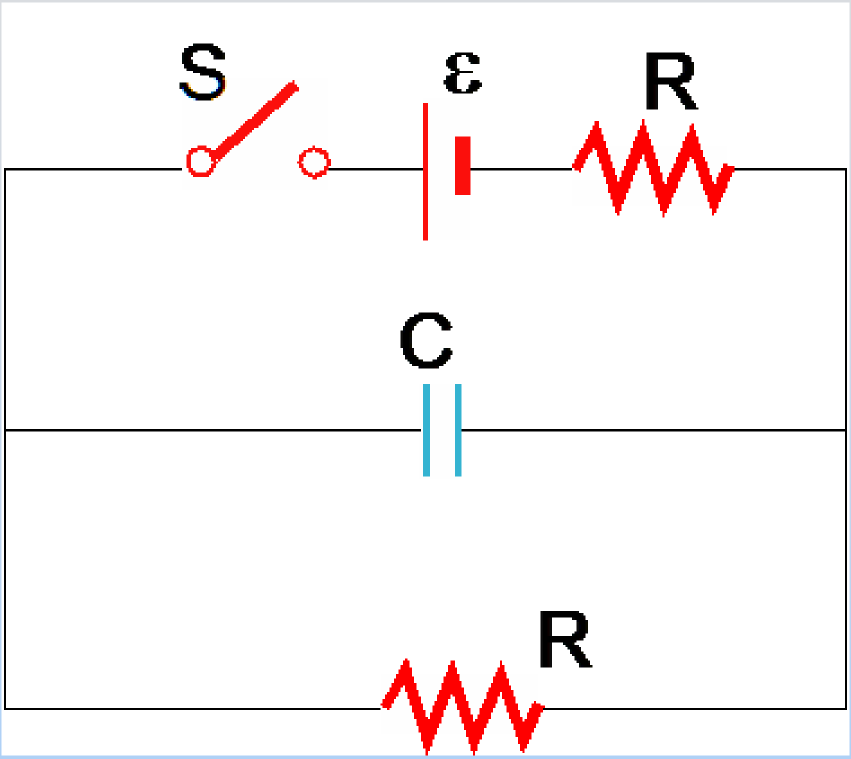


$$Q(t) = Q_o e^{-t/\tau}$$

$\tau = RC$: time constant

Demonstrations:
RC Time Constants

Problem: Circuits

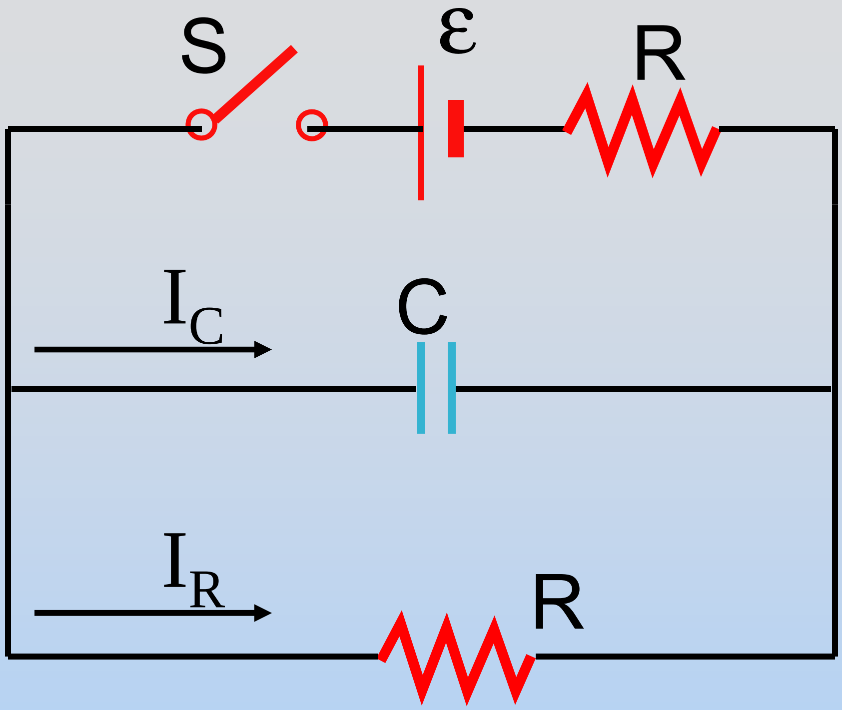


For the above circuit sketch the currents through the two bottom branches as a function of time (switch closes at $t = 0$, opens at $t = T$). State values at $t = 0^+$, T^-, T^+

Concept Question Questions: RC Circuit

Concept Question: RC Circuit

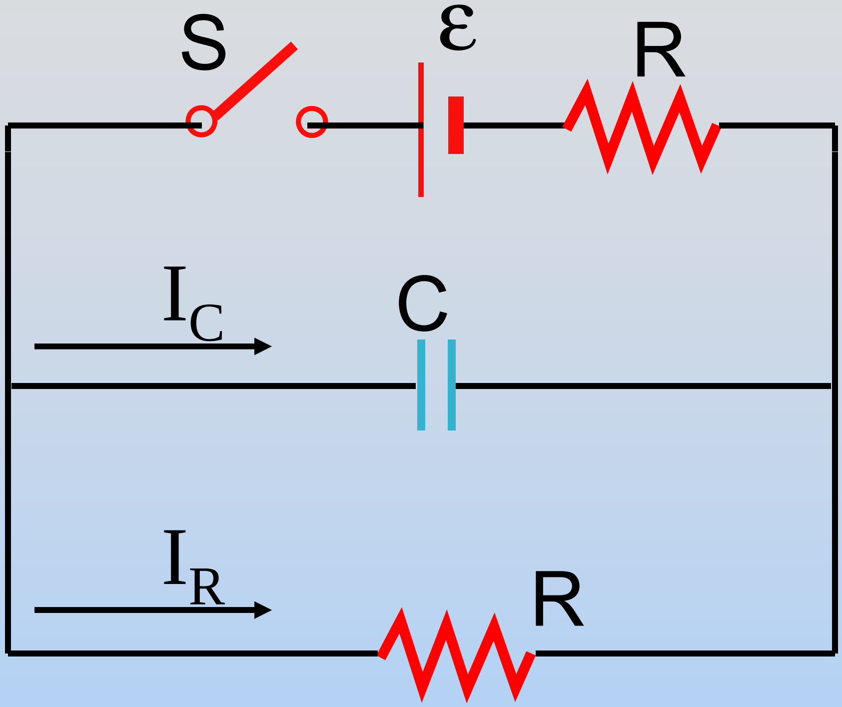
Now, after the switch has been closed for a very long time, it is opened. What happens to the current through the lower resistor?



1. It stays the same
2. Same magnitude, flips direction
3. It is cut in half, same direction
4. It is cut in half, flips direction
5. It doubles, same direction
6. It doubles, flips direction
7. None of the above

Concept Question: Current Thru Capacitor

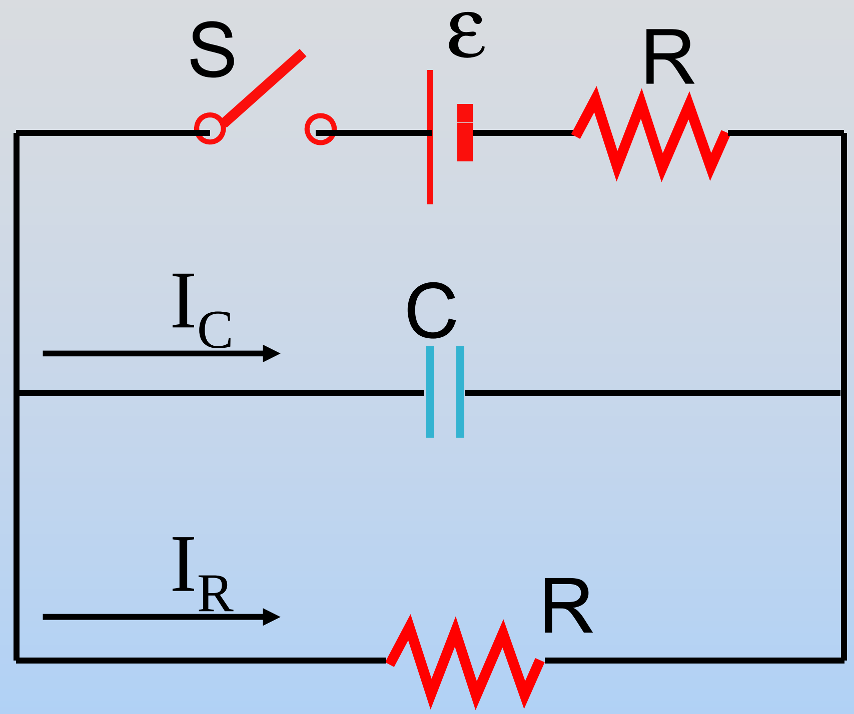
In the circuit at right the switch is closed at t = 0. At t = ∞ (long after) the *current through the capacitor* will be:



1. $I_C = 0$
2. $I_C = \varepsilon / R$
3. $I_C = \varepsilon / 2R$
4. I don't know

Concept Question: Current Thru Resistor

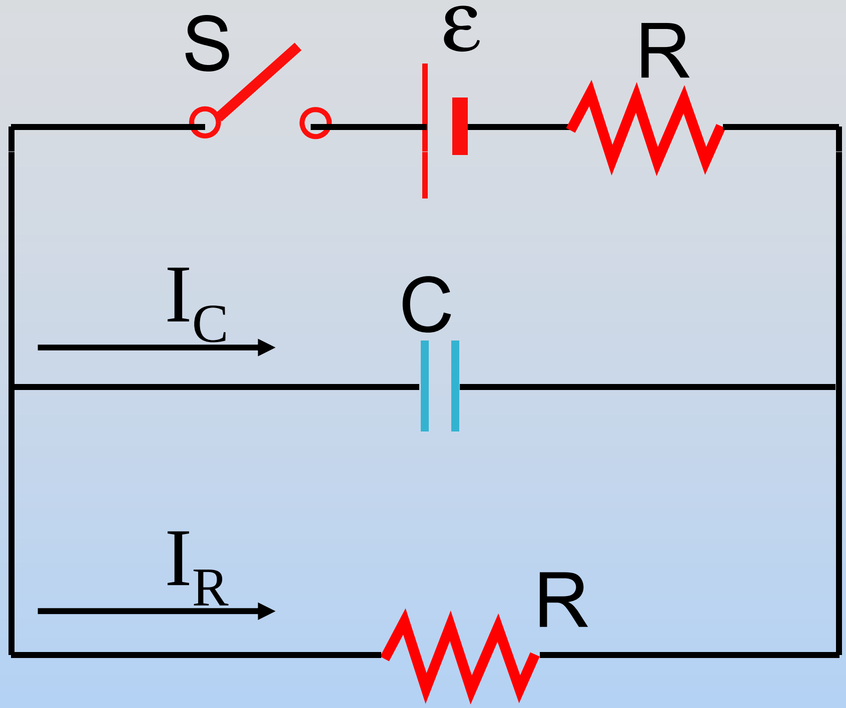
In the circuit at right the switch is closed at $t = 0$. At $t = \infty$ (long after) the *current through the lower resistor* will be:



1. $I_R = 0$
2. $I_R = \varepsilon / R$
3. $I_R = \varepsilon / 2R$
4. I don't know

Concept Question: Opening Switch in RC Circuit

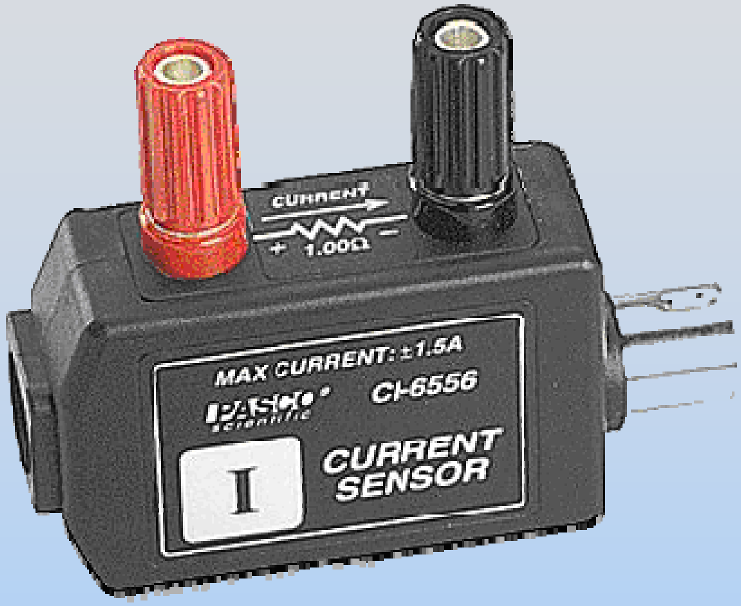
Now, after the switch has been closed for a very long time, it is opened. What happens to the current through the lower resistor?

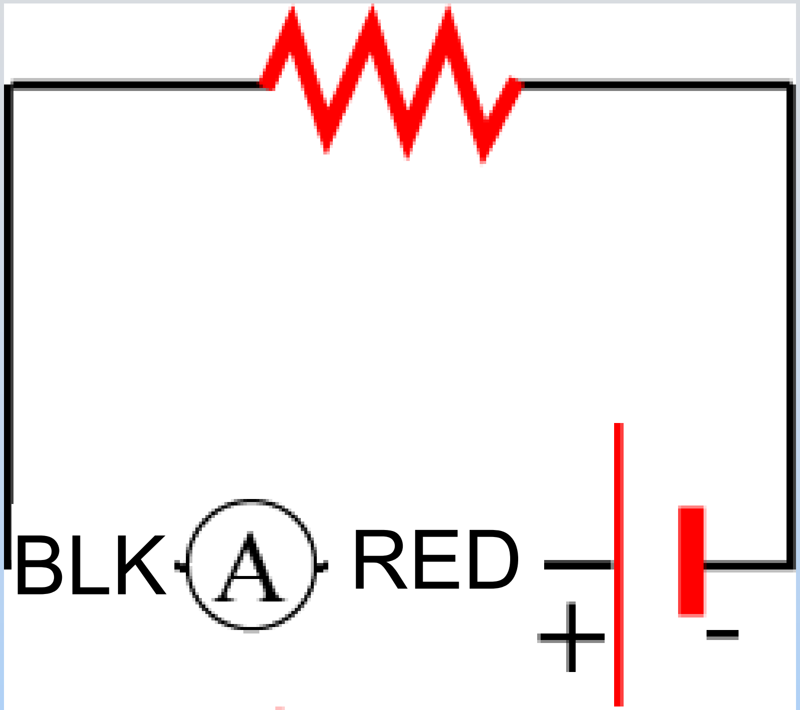


1. It stays the same
2. Same magnitude, flips direction
3. It is cut in half, same direction
4. It is cut in half, flips direction
5. It doubles, same direction
6. It doubles, flips direction
7. None of the above.

Experiment 4:
RC Circuits

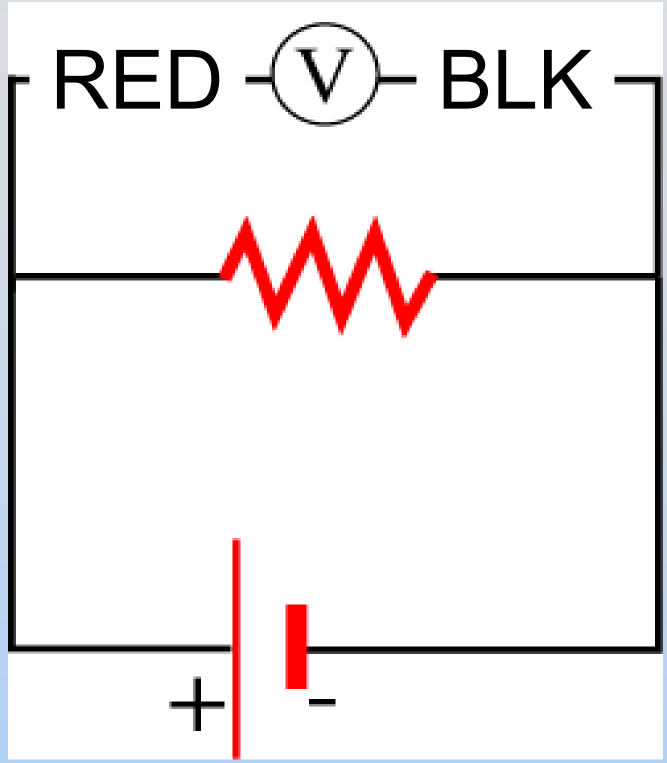
Measuring Current (THRU)





1. Hook in SERIES: current must go thru to measure
2. "Positive" if runs from Red to Black
3. Note: Not ideal – 1 Ω resistance. Does it matter?

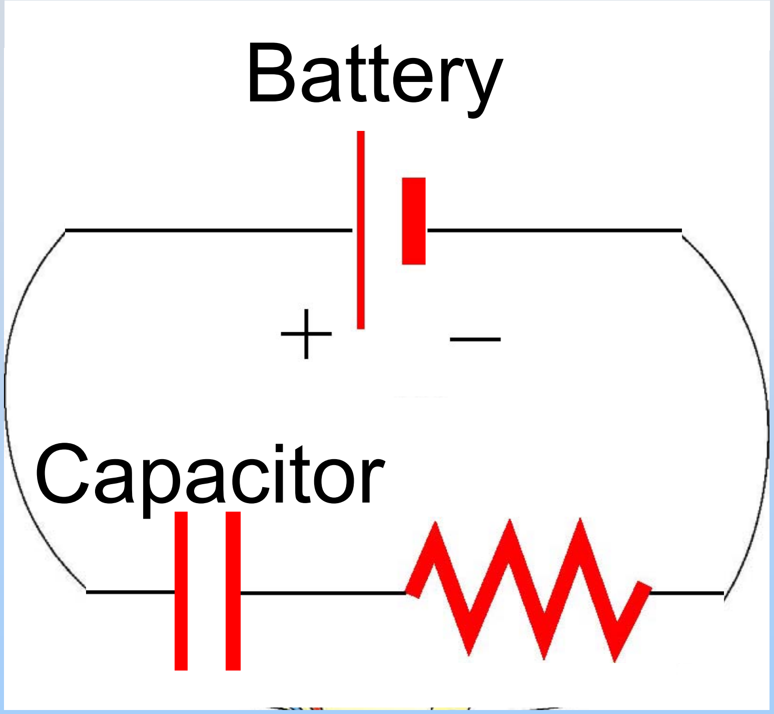
Measuring Voltage (ACROSS)



1. Hook in PARALLEL: reads V_{Red} - V_{Black}
2. Note: Not ideal – 1 MΩ resistance. Does it matter?

Expt. 4, Part I: RC Circuits

- Download and run Lab 4
- Build an RC circuit:
- Measure **current thru** and **voltage across** capacitor
- As battery 'turns on and off,' what happens to the capacitor? WHY?



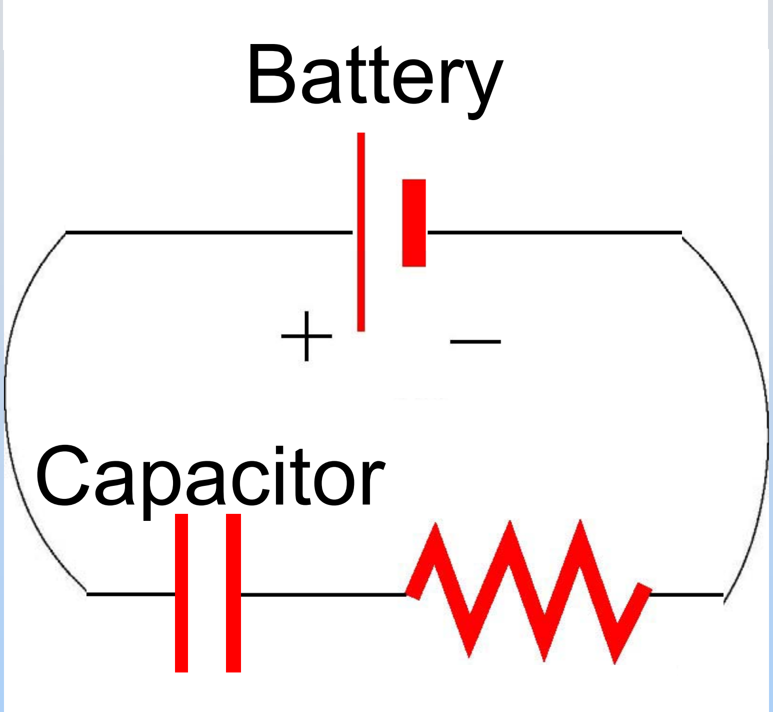
Concept Question: Voltage/Current in RC

Starting from a point in time where the voltage across the battery (V_B) & across the capacitor (V_C) as well as the current (I) are all zero, what happens when the battery is ‘turned on’?

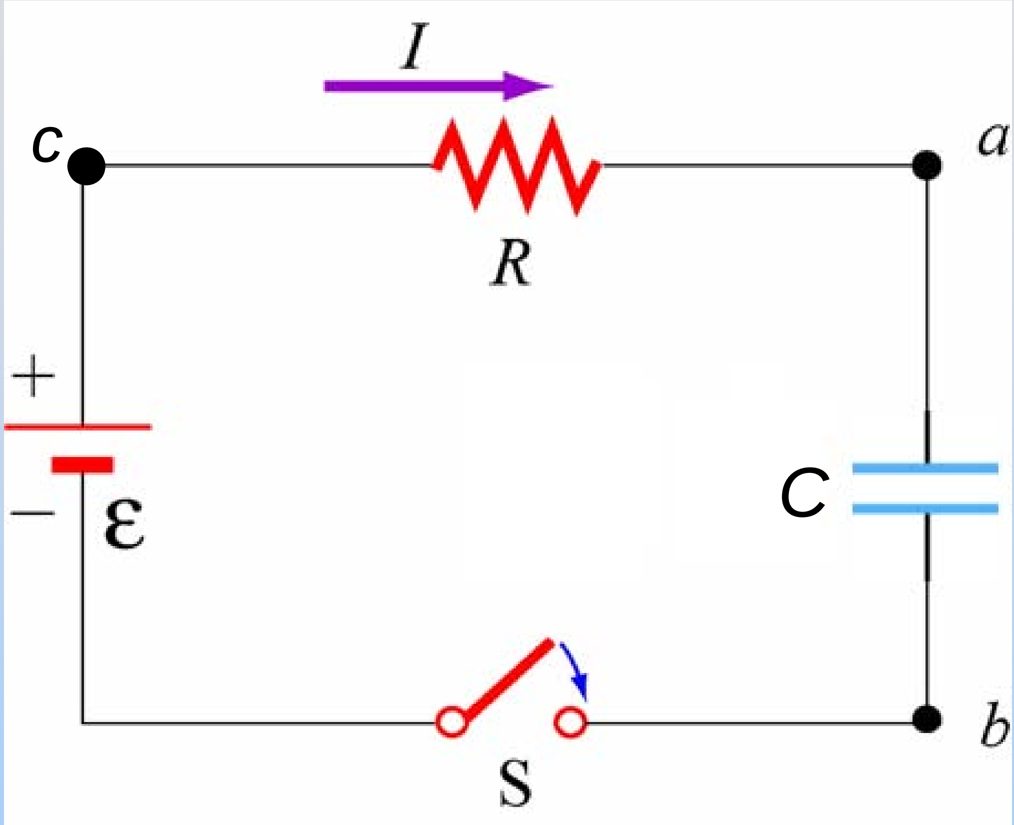
1. I jumps up then decays as V_C rises
2. V_C jumps up then decays as I rises
3. I & V_C both jump up then decay
4. I & V_C both gradually rise
5. I don’t know

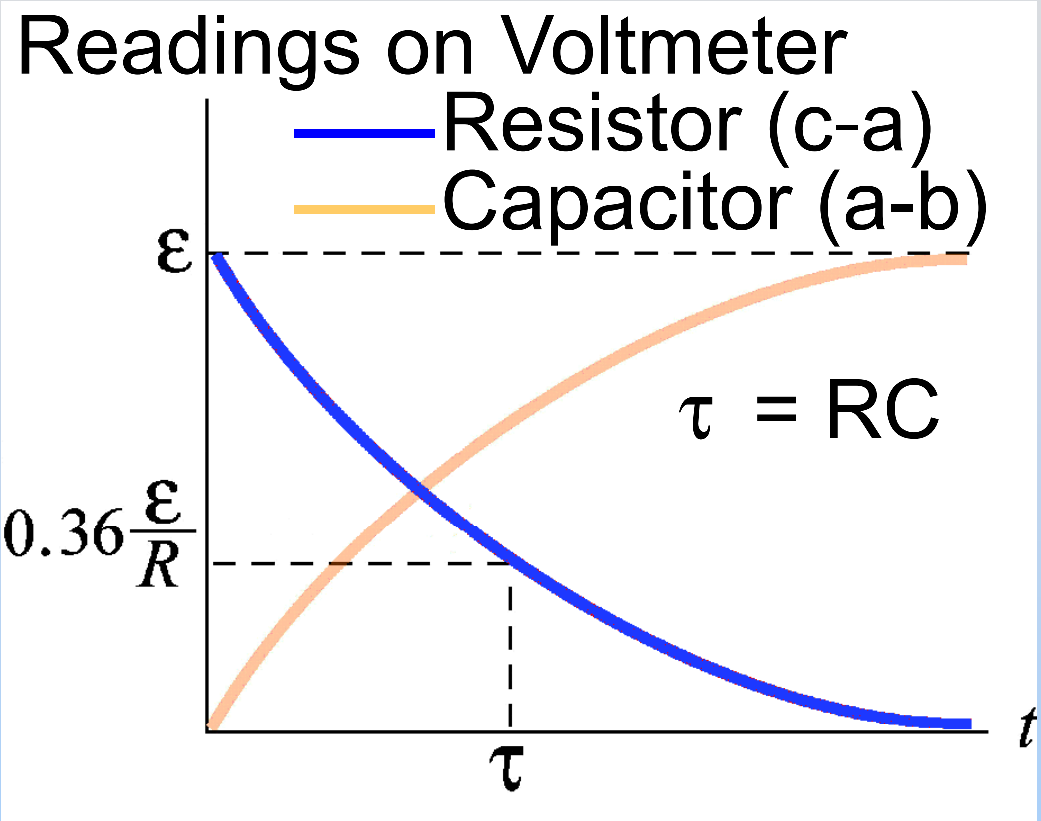
Expt. 4, part II: RC Circuits

- Same RC circuit
- Determine the resistance
- Measure the time constant to determine the capacitance
- You have a 2nd identical resistor. Where do you put it to make the TC as SHORT as possible?



RC Circuit

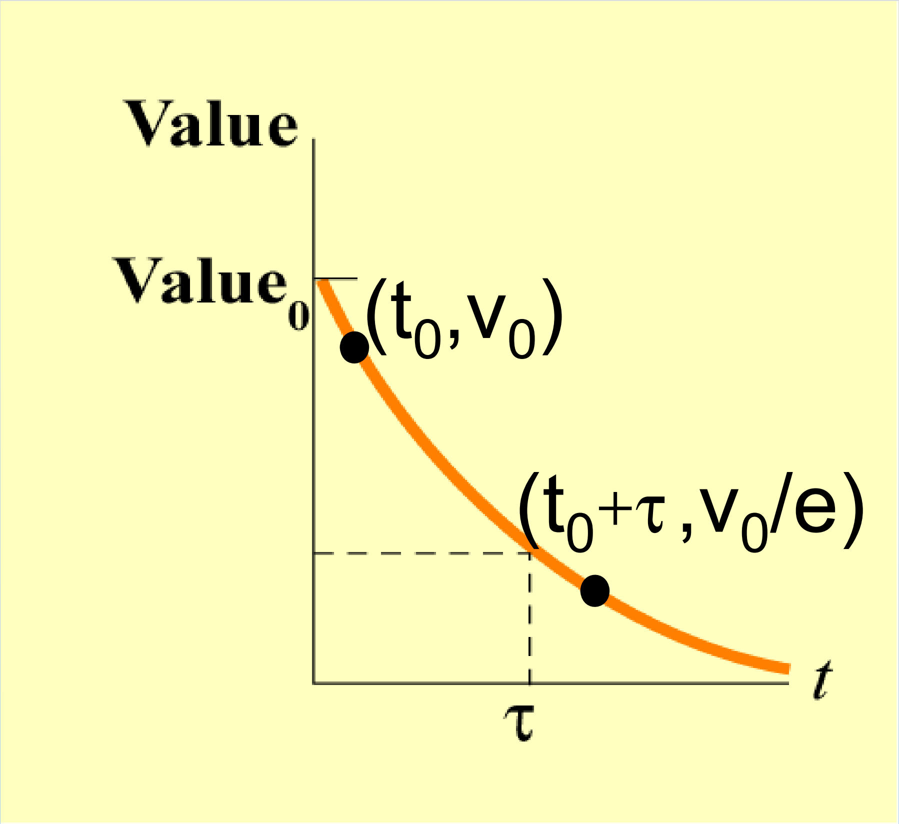




t=0⁺: Capacitor is uncharged so resistor sees full battery potential and current is largest

t=∞: Capacitor is "full." No current flows

Measuring Time Constant



$$\text{Value}(t) = \text{Value}_0 e^{-t/\tau}$$

How do you measure τ?

1) a) Pick a point
   b) Find point with "value" down by e
   c) Time difference is τ

2) Plot semi-log and fit curve (make sure you exclude data at both ends)

Read instructions about cursors. Right click to fit