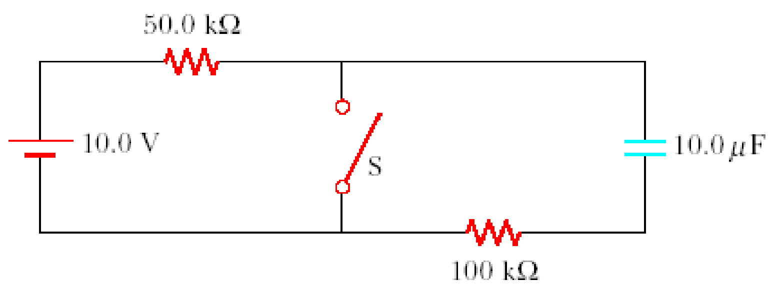
DC Circuits with Capacitors
Challenge Problems

Problem 1:

In the circuit shown, the switch S has been closed for a long time. At time t=0 the switch is opened. It remains open for “a long time” T, at which point it is closed again. Write an equation for (a) the voltage drop across the 100 kΩ resistor and (b) the charge stored on the capacitor as a function of time.

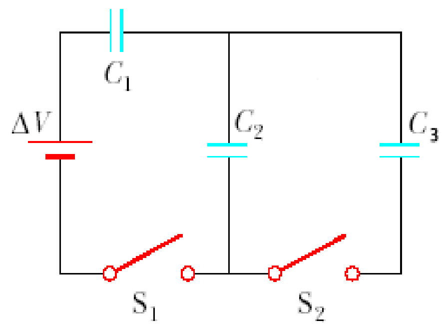


Problem 2:

You know that the power supplied by a battery is given by P = VI (the battery voltage times the current it is supplying). You also know (from the Friday problem solving) that a resistor dissipates power (turns it into heat) at a rate given by $P = I^2R$.

Consider a simple RC circuit (battery, resistor R, capacitor C). Determine an expression for the energy stored in the capacitor by integrating the difference between the power supplied by the battery and that consumed by the resistor. Should the energy be related to the current through the capacitor or the potential across it?

Problem 3:



In the circuit shown at right $C_1 = 2.0\ \mu$F, $C_2 = 6.0\ \mu$F, $C_3 = 3.0\ \mu$F and $\Delta V = 10.0\,\mathrm{V}$. Initially all capacitors are uncharged and the switches are open. At time $t = 0$ switch S_2 is closed. At time $t = T$ switch S_2 is then opened, proceeded nearly immediately by the closing of S_1. Finally at $t = 2T$ switch S_1 is opened, proceeded nearly immediately by the closing of S_2. Calculate the following:

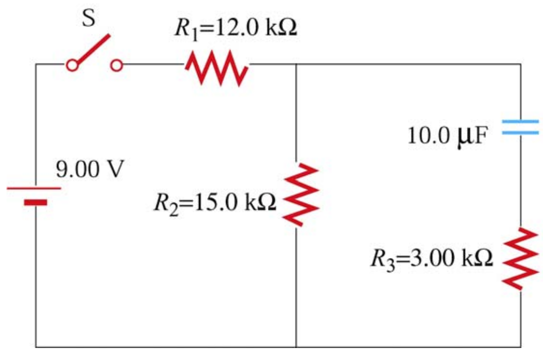
(a) the charge on C_2 for $0 < t < T$ (after S_2 is closed)

(b) the charge on C_1 for $T < t < 2T$

(c) the final charge on each capacitor (for $t > 2T$)

Problem 4:

Consider the *RC* circuit shown in the figure. Suppose that the switch has been closed for a length of time sufficiently long enough for the capacitor to be fully charged.



(a) Find the steady-state current in each resistor.

(b) Find the charge Q on the capacitor.

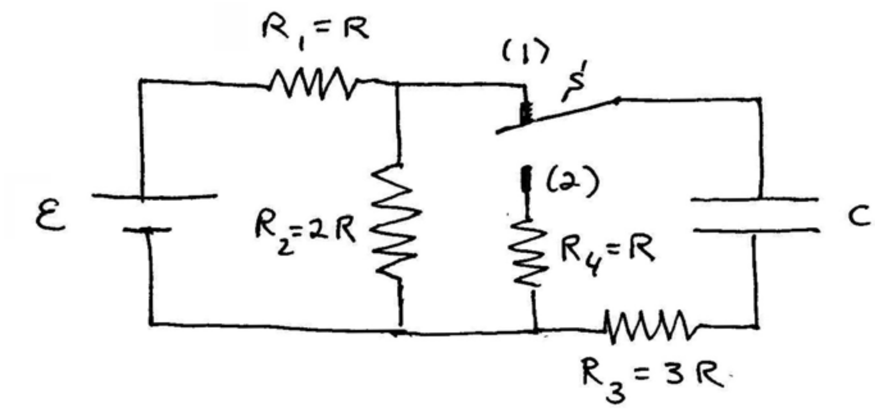
(c) The switch is opened at $t = 0$. Write an equation for the current I_2 in R_2 as a function of time.

(d) Find the time that it takes for the charge on the capacitor to fall to 1/e of its initial value.

Problem 5:

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

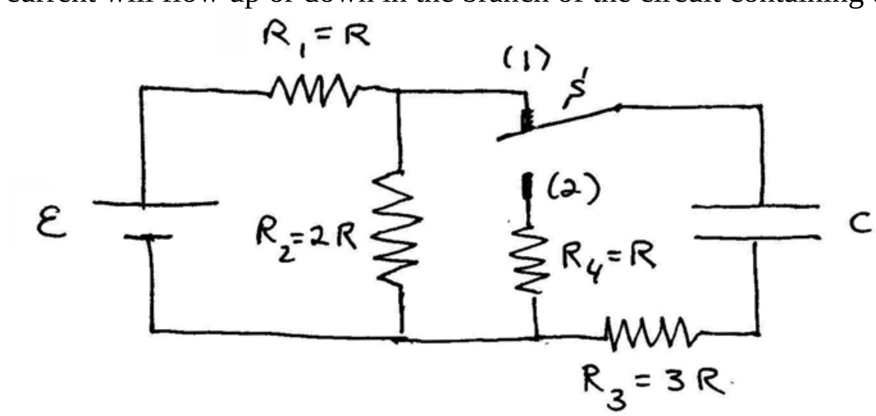
Consider the following circuit shown in the figure below. All questions can be answered without solving any differential equations.



a) Find the current through each of the four resistors, with resistances R_1, R_2, R_3, and R_4, a long time after the switch S has been in position (1).

b) Find the absolute value of the potential difference $|V_C|$ across the capacitor a long time after the switch S has been in position (1).

c) At $t = 0$ the switch is moved to position (2). What current will flow out of the capacitor at the instant the switch is moved to position (2)? Indicate whether the current will flow up or down in the branch of the circuit containing the capacitor.



d) Make a graph of current vs. time for the current that flows out of the capacitor after the switch is moved to position (2) at $t = 0$. Indicate the value of the current at time $t = 0$ on your graph.

e) Find an expression for how long it takes the current that flows out of the capacitor to reach a value equal to e^{-1} of the value of that current when the switch is moved to position (2) at $t = 0$. (You can answer this question without solving a differential equation.)

f) After a long period in position (2), the switch is thrown to position (1) again. Immediately after the switch has been thrown to position (1), find the current through the battery.