Conductors and Insulators, Conductors as Shields
Challenge Problem Solutions

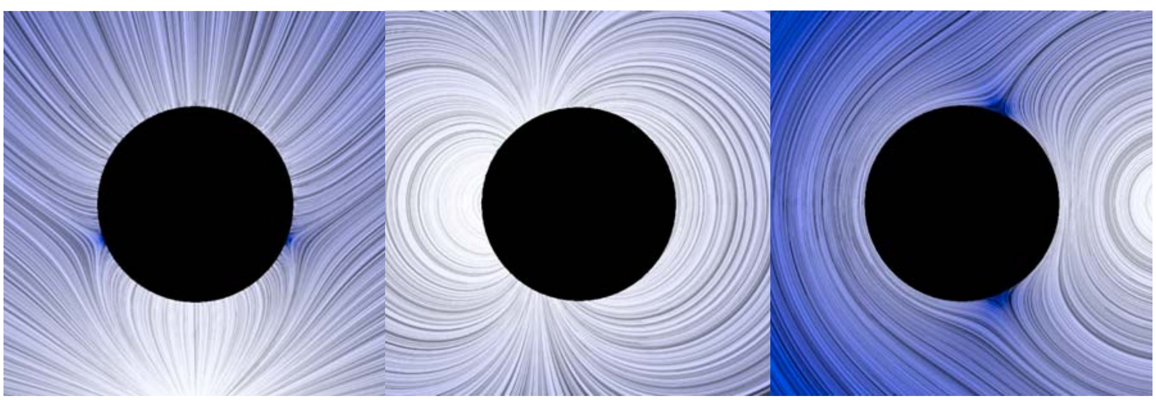
Problem 1:

Part of the lab this week involves shielding. We have a visualization to help you better understand this. Open it up:

http://web.mit.edu/viz/EM/visualizations/electrostatics/ChargingByInduction/shielding/shielding.htm

and play with it for a while. You can move the charge around the outside of the shield (or even inside) using the parameters “radius pc” and “angle pc.” You can change which field you are looking at – the total field, just the field of the external charge (“Free charge”) or just the field of the induced charge (on the shield). You can visualize it with grass seeds or display equipotential streaks by clicking “Electric Potential.”

Below are three captured images. I’ve blanked out the center so that you can’t see what is going on inside the conductor. For each describe where the charge is (ROUGH angle and distance), tell whether I am looking at field lines (grass seeds) or equipotential streaks (“Electric Potential”) and indicate whether I am doing so for the total field, or just the external or induced field. Also briefly explain HOW you know this (not just “I looked around until I was able to repeat the pattern”).



(a) (b) (c)

Problem 1 Solutions:

(a) These are electric fields lines (grass seeds) of the entire field. We can tell because they come in perpendicular to the equipotential surface of the conductor, which is only true for the total field (not the individual parts). The charge is clearly below the conductor ($\theta = 270^{o}$) and just off the screen (R = 11.5).

(b) Here the lines are neither perpendicular nor parallel to the conductor, so it can’t be for the entire field. They loop around, looking like a dipole, so they are associated with the induced charges, not the external charge. Are they field lines or equipotentials though? Without seeing the center this is non-trivial. If the charge were below, the field lines would look very much like this. But since the left and right “lobes” are not symmetric, it must be equipotentials created by a charge on the left (R = 6, $\theta = 180^{o}$).

(c) This one is easier. The lines wrap around the conductor, so they are clearly equipotential lines associated with the entire field. The charge is on the right (R=11, $\theta=0^{o}$)

Problem 2:

Consider two nested, spherical conducting shells. The first has inner radius *a* and outer radius *b*. The second has inner radius *c* and outer radius *d*.

In the following four situations, determine the total charge on each of the faces of the conducting spheres (inner and outer for each), as well as the electric field and potential everywhere in space (as a function of distance *r* from the center of the spherical shells). In all cases the shells begin uncharged, and a charge is then instantly introduced somewhere.

(a) Both shells are floating – that is, their net charge will remain fixed. A positive charge +Q is introduced into the center of the inner spherical shell. Take the zero of potential to be at infinity.

(b) The inner shell is floating but the outer shell is grounded – that is, it is fixed at V=0 and has whatever charge is necessary on it to maintain this potential. A negative charge –Q is introduced into the center of the inner spherical shell.

(c) The inner shell is grounded but the outer shell is floating. A positive charge +Q is introduced into the center of the inner spherical shell.

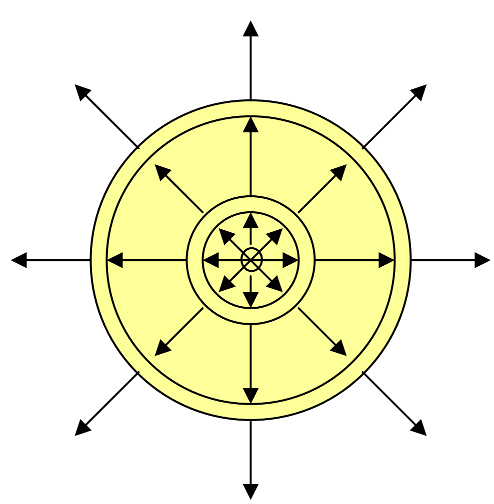
(d) Finally, the outer shell is grounded and the inner shell is floating. This time the positive charge +Q is introduced into the region in between the two shells. In this case the questions "What is **E**(r)/V(r)?" are not well defined in some regions of space. In the regions where these questions can be answered, answer them. In the regions where they can't be answered, explain why, and give as much information about the potential as possible (is it positive or negative, for example).

Problem 2 Solutions:

(a)There is no electric field inside a conductor. Also, the net charge on an isolated conductor is zero (i.e. $Q_a + Q_b = Q_c + Q_d = 0$).

$Q_a = -Q$, $Q_b = -Q_a = Q$, $Q_c = -Q$, $Q_d = -Q_c = Q$

Using the Gauss's law,



$$\vec{E}(r) = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, r > d \\ 0, c < r < d \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, b < r < c \\ 0, a < r < b \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, r < a \end{cases}$$

Since $V(r) = -\int_{\infty}^{r} E(r)dr$,

$$V(r) = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r}, r > d \\ \dfrac{Q}{4\pi\varepsilon_0 d}, c < r < d \\ \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{r} - \dfrac{1}{c} + \dfrac{1}{d}\right), b < r < c \\ \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{b} - \dfrac{1}{c} + \dfrac{1}{d}\right), a < r < b \\ \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{r} - \dfrac{1}{a} + \dfrac{1}{b} - \dfrac{1}{c} + \dfrac{1}{d}\right), r < a \end{cases}$$

(b)Since the outer shell is now grounded, $Q_d = 0$ to maintain $\vec{E}(r) = 0$ outside the outer shell. We have.

$Q_a = Q$, $Q_b = -Q_a = -Q$, $Q_c = Q$, $Q_d = 0$

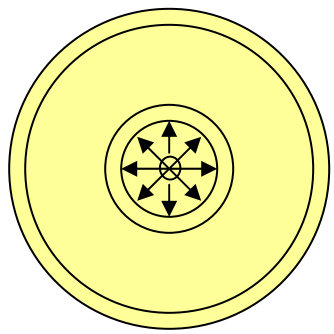
$$\vec{E}(r) = \begin{cases} 0, r > c \\ -\dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, b < r < c \\ 0, a < r < b \\ -\dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, r < a \end{cases}$$

$$V(r)=\begin{cases}0, r>c\\ -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{c}\right), b<r<c\\ -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{b}-\frac{1}{c}\right), a<r<b\\ -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right), r<a\end{cases}$$

(c)Since the inner shell is grounded and $Q_b = 0$ to maintain $\vec{E}(r) = 0$ outside the inner shell. Since there is no electric field on the outer shell, $Q_c = Q_d = 0$.

$Q_a = -Q$, $Q_b = Q_c = Q_d = 0$

$$\vec{E}(r)=\begin{cases}0, r>a\\ \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, r<a\end{cases}$$



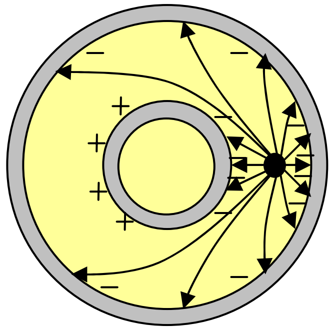
$$V(r)=\begin{cases}0, r>a\\ \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{a}\right), r<a\end{cases}$$

(d)The electric field within the cavity is zero. If there is any field line that began and ended on the inner wall, the integral $\oint \mathbf{E}\cdot d\vec{\mathbf{s}}$ over the closed loop that includes the field line would not be zero. This is impossible since the electrostatic field is conservative, and therefore the electric field must be zero inside the cavity. The charge Q between the two conductors pulls minus charges to the near side on the inner conducting shell and repels plus charges to the far side of that shell. However, the net charge on the outer surface of the inner shell (Q_b) must be zero since it was initially uncharged (floating). Since the outer shell is grounded, $Q_d = 0$ to maintain $\vec{E}(r) = 0$ outside the outer shell. Thus,

$$Q_a = Q_b = Q_d = 0,\ Q_c = -Q \text{ and } \vec{E}(r) = 0,\ r<b \text{ or } r>c$$

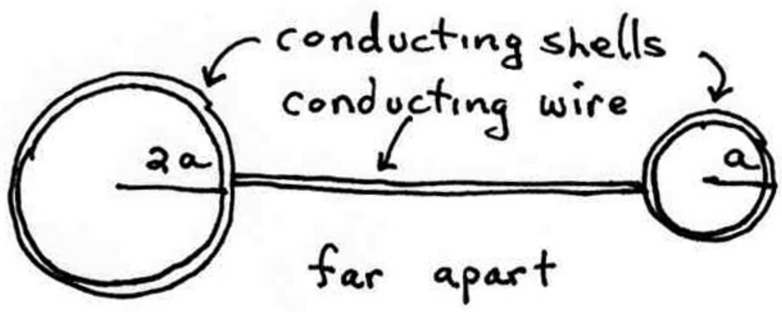
For $b<r<c$, $\vec{E}(r)$ is in fact well defined but it is very complicated. The filed lines are shown in the figure below.

What can we say about the electric potential? $V(r)=0$ **for** $r>c$**, and** $V(r)=\text{constant}$ **for** $r<b$ **but the potential is very complicated defined between the two shells.**



Problem 3:

A conducting wire is attached to an initially charged spherical conducting shell of radius $2a$. The other end of the wire is attached to the outer surface of a neutral conducting spherical shell of radius a that is located a very large distance away (at infinity). When electrostatic equilibrium is reached, the charge on the shell of radius $2a$ is equal to



a) one fourth the charge on the shell of radius a.
b) half the charge on the shell of radius a.
c) twice the charge on the shell of radius a.
d) four time the charge on the shell of radius a.
e) None of the above.

Problem 3 Solution:

c. When electrostatic equilibrium is reached, the two shells form one conducting surface and hence the potential on that surface is constant. Because the two shells are very far apart, the potential of each shell with respect to infinity can be calculated separated. Because the electric field outside each charged shell is identical to the electric field of a point-like object with the same charge located at the center of the shell. The potential on each shell with respect to infinity is just $Q/4\pi\varepsilon_0 r$. The potential difference between the two shells is zero, or $Q_{2a}/4\pi\varepsilon_0 2a - Q_a/4\pi\varepsilon_0 a = 0$, or $Q_{2a} = 2Q_a$. Therefore the charge on the shell of radius $2a$ is equal to twice the charge on the shell of radius a.