Capacitors and Dielectrics
Challenge Problems

Problem 1:

A parallel plate capacitor has capacitance C. It is connected to a battery of EMF ε until fully charged, and then disconnected. The plates are then pulled apart an extra distance d, during which the measured potential difference between them changed by a factor of 4. Below are a series of questions about how other quantities changed. Although they are related you do not need to rely on the answers to early questions in order to correctly answer the later ones.

a) Did the potential difference increase or decrease by a factor of 4?

INCREASE DECREASE

b) By what factor did the electric field change due to this increase in distance? Make sure that you indicate whether the field increased or decreased.

c) By what factor did the energy stored in the electric field change? Make sure that you indicate whether the energy increased or decreased.

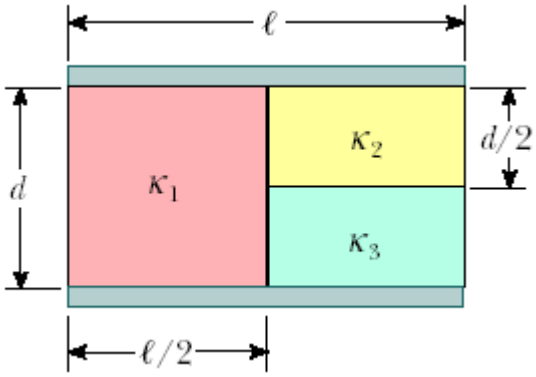
d) A dielectric of dielectric constant κ is now inserted to completely fill the volume between the plates. Now by what factor does the energy stored in the electric field change? Does it increase or decrease?

e) What is the volume of the dielectric necessary to fill the region between the plates? (Make sure that you give your answer only in terms of variables defined in the statement of this problem, fundamental constants and numbers)

Problem 2:

(a) Consider a plane-parallel capacitor completely filled with a dielectric material of dielectric constant κ. What is the capacitance of this system?

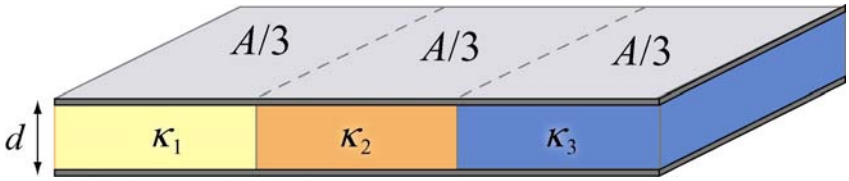
(b) A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in the figure below. You may assume that $\ell >> d$. Find an expression for the capacitance of the device in terms of the plate area A and d, $\boldsymbol{\kappa}_1$, $\boldsymbol{\kappa}_2$, and $\boldsymbol{\kappa}_3$.



Problem 3:

(a) Consider a plane-parallel capacitor completely filled with a dielectric material of dielectric constant κ. What is the capacitance of this system?

(b) A parallel-plate capacitor of area A and spacing d is filled with three dielectrics as shown in the figure below. Each occupies 1/3 of the volume. What is the capacitance of this system? [*Hint:* Consider an equivalent system to be three parallel capacitors, and justify this assumption.] Show that you obtain the proper limits as the dielectric constants approach unity, $\kappa_i \rightarrow 1$.]



(c) Suppose the capacitor is filled as shown in the figure below. What is its capacitance? Use Gauss's law to find the field in each dielectric, and then calculate ΔV across the entire capacitor. Again, check your answer as the dielectric constants approach unity, $\kappa_i \rightarrow 1$. Could you have assumed that this system is equivalent to three capacitors in series?

8.02SC Physics II: Electricity and Magnetism

Fall 2010