

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02

Problem Solving 11: Interference and Diffraction

OBJECTIVES

1. To understand the meaning of constructive and destructive interference
2. To understand how to determine the interference conditions for double slit interference
3. To understand how to determine the intensity of the light associated with double slit interference

REFERENCE: Sections 14-1 through 14-3 Course Notes.

Introduction

The *Huygens Principle* states that every unobstructed point on a wavefront will act a source of a secondary spherical wave. We add to this principle, the *Superposition Principle* that the amplitude of the wave at any point beyond the initial wave front is the superposition of the amplitudes of all the secondary waves.

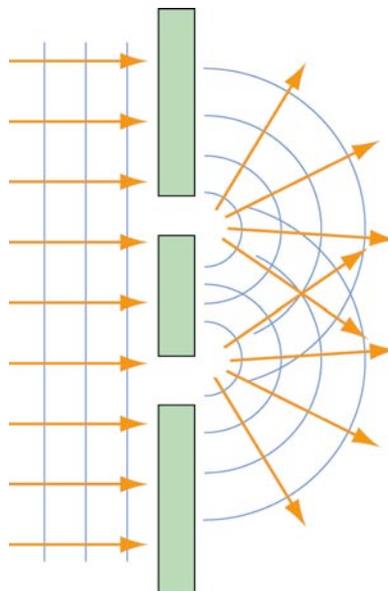


Figure 1: Huygens-Fresnel Principle applied to double slit

When ordinary light is emitted from two different sources and passes through two narrow slits, the plane waves do not maintain a constant phase relation and so the light will show no interference patterns in the region beyond the openings. In order for an interference pattern to develop, the incoming light must satisfy two conditions:

- The light sources must be coherent. This means that the plane waves from the sources must maintain a constant phase relation.
- The light must be monochromatic. This means that the light has just one wavelength.

When the coherent monochromatic laser light falls on two slits separated by a distance d , the emerging light will produce an interference pattern on a viewing screen a distance D away from the center of the slits. The geometry of the double slit interference is shown in the figure below.

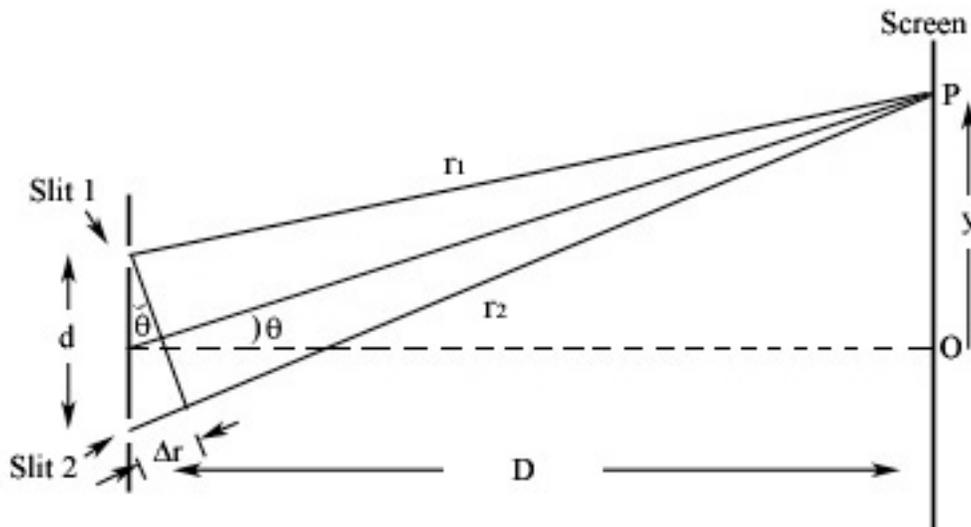


Figure 2: Double slit interference

Consider light that falls on the screen at a point P a distance y from the point O that lies on the screen a perpendicular distance D from the double slit system. The light from the slit 2 will travel an extra distance $r_2 - r_1 = \Delta r$ to the point P than the light from slit 1. This extra distance is called the path length.

Question 1: Draw a picture of two traveling waves that add up to form *constructive interference*.

Answer:

Question 2: Draw a picture of two traveling waves that add up to form *destructive interference*.

Answer:

Question 3: Explain why constructive interference will appear at the point P when the path length is equal to an integral number of wavelengths of the monochromatic light.

$$\Delta r = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \dots \quad \text{constructive interference}$$

Answer:

Question 4: Based on the geometry of the double slits, show that the condition for constructive interference becomes

$$d \sin \theta = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \dots \quad \text{constructive interference.}$$

Answer:

Question 5: Explain why destructive interference will appear at the point when the path length is equal to an odd integral number of half wavelengths

$$d \sin \theta = \left(m + \frac{1}{2} \right) \lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \dots \text{ destructive interference.}$$

Answer:

Question 6: Let y be the distance between the point P and the point O on the screen. Find a relation between the distance y , the wavelength λ , the distance between the slits d , and the distance to the screen D such that a constructive interference pattern will occur at the point P .

Answer:

Question 7: Find a similar relation such that destructive interference fringes will occur at the point P .

Answer:

Intensity of Double Slit Interference:

Suppose that the waves emerging from the slits are sinusoidal plane waves. The slits are located at the plane $x = -D$. The light that emerges from slit 1 and slit 2 at time t are in phase. Let the screen be placed at the plane $x = 0$. Suppose the component of the electric field of the wave from slit 1 at the point P is given by

$$E_1 = E_0 \sin(\omega t).$$

Let's assume that the plane wave from slit 2 has the same amplitude E_0 as the wave from slit 1. Since the plane wave from slit 2 has to travel an extra distance to the point P equal to the path length, this wave will have a phase shift ϕ relative to the wave from slit 1,

$$E_2 = E_0 \sin(\omega t + \phi).$$

Question 8: Why are the phase shift ϕ , the wavelength λ , the distance between the slits, and the angle related θ by

$$\phi = \frac{2\pi}{\lambda} d \sin \theta.$$

As a hint how are the ratio of the phase shift ϕ to 2π and the ratio of the path length $\Delta r = d \sin \theta$ to wavelength λ , related?

Answer:

Question 9: Use the trigonometric identity

$$\sin A + \sin(B) = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right).$$

To show that the total component of the electric field is

$$E_{total} = E_1 + E_2 = 2E_0 \sin\left(\omega t + \frac{\phi}{2}\right) \cos\left(\frac{\phi}{2}\right).$$

Answer:

The *intensity* of the light is equal to the time-averaged Poynting vector

$$I = \langle \vec{S} \rangle = \frac{1}{\mu_0} \langle \vec{E} \times \vec{B} \rangle.$$

Since the amplitude of the magnetic field is related to the amplitude of the electric field by $B_0 = E_0/c$. The intensity of the light is proportional to the time-averaged square of the electric field,

$$I \sim \langle E_{total}^2 \rangle = 4E_0^2 \cos^2\left(\frac{\phi}{2}\right) \left\langle \sin^2\left(\omega t + \frac{\phi}{2}\right) \right\rangle = 2E_0^2 \cos^2\left(\frac{\phi}{2}\right),$$

where the time-averaged value of the square of the sine function is

$$\left\langle \sin^2\left(\omega t + \frac{\phi}{2}\right) \right\rangle = \frac{1}{2}.$$

Let I_{\max} be the amplitude of the intensity. Then the intensity of the light at the point P is

$$I = I_{\max} \cos^2\left(\frac{\phi}{2}\right)$$

Question 10: Show that the intensity is maximal when $d \sin \theta = m\lambda$, $m = 0, \pm 1, \pm 2, \pm 3, \dots$

Answer:

Question 11: Graph the intensity pattern on the screen as a function of distance y from the point O for the case that $D \gg d$ and $d \gg \lambda$.

Question 12: Since the energy of the light is proportional to the square of the electric fields, is energy conserved for the time-averaged superposition of the electric fields i.e. does the following relation hold,

$$\langle (E_1 + E_2)^2 \rangle = \langle E_1^2 \rangle + \langle E_2^2 \rangle$$

Answer:

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Tear off this page and turn it in at the end of class !!!!

Note:

Writing in the name of a student who is not present is a Committee on Discipline offense.

Problem Solving 11: Interference

Group _____ (e.g. 6A Please Fill Out)

Names _____

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