

Problem Solving 5: Ampere's Law

OBJECTIVES

1. To learn how to use Ampere's Law for calculating magnetic fields from symmetric current distributions
2. To find an expression for the magnetic field of a cylindrical current-carrying shell of inner radius a and outer radius b using Ampere's Law.
3. To find an expression for the magnetic field of a slab of current using Ampere's Law.

REFERENCE: [Section 9-3, 8.02 Course Notes](#).

Problem-Solving Strategy for Ampere's Law (Section 9.10.2, 8.02 Course Notes)

Ampere's law states that the line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around any closed loop is proportional to the total steady current passing through any surface that is bounded by the closed loop:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{enc}}$$

To apply Ampere's law to calculate the magnetic field, we use the following procedure:

Step 1: Identify the 'symmetry' properties of the charge distribution.

Step 2: Determine the direction of the magnetic field

Step 3: Decide how many different spatial regions the current distribution determines

For each region of space...

Step 4: Choose an Amperian loop along each part of which the magnetic field is either constant or zero

Step 5: Calculate the current through the Amperian Loop

Step 6: Calculate the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around the closed loop.

Step 7: Equate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ with $\mu_0 I_{\text{enc}}$ and solve for $\vec{\mathbf{B}}$.

Example 1 : Magnetic Field of a Cylindrical Shell

We now apply this strategy to the following problem. Consider the cylindrical conductor with a hollow center and copper walls of thickness $b - a$ as shown. The radii of the inner and outer walls are a and b respectively, and the current I is uniformly spread over the cross section of the copper (shaded region). We want to calculate the magnetic field in the region $a < r < b$.

Problem Solving Strategy Step

(1) Identify Symmetry

Either circular or rectangular

(2) Determine Direction

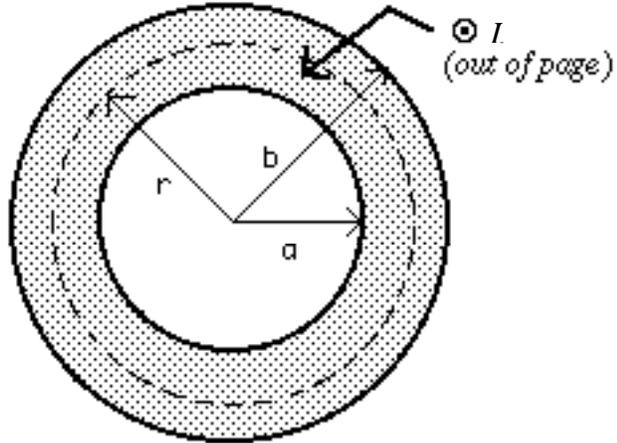
Clockwise or ccw?

(3) How many regions?

Three: $r < a$; $a < r < b$; $r > b$

(4) Draw Amperian Loop:

Here we take a loop that is a circle of radius r with $a < r < b$ (see figure).



(5) Current enclosed by Amperian Loop:

The next step is to calculate the current enclosed by this imaginary Amperian loop. There are typically two ways to do this. One way is to simply calculate it as a fraction of the total current. The second is to first calculate the current density J (current per unit area) and then multiply by the area enclosed. You should use both methods and compare.

Question 1 (*write your answer on the tear-sheet at the end*): What is the magnitude of the current per unit area J in the region $a < r < b$? Remember we are assuming that the current I is uniformly spread over the area $a < r < b$, and also remember that current density J is defined as the current per unit area. Once you have the current density, calculate the total current enclosed by the Amperian loop.

Question 2 (*write your answer on the tear-sheet at the end*): What is the fraction of the total area that is enclosed by the Amperian Loop? What is the total current it encloses?

Question 3 (*write your answer on the tear-sheet at the end*): Your answer above should be zero when $r = a$ and I when $r = b$ (why?). Does your answer have these properties?

(6): Calculate Line Integral $\oint \vec{B} \cdot d\vec{s}$:

Question 4 (*write your answer on the tear-sheet at the end*): What is $\oint \vec{B} \cdot d\vec{s}$?

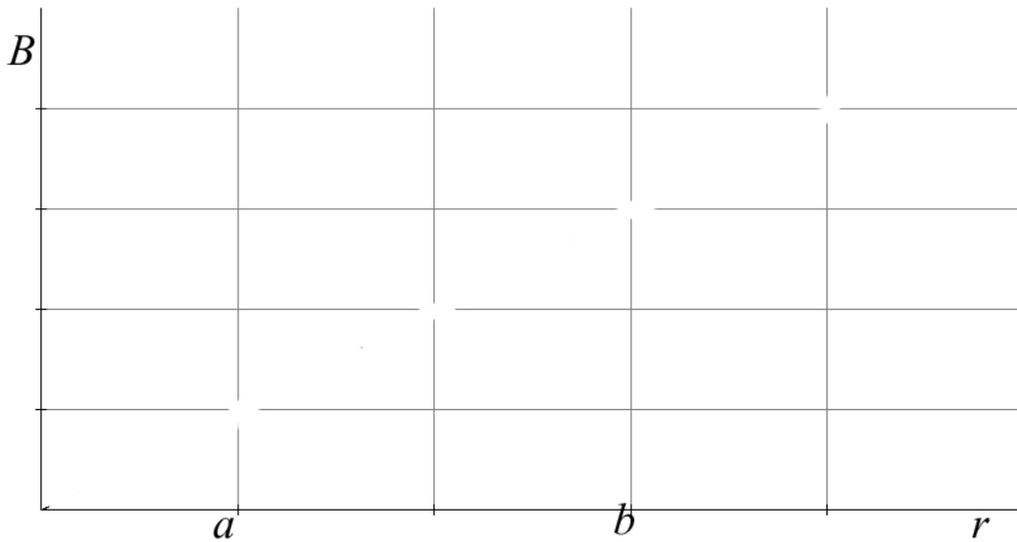
(7): Solve for \vec{B} :

Question 5 (*write your answer on the tear-sheet at the end*): If you equate your answer to Question 4 to your answer to Question 2 times μ_0 (i.e. use Ampere's Law), what do you get for the magnetic field in the region $a < r < b$?

Question 6 (*write your answer on the tear-sheet at the end*): Repeat the steps above to find the magnetic field in the region $r < a$.

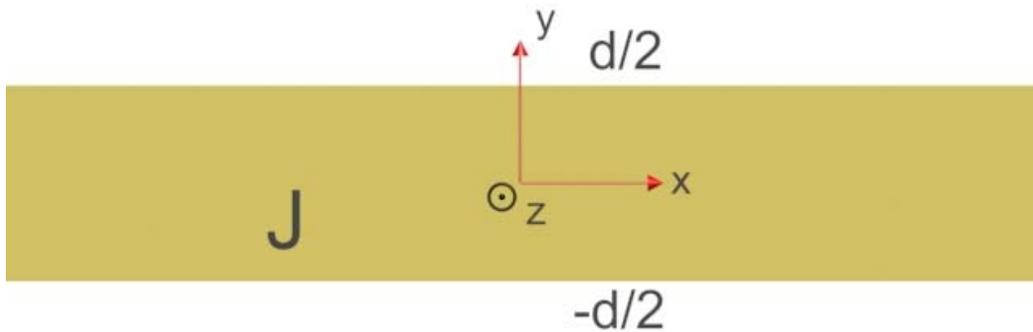
Question 7 (*write your answer on the tear-sheet at the end*): Repeat the steps above to find the magnetic field in the region $r > b$.

Question 8 (*put your answer on the tear-sheet at the end*): Plot B on the graph below.



Example 2: Magnetic Field of a Slab of Current

We want to find the magnetic field \vec{B} due to an infinite slab of current, using Ampere's Law. The figure shows a slab of current with current density $\vec{J} = 2J_e |y|/d \hat{z}$, where units of J_e are amps per square meter. The slab of current is infinite in the x and z directions, and has thickness d in the y -direction.



Question 9 (*write your answer on the tear-sheet at the end*): What is the magnetic field at $y = 0$, where $y = 0$ is the exact center of the slab?

Problem Solving Strategy Step

(1) Identify Symmetry

Either circular or rectangular. Which is it?

(2) Determine Direction

Make sure you determine the direction in all regions? Sketch on tear sheet figure of Q10.

(3) How many regions?

Two for this problem: in the slab and above it (we won't do below the slab).

(4) Draw Amperian Loop:

We want to find the magnetic field for $y > d/2$, and we have from the answer to Question 9 the magnetic field at $y = 0$. Therefore....

Question 10 (*write your answer on the tear-sheet at the end*): What Amperian loop do you take to find the magnetic field for $y > d/2$? Draw it on the figure above and also on the tear-sheet at the end, and indicate its dimensions.

(5) Current enclosed by Amperian Loop:

The next step is to calculate the current enclosed by this imaginary Amperian loop. Hint: the current enclosed is the integral of the current density over the enclosed area.

Question 11 (*write your answer on the tear-sheet at the end*): What is the total current enclosed by your Amperian loop from Question 10?

(6): Calculate Line Integral $\oint \vec{B} \cdot d\vec{s}$:

Question 12 (*write your answer on the tear-sheet at the end*): What is $\oint \vec{B} \cdot d\vec{s}$?

(7): Solve for B:

Question 13 (*write your answer on the tear-sheet at the end*): If you equate your answers in Question 12 to your answer in Question 11 times μ_0 using Ampere's Law, what do you get for the magnetic field in the region $y > d/2$?

We now want to find the magnetic field in the region $0 < y < d/2$.

(4) Draw Amperian Loop:

We want to find the magnetic field for $0 < y < d/2$, and we have from the answer to Question 9 that the magnetic field at $y = 0$. Therefore...

Question 14 (*write your answer on the tear-sheet at the end*): What Amperian loop do you take to find the magnetic field for $0 < y < d/2$? Draw it on the figure above and on the tear-sheet at the end, and indicate its dimensions.

(5) Current enclosed by Amperian Loop:

The next step is to calculate the current enclosed by this imaginary Amperian loop.

Question 15 (*write your answer on the tear-sheet at the end*): What is the total current enclosed by your Amperian loop from Question 14?

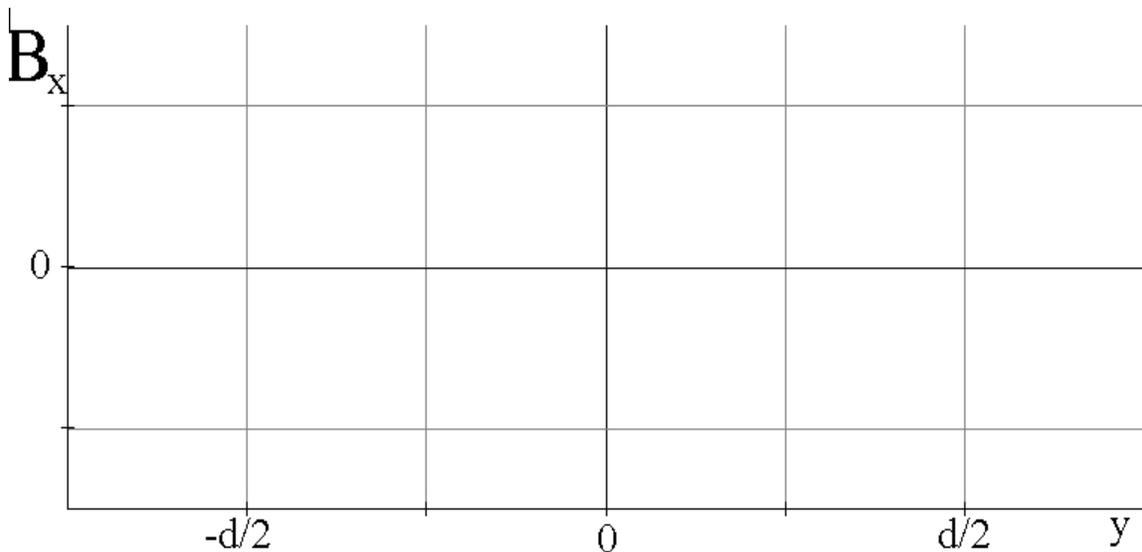
(6) Calculate Line Integral $\oint \vec{B} \cdot d\vec{s}$:

Question 16 (*write your answer on the tear-sheet at the end*): What is $\oint \vec{B} \cdot d\vec{s}$?

(7) Solve for B:

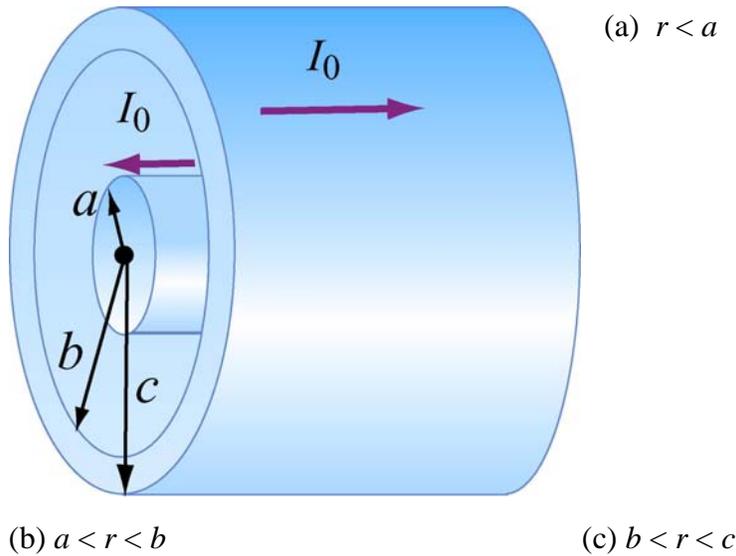
Question 17 (*write your answer on the tear-sheet at the end*): If you equate your answers in Question 16 to your answer in Question 15 times μ_0 using Ampere's Law, what do you get for the magnetic field in the region $0 < y < d/2$?

Question 18 (*put your answer on the tear-sheet at the end*): Plot B_x on the graph below. Use symmetry to determine B for $y < 0$. Label the y-axis.

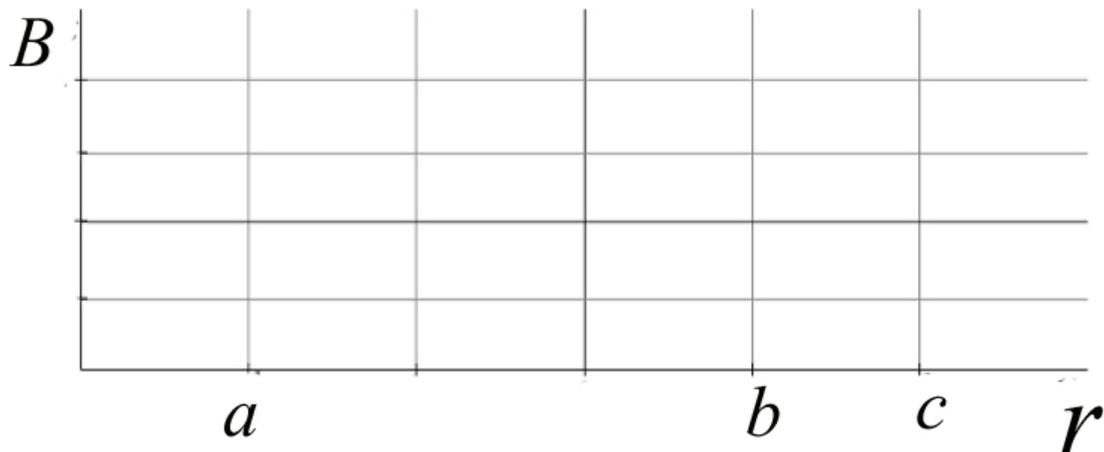


Sample Exam Question (If time, try to do this by yourself, closed notes)

A coaxial cable consists of a solid inner conductor of radius a , surrounded by a concentric cylindrical tube of inner radius b and outer radius c . The conductors carry equal and opposite currents I_0 distributed uniformly across their cross-sections. Determine the magnetic field at a distance r from the axis for the following ranges of radii. On the figure below, draw the Amperian loop you use in each case.



(d) Plot your answers for the magnitude of B above on the graph below. Label your vertical axis or you will lose points.



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Tear off this page and turn it in at the end of class !

Note:
Writing in the name of a student who is not present is a COD offense.

Problem Solving 6: Ampere's Law

Group _____ (e.g. L02 6A Please Fill Out)

Names _____

Question 1: What is J in the region $a < r < b$? What then is the current enclosed?

Question 2: What fraction of the current does your loop enclose? What is the current enclosed?

Question 3: Is your answer above zero when $r = a$ and I when $r = b$.

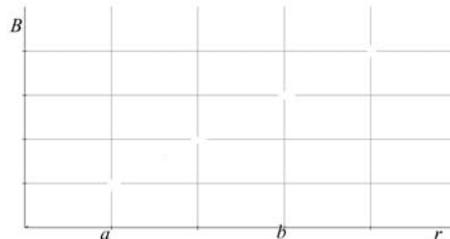
Question 4: What is the line integral $\oint \vec{B} \cdot d\vec{s}$ for your loop?

Question 5: What do you get for the magnetic field in the region $a < r < b$?

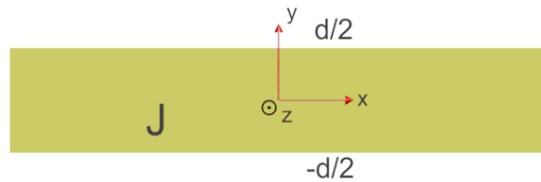
Question 6: What is the magnetic field in the region $r < a$.

Question 7: What is the magnetic field in the region $r > b$.

Question 8: Plot B on the graph.



Example 2: Magnetic Field of a Slab of Current



Question 9: What is the magnetic field at $y = 0$?

Question 10: Draw your Amperian loop for $y > d/2$ on the figure above, and indicate dimensions.

Question 11: What is the total current enclosed by your Amperian loop from Question 10?

Question 12: What is the line integral $\oint \vec{B} \cdot d\vec{s}$ for your loop?

Question 13: What is the magnetic field $\mathbf{B}(y > d/2)$?

Question 14: Draw your Amperian loop for $0 < y < d/2$ on the figure above.

Question 15: What is the total current enclosed by your Amperian loop from Question 14?

Question 16: What is the line integral $\oint \vec{B} \cdot d\vec{s}$ for your loop?

Question 17: What is the magnetic field $\mathbf{B}(0 < y < d/2)$?

Question 18: Plot B_x on the graph.

