

## Experiment 6: Ohm's Law, RC and RL Circuits

### OBJECTIVES

1. To explore the measurement of voltage & current in circuits
2. To see Ohm's law in action for resistors
3. To explore the time dependent behavior of RC and RL Circuits

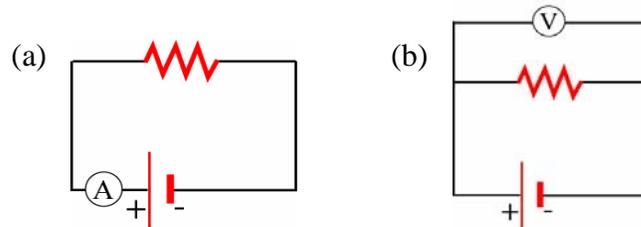
### PRE-LAB READING

### INTRODUCTION

When a battery is connected to a circuit consisting of wires and other circuit elements like resistors and capacitors, voltages can develop across those elements and currents can flow through them. In this lab we will investigate three types of circuits: those with only resistors in them and those with resistors and either capacitors (RC circuits) or inductors (RL circuits). We will confirm that there is a linear relationship between current through and potential difference across resistors (Ohm's law:  $V = IR$ ). We will also measure the very different relationship between current and voltage in a capacitor and an inductor, and study the time dependent behavior of RC and RL circuits.

### The Details: Measuring Voltage and Current

Imagine you wish to measure the voltage drop across and current through a resistor in a circuit. To do so, you would use a voltmeter and an ammeter – similar devices that measure the amount of current flowing in one lead, through the device, and out the other lead. But they have an important difference. An ammeter has a very low resistance, so when placed in series with the resistor, the current measured is not significantly affected (Fig. 1a). A voltmeter, on the other hand, has a very high resistance, so when placed in parallel with the resistor (thus seeing the same voltage drop) it will draw only a very small amount of current (which it can convert to voltage using Ohm's Law  $V_R = V_{meter} = I_{meter}R_{meter}$ ), and again will not appreciably change the circuit (Fig. 1b).



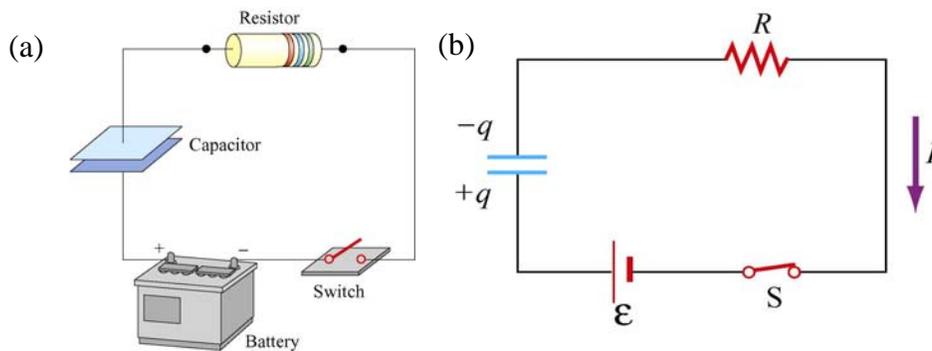
**Figure 1: Measuring current and voltage in a simple circuit.** To measure current *through* the resistor (a) the ammeter is placed in series with it. To measure the voltage drop *across* the resistor (b) the voltmeter is placed in parallel with it.

## The Details: Capacitors

Capacitors store charge, and develop a voltage drop  $V$  across them proportional to the amount of charge  $Q$  that they have stored:  $V = Q/C$ . The constant of proportionality  $C$  is the capacitance (measured in Farads = Coulombs/Volt), and determines how easily the capacitor can store charge. Typical circuit capacitors range from picofarads ( $1 \text{ pF} = 10^{-12} \text{ F}$ ) to millifarads ( $1 \text{ mF} = 10^{-3} \text{ F}$ ). In this lab we will use microfarad capacitors ( $1 \text{ }\mu\text{F} = 10^{-6} \text{ F}$ ).

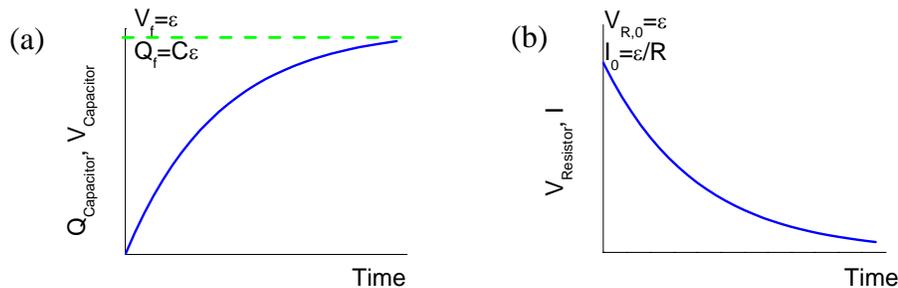
## RC Circuits

Consider the circuit shown in Figure 2. The capacitor (initially uncharged) is connected to a voltage source of constant emf  $\mathcal{E}$ . At  $t = 0$ , the switch  $S$  is closed.



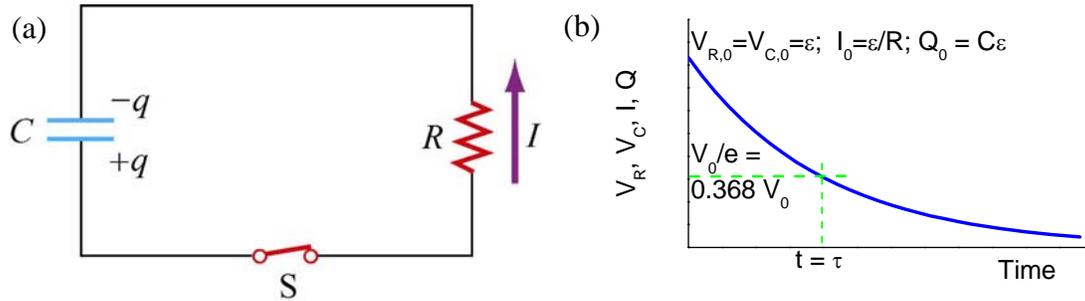
**Figure 2** (a) RC circuit (b) Circuit diagram for  $t > 0$

In class we derived expressions for the time-dependent charge on, voltage across, and current through the capacitor, but even without solving differential equations a little thought should allow us to get a good idea of what happens. Initially the capacitor is uncharged and hence has no voltage drop across it (it acts like a wire or “short circuit”). This means that the full voltage rise of the battery is dropped across the resistor, and hence current must be flowing in the circuit ( $V_R = IR$ ). As time goes on, this current will “charge up” the capacitor – the charge on it and the voltage drop across it will increase, and hence the voltage drop across the resistor and the current in the circuit will decrease. This idea is captured in the graphs of Fig. 3.



**Figure 3** (a) Voltage across and charge on the capacitor increase as a function of time while (b) the voltage across the resistor and hence current in the circuit decrease.

After the capacitor is “fully charged,” with its voltage essentially equal to the voltage of the battery, the capacitor acts like a break in the wire or “open circuit,” and the current is essentially zero. Now we “shut off” the battery (replace it with a wire). The capacitor will then release its charge, driving current through the circuit. In this case, the voltage across the capacitor and across the resistor are equal, and hence charge, voltage and current all do the same thing, decreasing with time. As you saw in class, this decay is exponential, characterized by a time constant  $t$ , as pictured in fig. 4.



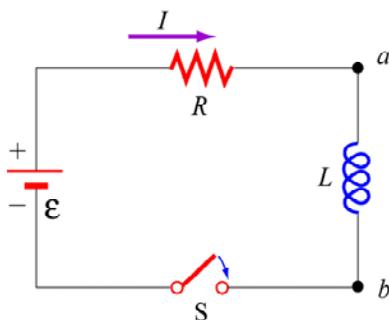
**Figure 4** Once (a) the battery is “turned off,” the voltages across the capacitor and resistor, and hence the charge on the capacitor and current in the circuit all (b) decay exponentially. The time constant  $\tau$  is how long it takes for a value to drop by  $e$ .

### The Details: Inductors

Inductors store energy in the form of an internal magnetic field, and find their behavior dominated by Faraday’s Law. In any circuit in which they are placed they create an EMF  $\epsilon$  proportional to the time rate of change of current  $I$  through them:  $\epsilon = L di/dt$ . The constant of proportionality  $L$  is the inductance (measured in Henries = Ohm s), and determines how strongly the inductor reacts to current changes (and how large a self energy it contains for a given current). Typical circuit inductors range from nanohenries to hundreds of millihenries. The direction of the induced EMF can be determined by Lenz’s Law: it will always oppose the change (inductors try to keep the current constant)

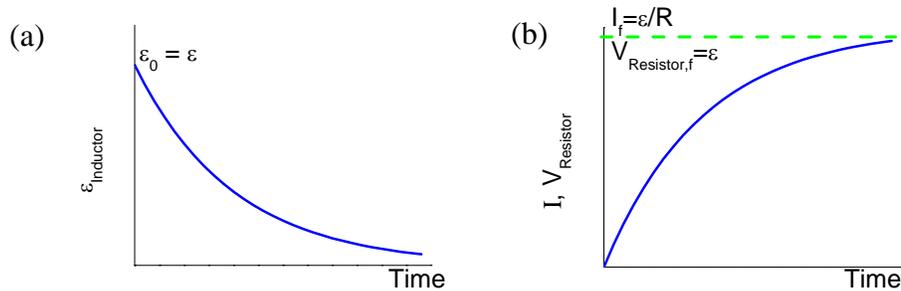
### RL Circuits

If we replace the capacitor of figure 2 with an inductor we arrive at figure 5. The inductor is connected to a voltage source of constant emf  $\mathcal{E}$ . At  $t = 0$ , the switch  $S$  is closed.



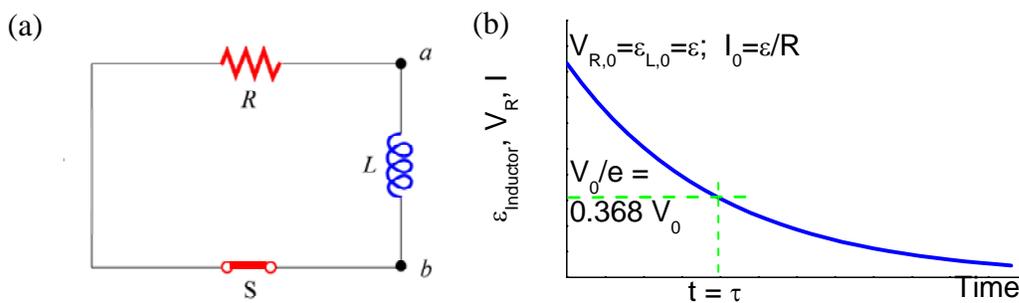
**Figure 5 RL circuit.** For  $t < 0$  the switch  $S$  is open and no current flows in the circuit. At  $t = 0$  the switch is closed and current  $I$  can begin to flow, as indicated by the arrow.

As we saw in class, before the switch is closed there is no current in the circuit. When the switch is closed the inductor wants to keep the same current as an instant ago – none. Thus it will set up an EMF that opposes the current flow. At first the EMF is identical to that of the battery (but in the opposite direction) and no current will flow. Then, as time passes, the inductor will gradually relent and current will begin to flow. After a long time a constant current ( $I = V/R$ ) will flow through the inductor, and it will be content (no changing current means no changing B field means no changing magnetic flux means no EMF). The resulting EMF and current are pictured in Fig. 6.



**Figure 6** (a) “EMF generated by the inductor” decreases with time (this is what a voltmeter hooked in parallel with the inductor would show) (b) the current and hence the voltage across the resistor increase with time, as the inductor ‘relaxes.’

After the inductor is “fully charged,” with the current essentially constant, we can shut off the battery (replace it with a wire). Without an inductor in the circuit the current would instantly drop to zero, but the inductor does not want this rapid change, and hence generates an EMF that will, for a moment, keep the current exactly the same as it was before the battery was shut off. In this case, the EMF generated by the inductor and voltage across the resistor are equal, and hence EMF, voltage and current all do the same thing, decreasing exponentially with time as pictured in fig. 7.



**Figure 7** Once (a) the battery is turned off, the EMF induced by the inductor and hence the voltage across the resistor and current in the circuit all (b) decay exponentially. The time constant  $\tau$  is how long it takes for a value to drop by  $e$ .

## The Details: Non-Ideal Inductors

So far we have always assumed that circuit elements are ideal, for example, that inductors only have inductance and not capacitance or resistance. This is generally a decent assumption, but in reality no circuit element is truly ideal, and today we will need to consider this. In particular, today's "inductor" has both inductance and resistance (real inductor = ideal inductor in series with resistor). Although there is no way to physically separate the inductor from the resistor in this circuit element, with a little thought (which you will do in the pre-lab) you will be able to measure both the resistance and inductance.

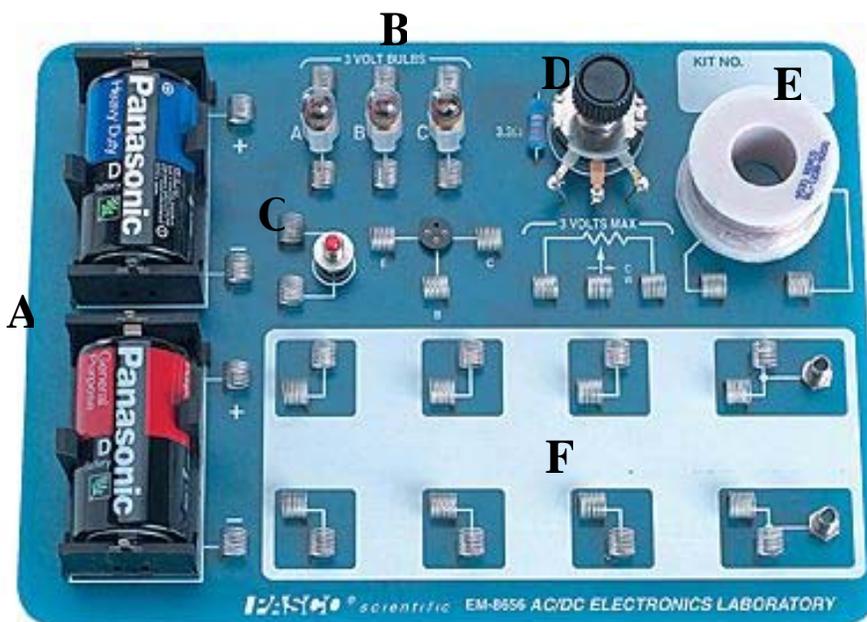
## APPARATUS

### 1. Science Workshop 750 Interface

In this lab we will again use the Science Workshop 750 interface to create a "variable battery" which we can turn on and off, whose voltage we can change and whose current we can measure.

### 2. AC/DC Electronics Lab Circuit Board

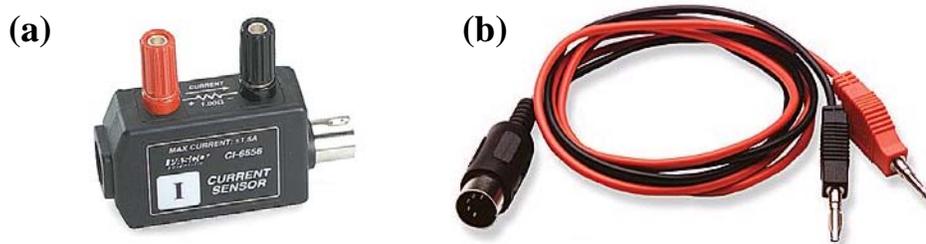
We will also use, for the first of several times, the circuit board pictured in Fig. 8. This is a general purpose board, with (A) battery holders, (B) light bulbs, (C) a push button switch, (D) a variable resistor called a potentiometer, and (E) an inductor. It also has (F) a set of 8 isolated pads with spring connectors that circuit components like resistors and capacitors can easily be pushed into. Each pad has two spring connectors connected by a wire (as indicated by the white lines). The right-most pads also have banana plug receptacles, which we will use to connect to the output of the 750.



**Figure 8** The AC/DC Electronics Lab Circuit Board, with (A) Battery holders, (B) light bulbs, (C) push button switch, (D) potentiometer, (E) inductor and (F) connector pads

### 3. Current & Voltage Sensors

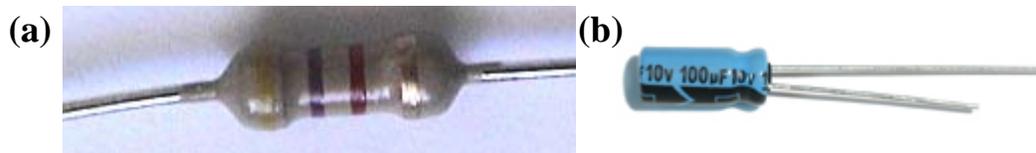
Recall that both current and voltage sensors follow the convention that red is “positive” and black “negative.” That is, the current sensor records currents flowing in the red lead and out the black as positive. The voltage sensor measures the potential at the red lead minus that at the black lead.



**Figure 9** (a) Current and (b) Voltage Sensors

### 4. Resistors & Capacitors

We will work with resistors and capacitors in this lab. Resistors (Fig. 8a) have color bands that indicate their value (see appendix A if you are interested in learning to read this code), whereas capacitors (Fig. 8b) are typically stamped with a numerical value.



**Figure 10** Examples of a (a) resistor and (b) capacitor. Aside from their size, most resistors look the same, with 4 or 5 colored bands indicating the resistance. Capacitors on the other hand come in a wide variety of packages and are typically stamped both with their capacitance and with a maximum working voltage.

## **GENERALIZED PROCEDURE**

This lab consists of five main parts. In each you will set up a circuit and measure voltage and current while the battery periodically turns on and off. In the last two parts you are encouraged to develop your own methodology for measuring the resistance and inductance of the coil on the AC/DC Electronics Lab Circuit Board both with and without a core inserted. The core is a metal cylinder which is designed to slide into the coil and affect its properties in some way that you will measure.

### **Part 1: Measure Voltage Across & Current Through a Resistor**

Here you will measure the voltage drop across and current through a single resistor attached to the output of the 750.

### **Part 2: Resistors in Parallel**

Now attach a second resistor in parallel to the first and see what happens to the voltage drop across and current through the first.

### **Part 3: Measuring Voltage and Current in an RC Circuit**

In this part you will create a series RC (resistor/capacitor) circuit with the battery turning on and off so that the capacitor charges then discharges. You will measure the time constant in two different ways (see Pre-Lab #5) and use this measurement to determine the capacitance of the capacitor.

### **Part 4: Measure Resistance and Inductance Without a Core**

The battery will alternately turn on and turn off. You will need to hook up this source to the coil and, by measuring the voltage supplied by and current through the battery, determine the resistance and inductance of the coil.

### **Part 5: Measure Resistance and Inductance With a Core**

In this section you will insert a core into the coil and repeat your measurements from part 1 (or choose a different way to make the measurements).

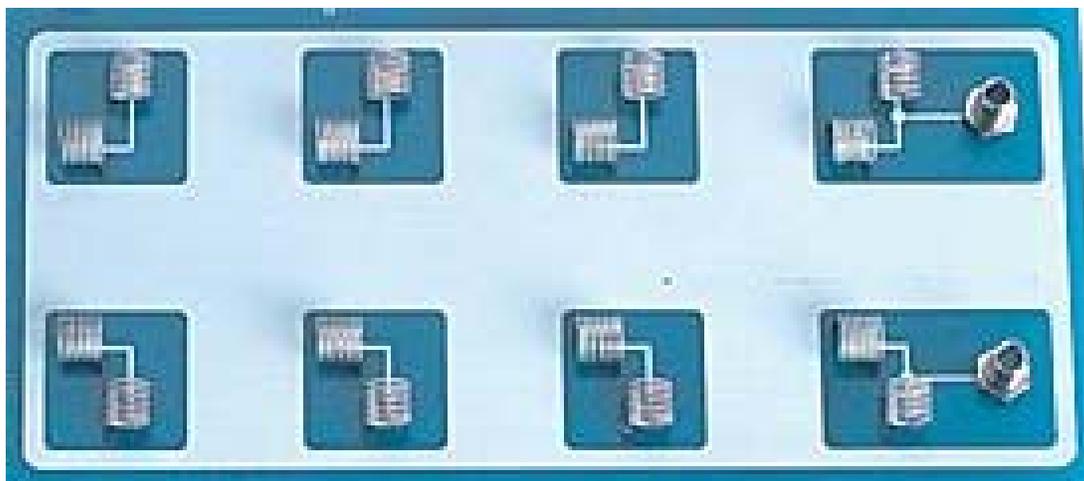
**END OF PRE-LAB READING**

## Experiment 6: Ohm's Law, RC and RL Circuits

**Answer these questions on a separate sheet of paper and turn them in before the lab**

### 1. Measuring Voltage and Current

In Part 1 of this experiment you will measure the potential drop across and current through a single resistor attached to the “variable battery.” On a diagram similar to the one below, indicate where you will attach the leads to the resistor, the battery, the voltage sensor  $\text{V}$ , and the current sensor  $\text{A}$ . For the battery and sensors make sure that you indicate which color lead goes where, using the convention that red is “high” (or the positive input) and black is “ground.” Reread the pre-lab description of this board carefully to understand the various parts. When you draw a resistor or other circuit element it should go between two pads (dark green areas) with each end touching one of the spring clips (the metal coils). Do NOT just draw a typical circuit diagram. You need to think about how you will actually wire this board during the lab. RECALL: ammeters must be in series with the element they are measuring current through, while voltmeters must be in parallel.



### 2. Resistors in Parallel

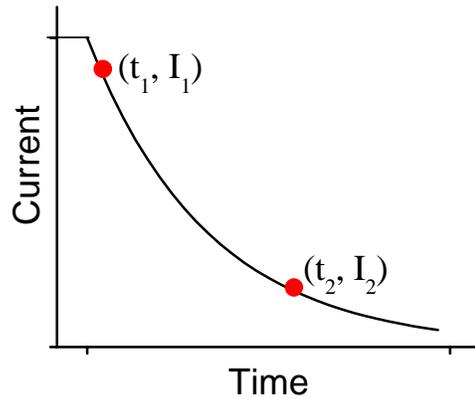
In Part 2 you will add a second resistor in parallel with the first. Show where you would attach this second resistor in the diagram you drew for question 1, making sure that the ammeter continues to measure the current through the first resistor and the voltmeter measures the voltage across the first resistor.

### 3. Measuring the Time Constant $\tau$

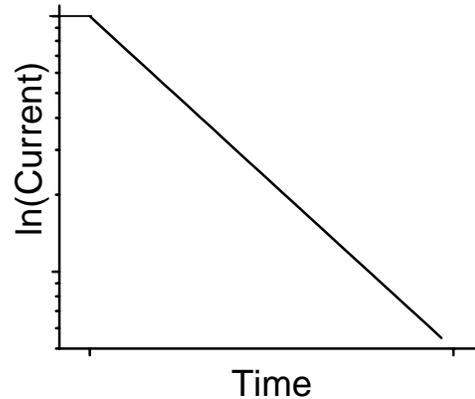
As you have seen, current always decays exponentially in RC circuits with a time constant  $\tau$ :  $I = I_0 \exp(-t/\tau)$ .

We will measure this time constant in two different ways.

- (a) After measuring the current as a function of time we choose two points on the curve  $(t_1, I_1)$  and  $(t_2, I_2)$ . What relationship must we choose between  $I_2$  and  $I_1$  in order to determine the time constant by subtraction:  $\tau = t_2 - t_1$ ? Should we be able to find a  $t_2$  that satisfies this for any choice of  $t_1$ ?



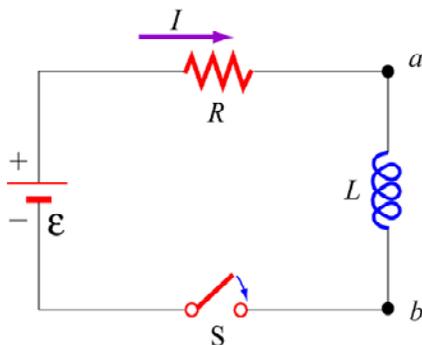
- (b) We can also plot the natural log of the current vs. time, as shown at right. If we fit a line to this curve we will obtain a slope  $m$  and a y-intercept  $b$ . From these fitting parameters, how can we calculate the time constant?



- (c) Which of these two methods is more likely to help us obtain an accurate measurement of the time constant? Why?

***Make sure that you record your answers to question 3 in your notes as you will need them for the lab.***

### 4. RL Circuits



Consider the circuit at left, consisting of a battery (emf  $\epsilon$ ), an inductor  $L$ , resistor  $R$  and switch  $S$ .

For times  $t < 0$  the switch is open and there is no current in the circuit. At  $t = 0$  the switch is closed.

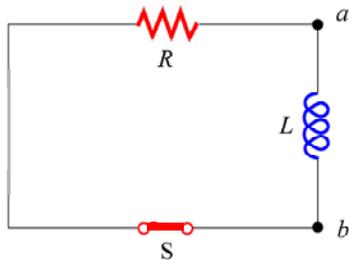
- (a) Using Kirchhoff's loop rules (really Faraday's law now), write an equation relating the emf on the battery, the current in the circuit and the time derivative of the current in the circuit.

In class we stated that this equation was solved by an exponential. In other words:

$$I = A(X - \exp(-t/\tau))$$

- (b) Plug this expression into the differential equation you obtained in (a) in order to confirm that it indeed is a solution and to determine what the time constant  $\tau$  and the constants  $A$  and  $X$  are. What would be a better label for  $A$ ? (HINT: You will also need to use the initial condition for current. What is  $I(t=0)$ ?).
- (c) Now that you know the time dependence for the current  $I$  in the circuit you can also determine the voltage drop  $V_R$  across resistor and the EMF generated by the inductor. Do so, and confirm that your expressions match the plots in Fig. 6a or 2b.

### 5. 'Discharging' an Inductor



After a long time  $T$  the current will reach an equilibrium value and inductor will be “fully charged.” At this point we turn off the battery ( $\mathcal{E}=0$ ), allowing the inductor to ‘discharge,’ as pictured at left. Repeat each of the steps a-c in problem 4, noting that instead of  $\exp(-t/\tau)$ , our expression for current will now contain  $\exp(-(t-T)/\tau)$ .

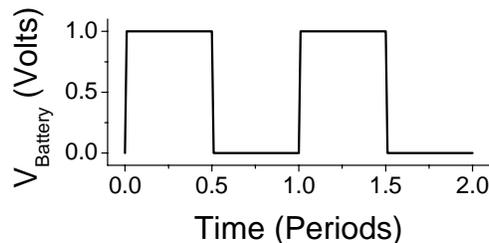
### 6. The Coil

The coil you will be measuring has is made of thin copper wire (radius  $\sim 0.25$  mm) and has about 600 turns of average diameter 25 mm over a length of 25 mm. What approximately should the resistance and inductance of the coil be? The resistivity of copper at room temperature is around  $20$  n $\Omega$ -m. Note that your calculations can only be approximate because this is not at all an ideal solenoid (where length  $\gg$  diameter).

## 7. A Real Inductor

As mentioned above, in this lab you will work with a coil that does not behave as an ideal inductor, but rather as an ideal inductor in series with a resistor. For this reason you have no way to independently measure the voltage drop across the resistor or the EMF induced by the inductor, but instead must measure them together. None-the-less, you want to get information about both. In this problem you will figure out how.

- (a) In the lab you will hook up the circuit of problem 4 with the ideal inductor  $L$  of that problem now replaced by a coil that is a non-ideal inductor – an inductor  $L$  and resistor  $r$  in series. The battery will periodically turn on and off, displaying a voltage as shown here:



Sketch the current through the battery as well as what a voltmeter hooked across the coil would show versus time for the two periods shown above. Assume that the period of the battery turning off and on is comparable to but longer than several time constants of the circuit.

- (b) How can you tell from your plot of the voltmeter across the coil that the coil is not an ideal inductor? Indicate the relevant feature clearly on the plot. Can you determine the resistance of the coil,  $r$ , from this feature?
- (c) In the lab you will find it easier to make measurements if you do NOT use an additional resistor  $R$ , but instead simply hook the battery directly to the coil. (Why? Because the time constant is difficult to measure with extra resistance in the circuit). Plot the current through the battery and the reading on a voltmeter across the coil for this case. We will only bother to measure the current. Why?
- (d) For this case (only a battery & coil) how will you determine the resistance of the coil,  $r$ ? How will you determine its inductance  $L$ ?

***Make sure that you record your answer to 7d in your notes as you will need it for the lab.***

## IN-LAB ACTIVITIES

### EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop (right click on the link and choose “Save Target As” to the desktop. Overwrite any file by this name that is already there). Start LabView by double clicking on this file.
2. Connect the Voltage Sensor to Analog Channel A on the 750 Interface and the Current Sensor to Analog Channel B.
3. Connect cables from the output of the 750 to the banana plug receptacles on the lower right side of the circuit board (red to the sin wave marked output, black to ground).

### MEASUREMENTS

#### **Part 1: Measuring the Resistance of a Single Resistor**

1. Hook up the circuit as you determined it should be set up in Pre-Lab #1 (to measure the voltage across and current through a single resistor driven by the “variable battery.”)
2. Record  $V$  and  $I$  for 1 second. (Press the green “Go” button above the graph). During this time the battery will switch between putting out 1 Volt and 0 Volts.

#### **Question 1:**

When the battery is “on” what is the voltage drop across the resistor and what is the current through it? What is the resistance of the resistor (calculate it from what you just measured, do NOT figure it out from the color code).

#### **Part 2: Resistors in Parallel**

1. Hook up the circuit as you determined it should be set up in Pre-Lab #2 (to measure the voltage across and current through the first resistor connected in parallel to a second resistor)
2. Record  $V$  and  $I$  for 1 second. (Press the green “Go” button above the graph). During this time the battery will switch between putting out 1 Volt and 0 Volts.

**Question 2:**

When the battery is “on” what is the voltage drop across the resistor and what is the current through it? Did these values change from Part 1? Why or why not?

**Part 3: Measuring Voltage and Current in an RC Circuit****3A: Using a Single Resistor**

1. Create a circuit with the first resistor and the capacitor in series with the battery
2. Connect the voltage sensor (still in channel A) across the capacitor
3. Record the voltage across the capacitor  $V$  and the current sourced by the battery  $I$  (Press the green “Go” button above the graph). During this time the battery will switch between putting out 1 Volt and 0 Volts.

**Question 4:**

Using the two-point method (which you calculated in Pre-Lab #3a), what is the time constant of this circuit? Using this time constant, the resistance you measured in Question 1 and the typical expression for an RC time constant, what is the capacitance of the capacitor?

**Question 5:**

Using the logarithmic method (which you calculated in Pre-Lab #3b), what is the time constant of this circuit? Using this time constant, what is the capacitance of the capacitor?

### **3B: Using Two Resistors in Series**

1. Put the second resistor in series with the first resistor and capacitor
2. Connect the voltage sensor (still in channel A) across the capacitor
3. Measure the current sourced by the battery (Press the green “Go” button above the graph).

#### **Question 6:**

Using one of the two methods used above, what is the time constant of this new circuit? What must the resistance of the second resistor be?

### **Part 4: Measure Resistance and Inductance Without a Core**

1. Connect cables from the output of the 750 to either side of the coil (using the clips)
2. Make sure that the core is removed from the coil
3. Record the current through and voltage across the battery for a fraction of a second. (Press the green “Go” button above the graph).

#### **Question 7:**

What is the maximum current during the cycle? What is the EMF generated by the inductor at the time this current is reached?

#### **Question 8:**

What is the time constant  $\tau$  of the circuit?

**Question 9:**

What are the resistance  $r$  and inductance  $L$  of the coil? Calculate this using your answer to Pre-Lab #7d.

**Part 5: Measure Resistance and Inductance With a Core**

1. Insert the core into the center of the coil
2. Record the current through and voltage across the battery for a fraction of a second. (Press the green “Go” button above the graph).

**Question 10:**

Does the maximum current in the circuit change due to the introduction of the core? If it does, try to explain as clearly as possible why this happens (including why the change to bigger or smaller makes sense)

**Question 11:**

Does the time constant  $\tau$  of the circuit change due to the introduction of the core? If it does, try to explain as clearly as possible why this happens (including why the change to longer or shorter makes sense)

**Question 12:**

What are the new resistance  $r$  and inductance  $L$  of the coil?

### **Further Questions (for experiment, thought, future exam questions...)**

- What happens if we instead put the second resistor in parallel with the first?
- What if we instead put the second resistor in parallel with the capacitor? Does the initial current change? The final current? The final voltage (and hence charge) on the capacitor?
- What if we change the order of the elements in the circuit (e.g. put the capacitor between the two resistors, or switch the leads from the battery)?
- The ammeter is marked as having a 1 ohm resistance, small, but not tiny. Can you see the effects of the ammeter resistance in the circuits of part 1 and 2? Can you measure the voltage drop across the ammeter? Does this make the measurement of the current through the resistor inaccurate?
- What happens if we put a resistor  $R$  in series with the coil? In parallel with the coil?
- What happens if you make the battery switch on and off with a period shorter than the time constant of the circuit? Would you still be able to determine the inductance  $L$  and resistance  $r$  of the coil using the same method?
- What happens if you only partially insert the core into the coil? Can you continuously adjust the core's effects or there an abrupt jump from one behavior to another? Would another core (like your finger) have the same effects?
- If the coil were made of some superconducting material, what would its resistance be? Would the EMF you measure be any different? Would the potential difference from one side of the inductor to the other  $\left(\Delta V = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}\right)$  be any different?