MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2006**

Experiment 1: Equipotential Lines and Electric Fields

OBJECTIVES

1. To develop an understanding of electric potential and electric fields
2. To better understand the relationship between equipotentials and electric fields
3. To become familiar with the effect of conductors on equipotentials and E fields

PRE-LAB READING

INTRODUCTION

Thus far in class we have talked about fields, both gravitational and electric, and how we can use them to understand how objects can interact at a distance. A charge, for example, creates an electric field around it, which can then exert a force on a second charge which enters that field. In this lab we will study another way of thinking about this interaction through electric potentials.

The Details: Electric Potential (Voltage)

Before discussing electric potential, it is useful to recall the more intuitive concept of potential energy, in particular *gravitational* potential energy. This energy is associated with a mass's position in a gravitational field (its height). The potential energy *difference* between being at two points is defined as the amount of work that must be done to move between them. This then sets the relationship between potential energy and force (and hence field):

$$\Delta U = U_B - U_A = -\int_A^B \vec{\mathbf{F}} \cdot d\,\vec{\mathbf{s}} \quad \Rightarrow \quad (\text{in 1D}) \;\; F = -\frac{dU}{dz} \tag{1}$$

We earlier defined fields by breaking a two particle interaction, force, into two single particle interactions, the creation of a field and the "feeling" of that field. In the same way, we can define a potential which is created by a particle (gravitational potential is created by mass, electric potential by charge) and which then gives to other particles a potential energy. So, we define electric potential, V, and given the potential can calculate the field:

$$\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} \quad \Rightarrow \quad (\text{in 1D}) \;\; E = -\frac{dV}{dz}\,. \tag{2}$$

Noting the similarity between (1) and (2) and recalling that $\boldsymbol{F} = q\boldsymbol{E}$, the potential energy of a charge in this electric potential must be simply given by $U = qV$.

When thinking about potential it is convenient to think of it as “height” (for gravitational potential in a uniform field, this is nearly precise, since $U = mgh$ and thus the gravitational potential $V = gh$). Electric potential is measured in Volts, and the word “voltage” is often used interchangeably with “potential.” You are probably familiar with this terminology from batteries, which maintain fixed potential differences between their two ends (e.g. 9 V in 9 volt batteries, 1.5 V in AAA-D batteries).

Equipotentials and Electric Fields

When trying to picture a potential landscape, a map of equipotential curves – curves along which the potential is equal – can be very helpful. For gravitational potentials these maps are called topographic maps. An example is shown in Fig. 1b.

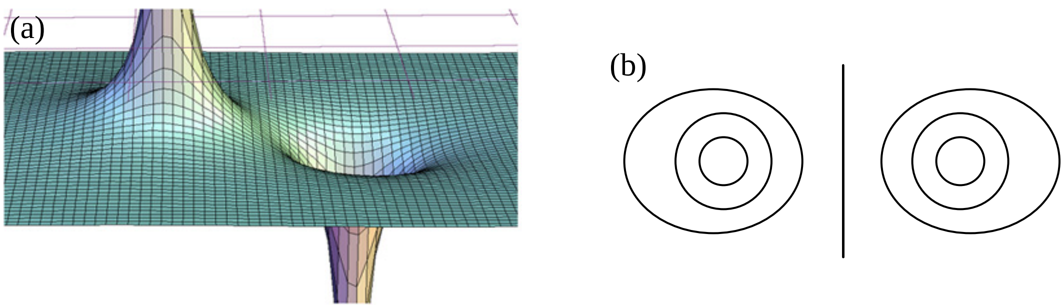


Figure 1: Equipotentials. A potential landscape (pictured in 3D in (a)) can be represented by a series of equipotential lines (b), creating a topographic map of the landscape. The potential (“height”) is constant along each of the curves.

Now consider the relationship between equipotentials and fields. At any point in the potential landscape, the field points in the direction that a mass would feel a force if placed there (or that a positive charge would feel a force for electric potentials and fields). So, place a ball at the top of the hill (near the center of the left set of circles in the topographic map of Fig. 1b). Which way does it roll? Downhill! But what direction is that? Perpendicular to the equipotential lines. Why? Equipotential lines are lines of constant height, so moving along them at all does not achieve the objective of going downhill. So the force (and hence field) must point across them, pushing the object downhill. But why exactly perpendicular? Work done on an object changes its potential, so it can take no work to move along an equipotential line. Work is given by the dot product of force and displacement. For this to be zero, the force must be perpendicular to the displacement, that is, force (and hence fields) must be perpendicular to equipotentials.

Note: Potential vs. Potential Difference

Note that in equation (2) we only defined ΔV, the potential *difference* between two points, and not the potential V. This is because potential is like height – the location we choose to call “zero” is completely arbitrary. In this lab we will choose one location to call zero (the “ground”), and measure potentials relative to the potential at that location.

APPARATUS

1. Conducting Paper Landscapes

To get a better feeling for what equipotential curves look like and how they are related to electric field lines, we will measure sets of equipotential curves for several different potential landscapes. These landscapes are created on special paper (on which you can measure electric potentials) by fixing a potential difference between two conducting shapes on the paper. For reasons that we will discuss later, these conducting shapes are themselves equipotential surfaces, and their shape and relative position determines the electric field and potential everywhere in the landscape. One purpose of this lab is to develop an intuition for how this works. There are four landscapes to choose from (Fig. 2), and you will measure equipotentials on two of them (one from Fig. 1a, b and one from Fig. 1c, d).

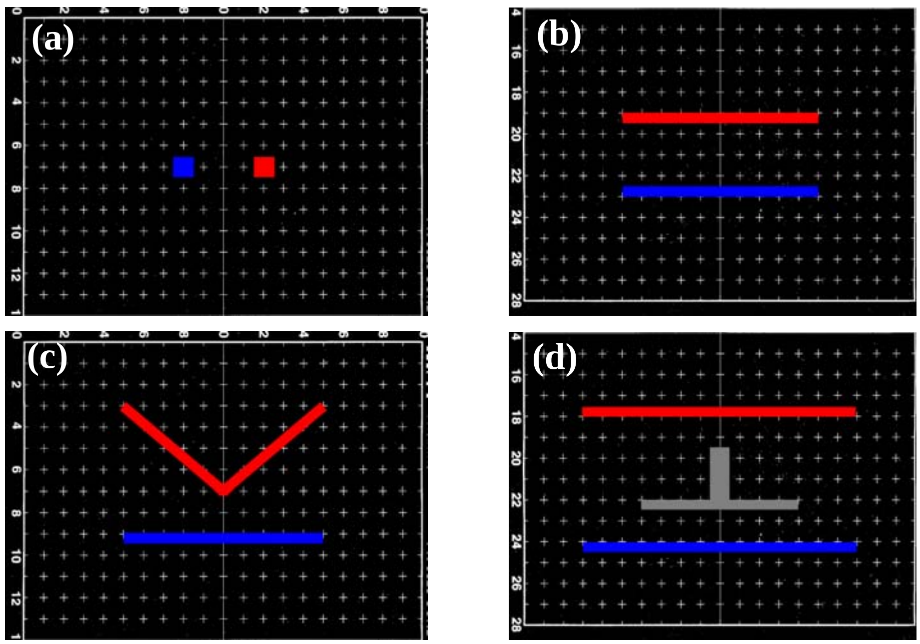


Figure 2 Conducting Paper Landscapes. Each of the four landscapes – the "standard" (a) dipole and (b) parallel plates, and the "non-standard" (c) bent plate and (d) filled plates – consists of two conductors which will be connected to the positive (red) and ground (blue) terminals of a battery. In (d) there is an additional conductor which is free to float to whatever potential is required. The pads are painted on conducting paper with a 1 cm grid.

2. Science Workshop 750 Interface

In this lab we will use the Science Workshop 750 interface (Fig. 3) both to create the potential landscapes (using the "OUPUT" connections that act like a battery) and to measure the potential at various locations in that landscape using a voltage sensor (see below). There are two connections to the output, just like there are two sides of a battery.

The one marked with a sin wave is the positive terminal and the one marked with a triangle (the "ground symbol") is the negative terminal. The potential of this terminal is what we will call zero. We will set the potential difference between the two to 5 V.

Figure 3 The *Science Workshop* 750 Interface.

3. Voltage Sensor

In order to measure the potential as a function of position we use a voltage sensor (Fig. 4). One side of the sensor plugs into Channel A on the 750, the other has two leads, red and black. When the 750 records the "potential," it really measures the potential difference between the two leads, the potential at the red lead minus that at the black lead.

Figure 4 Voltage Sensor

GENERALIZED PROCEDURE

For each of the two landscapes that you choose, you will find at least four equipotential contours by searching for points in the landscape at the same potential using the voltage sensor. After recording these curves, you will draw several electric field lines, making use of the fact that they are everywhere perpendicular to equipotential contours.

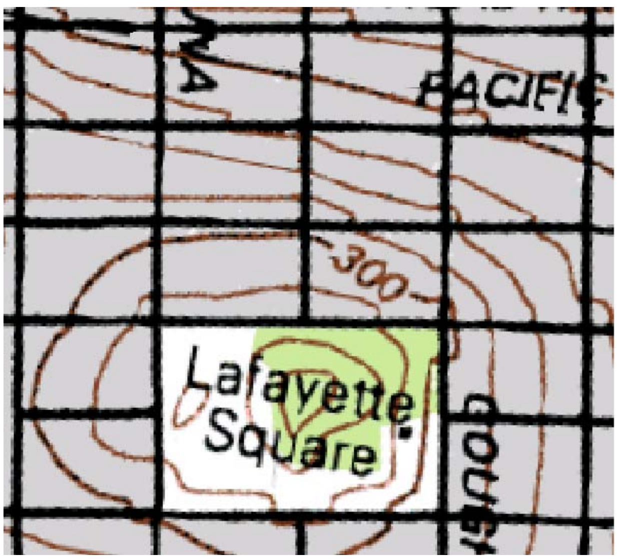
END OF PRE-LAB READING

Expt. 1: Equipotential Lines and Electric Fields Pre-Lab Questions

Answer these questions on a separate sheet of paper and turn them in before the lab

1. Equipotentials Curves – Reading Topographic Maps

Below is a topographic map of a 0.4 mi square region of San Francisco. The contours shown are separated by heights of 25 feet (so from 375 feet to 175 feet above sea level for the region shown)



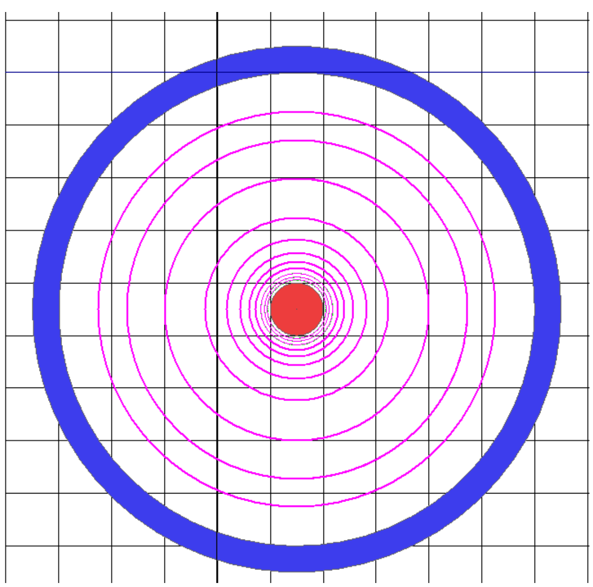
From left to right, the NS streets shown are Buchanan, Laguna, Octavia, Gough and Franklin. From top to bottom, the EW streets shown are Broadway, Pacific, Jackson, Washington, Clay (which stops on either side of the park) and Sacramento.

(a) In the part of town shown in the above map, which street(s) have the steepest runs? Which have the most level sections? How do you know?

(b) How steep is the steepest street at its steepest (what is its slope in ft/mi)?

(c) Which would take more work (in the physics sense): walking 3 blocks south from Laguna and Jackson or 1 block west from Clay and Franklin?

2. Equipotentials, Electric Fields and Charge



One group did this lab and measured the equipotentials for a slightly different potential landscape then the ones you have been given (although still on a 1 cm grid) and using +10 V rather than +5V.

Note that they went a little overboard and marked equipotential curves (the magenta circles) at V = 0.25 V, 0.5V and then from V = 1 V to V = 10 V in 1 V increments.

They followed the convention that red was their positive electrode (V = +10 V) and blue was ground (V = 0 V).

(a) Copy the above figure and sketch eight electric field lines on it (equally spaced around the inner conductor).

(b) What, approximately, is the magnitude of the electric field at r = 1 cm, 2 cm, and 3 cm, where r is measured from the center of the inner conductor? You should express the field in V/cm. (HINT: The field is the local slope (derivative) of the potential. Also, if you choose to use a ruler realize that the above reproduction of this group's results is not the same size as the original, where the grid size was 1 cm).

(c) What is the relationship between the density of the equipotential lines, the density of the electric field lines, and the strength of the electric field?

(d) Plot the field strength vs. $1/r^2$ for the three points from part (b). If the field were created by a single point charge what shape should this sketch be? Is it?

(e) Approximately how much charge was on the inner conductor when the group made their measurements? Make sure you include the sign as well as the amplitude. (HINT: Use the sketch of (d))

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop (right click on the link and choose "Save Target As"). Start LabView by double clicking
2. Connect cables to the output of the 750 (red to the sin wave marked output, black to ground). One member of the group will hold these wires to the two conductors while another maps out the equipotentials.
3. Connect the Voltage Sensor to Analog Channel A on the 750 Interface
4. Connect the black lead of the voltage sensor to the black output (the ground). You will use the red lead to measure the potential around your landscapes.

MEASUREMENTS

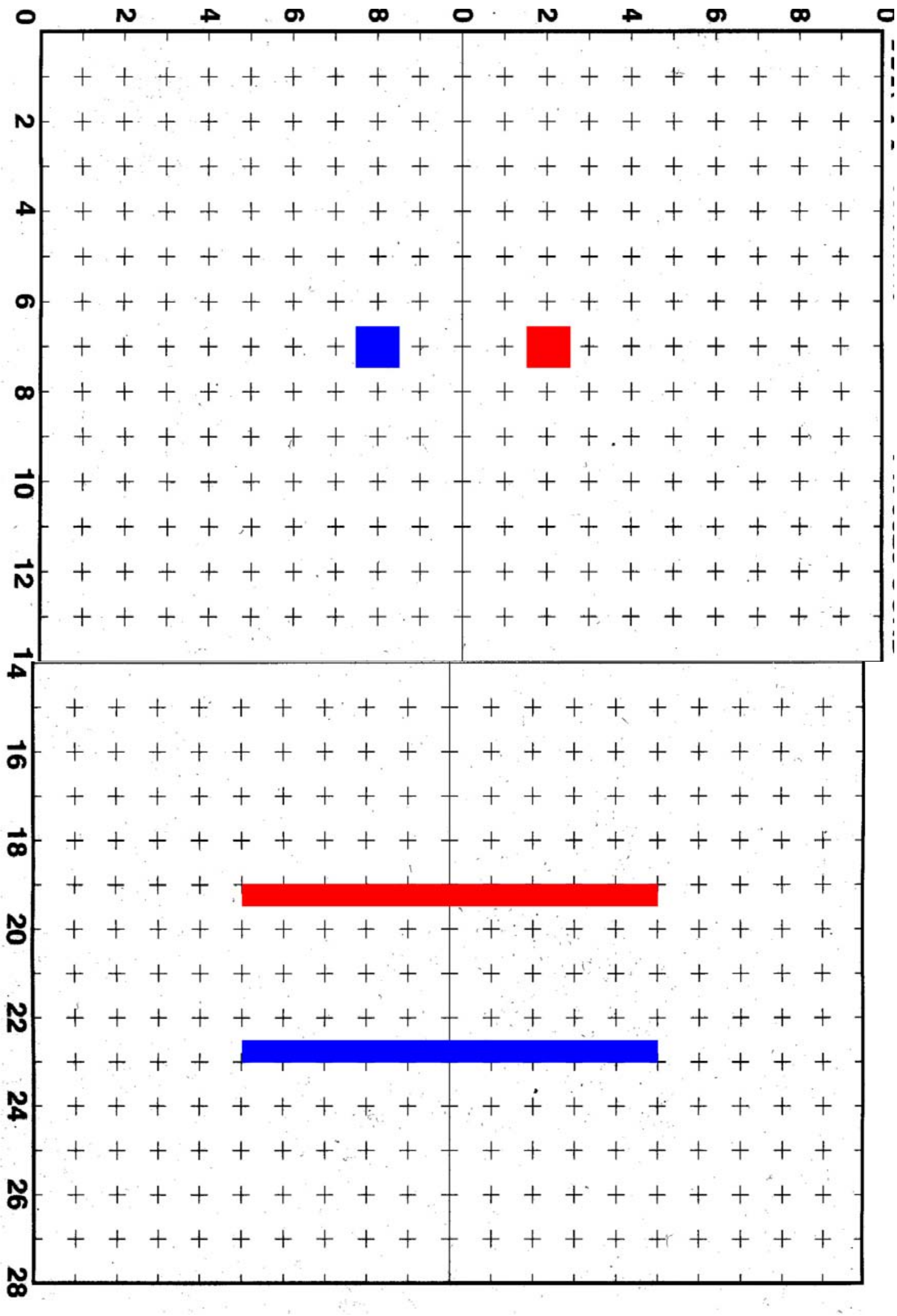
Part 1: "Standard" Configuration

1. Choose one of the two "standard" conducting paper landscapes (the dipole or parallel plate configuration)
2. Use the voltage connectors to make contacts to the two conducting pads
3. Press the green "Go" button above the graph to energize the battery and begin recording the potential of the red lead (relative to the black lead = ground).
4. Measure the potential of both conducting pads to confirm that they are properly connected (one should be at +5 V, the other at 0 V), and that they are indeed equipotential objects (we will explain why next week).
5. Now, try to find some location on the paper that is at about +1 V (don't worry about being too precise). Mark this point on the plot on the next page.

Do NOT write on the conducting paper

6. Find another 1 V point, about 1 cm away. Continue until you have closed the curve or left the page. Sketch and label this equipotential curve.
7. Repeat this process to find equipotentials at 2 V, 3 V, and 4 V. Work pretty fast; it's more important to think about what these lines mean than it is to draw them perfectly. Think about what you are doing – are there symmetries that you can exploit to make this task easier?

“Standard” Configurations



Question 1:

Sketch in a set of electric field lines (~ ten) on your plot of equipotentials on the previous page. Where do the field lines begin and end? If they are equally spaced at their beginning, are they equally spaced at the end? Along the way? Why?

Question 2:

What, approximately, is the potential midway between the two conductors?

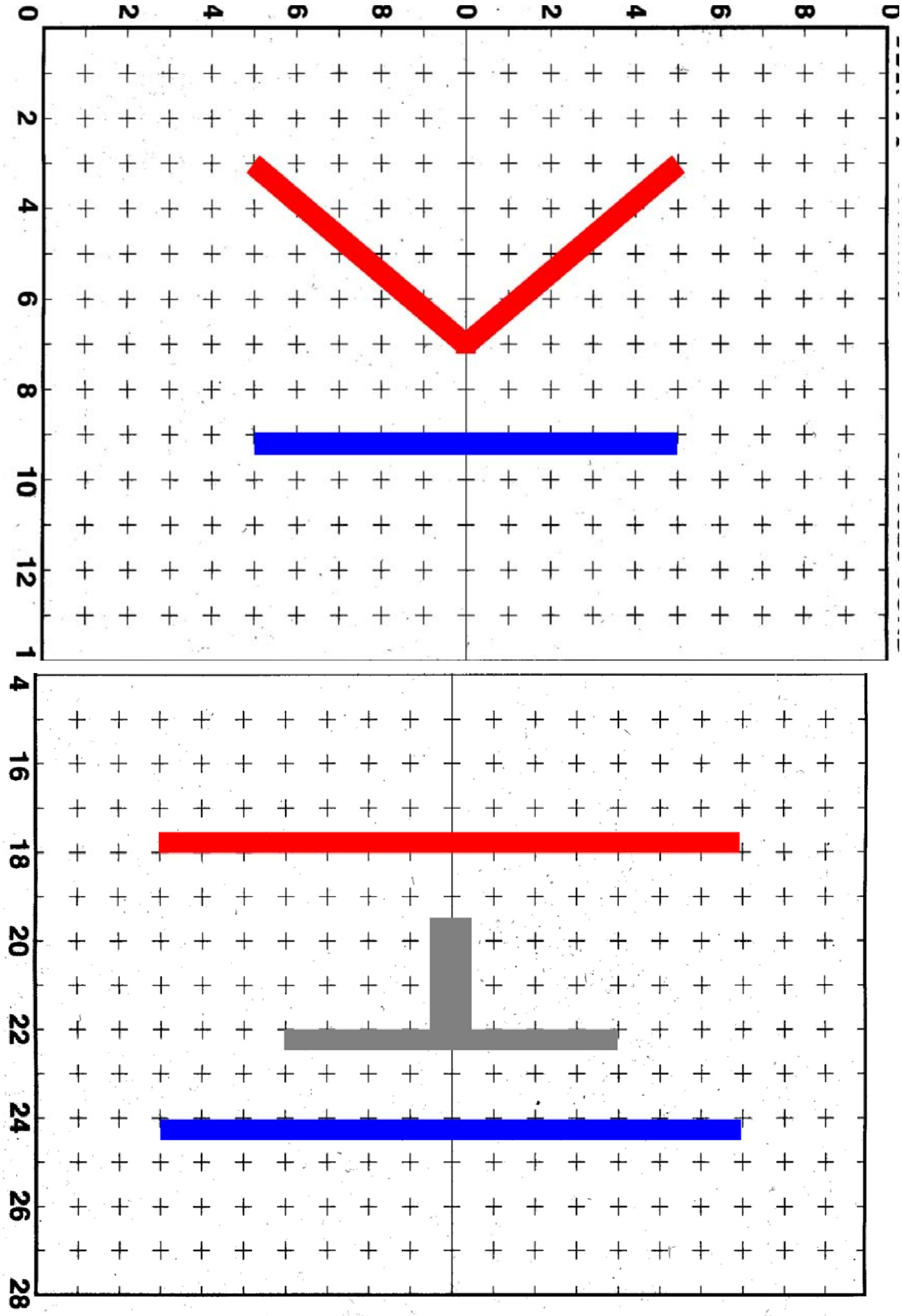
Question 3:

What, approximately, is the strength of the electric field midway between the two conductors? You may find it easier to answer this question if you just measure the potential at a few points near the center.

Part 2: "Non-Standard" Configuration

1. Choose one of the two "non-standard" conducting paper landscapes (the bent plate or filled plates configuration)
2. Use the voltage connectors to make contacts to the two conducting pads (for the filled plates, the center pad does *not* have a connection to it)
3. Press the green "Go" button above the graph to energize the battery and begin recording the potential of the red lead (relative to the black lead = ground).
4. Confirm that everything is properly connected by measuring the potential on the two connected pads, then record a set of equipotential curves following the same procedure of part 1.

“Non-Standard” Configurations



Question 4:

Sketch in a set of electric field lines on your plot of equipotentials on the previous page. Where is the electric field the strongest? What, approximately, is its magnitude?

Question 5:

Where is the electric field the most uniform? How can you tell?

<u>Further Questions (for experimentation, thought, future exam questions…)</u>

- What changes if you switch which conducting pad is at +5 V and which is ground?
- What if you forget to connect the ground lead?
- If you rest your hand on the paper while making measurements, does it affect the readings? Why or why not?
- If you wanted to push a charge along one of the field lines from one conductor to the other, how does the choice of field line affect the amount of work required?
- The potential is everywhere the same on an equipotential line. Is the electric field everywhere the same on an electric field line?

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2006**

Experiment 2: Faraday Ice Pail

OBJECTIVES

1. To explore the charging of objects by friction and by contact.
2. To explore the charging of objects by electrostatic induction.
3. To explore the concept of electrostatic shielding.

PRE-LAB READING

INTRODUCTION

When a charged object is placed near a conductor, electric fields exert forces on the free charge carriers in the conductor which cause them to move. This process occurs rapidly, and ends when there is no longer an electric field inside the conductor ($\mathbf{E}_{\text{inside conductor}}=0$). The surface of the conductor ends up with regions where there is an excess of one type of charge over the other. For example, if a positive charge is placed near a metal, electrons will move to the surface nearest the charge, leaving a net positive charge on the opposite surface[1]. This charge distribution is called an *induced charge distribution*. The process of separating positive from negative charges on a conductor by the presence of a charged object is called *electrostatic induction*.

Michael Faraday used a metal ice pail as a conducting object to study how charges distributed themselves when a charged object was brought inside the pail. Suppose we lower a positively charged metal ball into the pail ***without touching it to the pail***. When we do this, positive charges move as far away from the ball as possible – to the outer surface of the pail, leaving a net negative charge on the inner surface. If at this point we provide some way for the positive charges to flow away from the pail, for example by touching our hand to it, they will run off through our hand. If we then remove our hand from the pail and then remove the positively charged metal ball from inside the pail, the pail will be left with a net negative charge. This is called *charging by induction*.

In contrast, if we touch the positively charged ball to the uncharged pail, electrons flow from the pail into the ball, trying to neutralize the positive charge on it. This leaves the pail with a net positive charge. This is called *charging by contact*.

Finally, when a positively charged ball approaches the ice pail from outside of the pail, charges will redistribute themselves on the outside surface of the pail and will exactly cancel the electric field inside the pail. This is called *electrostatic shielding*.

---

[1] We will typically say that “positive charge flows outward” even though in metals it’s really electrons moving inward. This is a completely equivalent way of thinking about it for our purposes.

You will investigate all three of these phenomena—charging by induction, charging by contact, and electrostatic shielding—in this experiment.

The Details: Gauss's Law

In the above situations, the excess charge on the conductor resides entirely on the surface, a fact that may be explained by Gauss's Law. Gauss's Law[2] states that the electric flux through any closed surface is proportional to the charge enclosed inside that surface,

$$\oiint_{\substack{\text{closed}\\\text{surface}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\varepsilon_0} . \tag{2.1}$$

Consider a mathematical, closed Gaussian surface that is *inside* the ice pail:

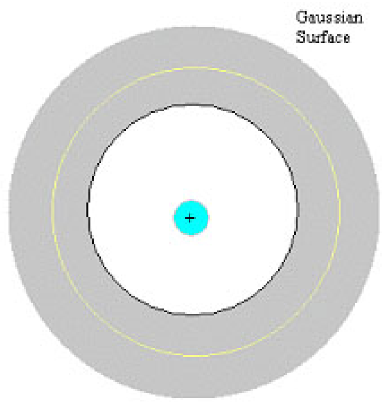


Figure 1 Top View of Gaussian surface for the Faraday Ice Pail (a thick walled cylinder)

Once static equilibrium has been reached, the electric field inside the conducting metal walls of the ice pail is zero. Since the Gaussian surface is in a conducting region where there is zero electric field, the electric flux through the Gaussian surface is zero. Therefore, by Gauss's Law, the net charge inside the Gaussian surface must be zero. For the Faraday ice pail, the positively charged ball is inside the Gaussian surface. Therefore, there must be an additional induced negative charge on the inner surface of the ice pail that exactly cancels the positive charge on the ball. It must reside on the surface because we could make the same argument with any Gaussian surface, including one which is just barely outside the inner surface. Since the pail is uncharged, by charge conservation there must be a positive induced charge on the pail which has the same magnitude as the negative induced charge. This positive charge must reside outside the Gaussian surface, hence on the outer surface of the ice pail.

Note that the electric field in the hollow region inside the ice pail is not zero due to the presence of the charged ball, and that the electric field outside the pail is also not zero, due to the positive charge on its outer surface.

2 For more details on Gauss's Law, see Chapter 4 of the *Course Note*, Section 4.3 for info on conductors.

Now suppose the ice pail is connected to a large conducting object ("ground"):

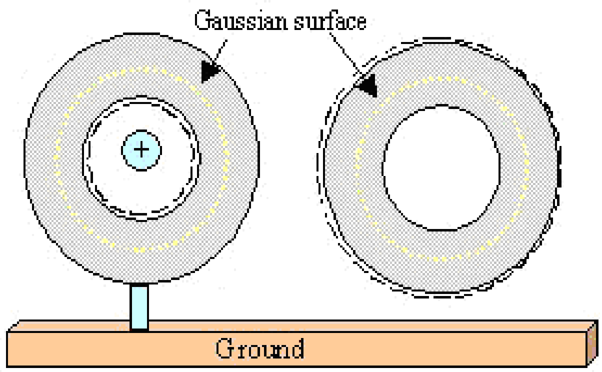


Figure 2 Grounding the ice pail (left) and after removing the ground & ball (right)

Now the positive charges that had moved to the surface of the ice pail can get even further away from the positively charged metal ball by flowing into the ground. Now that there are no charges on the outer surface of the pail, the electric field outside the pail is zero and the pail is at the same "zero" potential as the ground (and infinity). If the wire to ground is then disconnected, the pail will be left with an overall negative charge. Once the positively charged ball is removed, this negative charge will redistribute itself over the outer surface of the pail.

Finally, when a charged ball approaches the ice pail from outside of the pail, charges will redistribute themselves *on the outside surface* of the pail while the electric field inside the pail will remain zero, cut-off from any knowledge of what is going on outside by the enforced zero electric field inside the conductor. This effect is called shielding or "screening" and explains popular science demonstrations in which a person sits safely inside a cage while an enormous voltage is applied to the cage. This same effect explains why metal boxes are used to screen out undesirable electric fields from sensitive equipment.

APPARATUS

1. Ice Pail

Our primary apparatus consists of two concentric wire-mesh cylinders. The inner cylinder (the "pail") is electrically isolated by three insulating rods. The outer cylinder (the "shield") will be attached to ground – charge can flow to or from it as necessary. This cylinder will act both as a screen to eliminate the effect of any external charges and other external fields and as a "zero potential" point, relative to which you will measure the potential of the pail.

Figure 3 The Ice Pail

2. Charge Producers

To replace the positively charged metal ball of Faraday’s experiment, you will use charge producers (Figure 4). When rubbed together a net positive charge will move to one of them and a net negative charge to the other.

Figure 4 One of two charge producers (the other has a blue charged pad)

3. Charge Sensor

The Charge Sensor does not directly measure charge, but instead measures the voltage difference between its positive (red) and negative (black) leads. Furthermore, it connects the black lead to ground, meaning that as much charge can flow into or out of that lead as is necessary to keep it at “zero potential” (ideally the same voltage as at infinity).

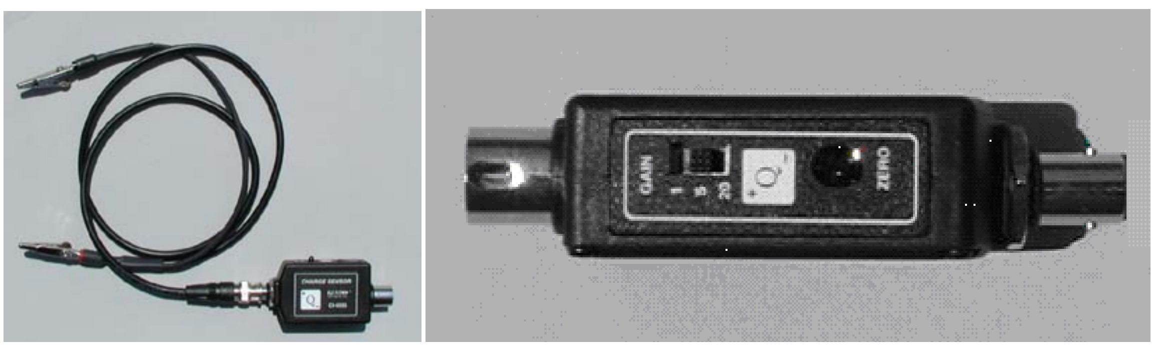


Figure 5 Charge Sensor – measures voltage difference between its red and black leads. Left: Shown attached to the lead assembly. Right: The gain switch (used to amplify small signals) should be set at 1. The zero button sets the output signal to zero.

The red lead is free to be at any potential, although by pushing the “zero” button on the sensor (Fig. 5, right), it too can be attached to ground (the potential difference between the red and black leads is set to zero).

Even though this is really a potential difference sensor, we none-the-less call it a “Charge Sensor” because the voltages measured arise from the presence of charges on the ice pail, as you will calculate in Pre-Lab problem 1.

4. Science Workshop 750 Interface

In this lab we will for the first time use the Science Workshop 750 interface (Fig. 6). This allows signal measurements (in this lab, from the charge sensor) to be sent to the computer where they can be recorded and plotted.

Figure 6 The *Science Workshop* 750 Interface.

GENERALIZED PROCEDURE

This lab consists of four main parts. In each you will measure the voltage between the inner and outer cylinder to determine what is happening on the inner cylinder.

Part 1: Determine Polarity of (Sign of Charge on) Charge Producers
Here you will lower the charge producers into the center of the pail (the inner cylinder) and determine which producer is positively charged and which is negatively charged

Part 2: Charging by Contact
You will now rub the charge producer against the inner surface of the pail and see if the charge is transferred to it.

Part 3: Charging by Induction
In this part you will not let the charge producer touch the pail, but will instead briefly ground the pail by connecting it to the shield (the outer cylinder) while the charge producer is inside. Then you will remove the charge producer and observe the induced charge on the pail.

Part 4: Electrostatic Shielding
In this part you will measure the effects of placing a charge producer outside of the grounded shield.

END OF PRE-LAB READING

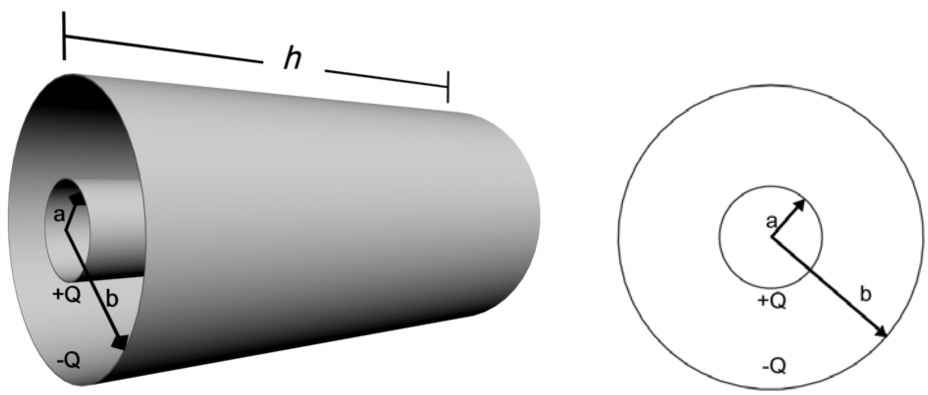
Experiment 2: Faraday Ice Pail Pre-Lab Questions

Answer these questions on a separate sheet of paper and turn them in before the lab

1. Capacitance of our Experimental Set-Up

In this experiment we will measure the potential difference between the pail and the shield, and make statements about the charge on the pail based on this. Here you will calculate the relationship between charge and potential.

Consider two nested cylindrical conductors of height h and radii a & b respectively. A charge $+Q$ is evenly distributed on the outer surface of the pail (the inner cylinder), $-Q$ on the inner surface of the shield (the outer cylinder).



(a) Calculate the electric field between the two cylinders ($a < r < b$).

(b) Calculate the potential difference between the two cylinders:

$$\Delta V = V(a) - V(b)$$

(c) Calculate the capacitance of this system, $C = Q/\Delta V$

(d) Numerically evaluate the capacitance for your experimental setup, given:
$h \cong 15$ cm, $a \cong 4.75$ cm and $b \cong 7.25$ cm

The capacitance should be given in Farads (1 F = 1 Coulomb/Volt) or some fraction thereof (mF, μF, …) ***<u>Write this number in your notes</u>***, as you will use it to convert from the measured voltage difference ΔV to a charge on the outer surface of the pail (inner cylinder) Q, using $Q = C\,\Delta V$

2. What about charge on other surface?

In the previous problem we assumed that charge was located on the outer surface of the inner cylinder and the inner surface of the outer one, in other words, if both cylinders were charged. What if instead the cylinders were both neutral but exhibited charge separation due to the presence of a positive charge Q sitting in the interior of the inner cylinder? What now is the potential difference between the two cylinders?

For the next three questions you are asked to sketch potential difference versus time as you move charges into or around the system. You should mark the given actions on the time axis (assume they are each done very quickly but with some time in between) and indicate what happens to the potential difference when each action is made. In each case the potential difference should start at zero volts. NOTE: Of course you can't give any numbers, we just want you to indicate if the potential increases or decreases or goes to zero or back to some previous value.

3. Prediction: Charging by Contact

Sketch your prediction for the graph of potential difference vs. time for part 2 of this experiment. Indicate the following events on the time axis:

- (a) Insert positive charge producer into pail
- (b) Rub charge producer against inner surface of pail
- (c) Remove charge producer

4. Prediction: Charging by Induction

Sketch your prediction for the graph of potential difference vs. time for part 3 of this experiment. Indicate the following events on the time axis:

- (a) Insert positive charge producer into pail
- (b) Ground pail to shield
- (c) Remove ground contact between pail and shield
- (d) Remove charge producer

5. Prediction: Electrostatic Shielding

Sketch your prediction for the graph of potential difference vs. time for part 4 of this experiment. Indicate the following events on the time axis:

- (a) Bring positive charge producer near (but outside of) shield
- (b) Move charge producer away

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop (right click on the link and choose "Save Target As" to the desktop. Overwrite any file by this name that is already there). Start LabView by double clicking on this file.
2. Using the multi-pin cable, connect the Charge Sensor to Analog Channel A on the 750 Interface. The cable runs from the left end of the sensor (in Fig. 5) to Channel A.
3. Connect the lead assembly to the BNC port on the Charge Sensor (right end of the sensor in Fig. 5). Line up the connector on the end of the cable with the pin on the BNC port. Push the connector onto the port and twist it clockwise about one-quarter turn until it clicks into place. Set the Charge Sensor gain to 1x.
4. Connect the charge sensor input lead (red alligator clip) to the pail (the inner wire mesh cylinder), and the ground lead (black alligator clip) to the shield (the outer wire mesh cylinder).

MEASUREMENTS

Important Notes:

The charge producers are delicate. When rubbing them together do so briskly but gently.

Each experiment should begin with completely discharged cylinders. **To discharge them, ground the pail by touching both it and the shield at the same time with a conductor (e.g. the finger of one hand). You also will always want to zero the charge sensor before starting by pressing the "Zero" button.**

<u>Part 1: Polarity of the Charge Producers</u>

1. Ground the pail and zero the charge sensor
2. Start recording data. (Press the green "Go" button above the graph).
3. Rub the blue and white surfaces of the charge producers together several times.
4. Without touching the pail, lower the white charge producer into the pail.
5. Remove the white charge producer and then lower in the blue charge producer

Question 1 (Don't forget to submit answers in the software!):

What are the polarities of the white and the blue charge producers?

Note: There may be some variations in this from group to group.

Part 2: Charging By Contact

Part 2A: Using the White Charge Producer

1. Ground & zero; Start recording; Rub the producers
2. Lower the *white* charge producer into the pail
3. Rub the charge producer against the inner surface of the pail
4. Remove the charge producer

Question 2: Sketch the plot of voltage vs. time and indicate the charge on the inner and outer surfaces of the pail after each of the above steps

Answer:

Charge on inner & outer surfaces of the inner cylinder (indicate sign, and use a variable like q for non-zero magnitudes – do NOT simply record numerical values)

After Step 1: Q_{inner} = Q_{outer} =

After Step 2: Q_{inner} = Q_{outer} =

After Step 3: Q_{inner} = Q_{outer} =

After Step 4: Q_{inner} = Q_{outer} =

Part 2B: Using the Blue Charge Producer

1. Ground & zero; Start recording; Rub the producers
2. Lower the *blue* charge producer into the inner cylinder
3. Rub the charge producer against the inner surface of the inner cylinder
4. Remove the charge producer

Question 3:

What happens to the charge on the pail when you rub it with the blue charge producer?

Part 3: Charging By Induction

Part 3A: Using the White Charge Producer

1. Ground & zero; Start recording; Rub the producers
2. Lower the *white* charge producer into the pail, without touching it
3. Ground the pail by connecting it to the shield with your finger
4. Remove the ground connection (your finger)
5. Remove the charge producer

Question 4:

Sketch the plot of voltage vs. time and indicate the charge on the inner and outer surfaces of the pail after each of the above steps

Answer:

Charge on inner & outer surfaces of the inner cylinder (indicate sign, and use a variable like q for non-zero magnitudes – do NOT simply record numerical values)

After Step 1: Q_{inner} = Q_{outer} =

After Step 2: Q_{inner} = Q_{outer} =

After Step 3: Q_{inner} = Q_{outer} =

After Step 4: Q_{inner} = Q_{outer} =

After Step 5: Q_{inner} = Q_{outer} =

3B: Using the Blue Charge Producer

1. Ground & zero; Start recording; Rub the producers
2. Lower the *white* charge producer into the pail, without touching it
3. Ground the pail by connecting it to the shield with your finger
4. Remove the ground connection (your finger)
5. Remove the charge producer

Question 5:

What happens to the charge on the pail when you do the above steps?

Part 4: Testing the shield

1. Ground & zero; Start recording; Rub the producers
2. Bring the *white* charge producer to just outside the shield (the outer cylinder) ***Do Not Touch it!***
3. Repeat, bringing the *blue* charge producer just outside the shield.

Question 6:

What happens to the charge on the pail when the white charge producer is placed just outside the shield? Will an induced charge distribution appear on the pail? Explain your reasoning. Will an induced charge distribution appear on the shield? Are we sensitive to this? What about the blue charge producer?

Further Questions (for experiment, thought, future exam questions…)

- What happens if we repeat the above measurements with the ground (black clip) attached to the pail and the red clip attached to the shield? Does anything change aside from the sign of the voltage difference?
- What happens if in part 2 we touch the charge producer to the outside of the pail rather than the inside?
- What happens if we place the charge producer between the pail & shield rather than inside the pail?
- What happens if we put both the white & blue charge producers inside the pail together (not touching, just both inside). Is the cancellation exact? Should it be?
- What if in part 2 we touch the white producer and then the blue producer to the pail? What if we touch the white producer, then recharge it and touch again? Doing this repeatedly, is there a difference between touching the inside of the pail and the outside of the pail?

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2006**

Experiment 3: Magnetic Fields of a Bar Magnet and Helmholtz Coil

OBJECTIVES

1. To learn how to visualize magnetic field lines using compasses and a gauss meter
2. To examine the field lines from bar magnets and see how they add
3. To examine the field lines from a Helmholtz coil and understand the difference between using it in Helmholtz and anti-Helmholtz configurations.

PRE-LAB READING

INTRODUCTION

In this lab we will measure magnetic field lines using two methods. First, we will use small compasses that show the direction, but not magnitude, of the local magnetic field. Next we will use a gauss meter, which measures the magnitude of the magnetic field along a single, specific axis and thus does not allow as easy a visualization of the magnetic field direction. We will measure fields both from bar magnets and from a Helmholtz coil.

APPARATUS

1. Mini-Compass

You will receive a bag of mini-compasses (Fig. 1a) that indicate the magnetic field direction by aligning with it, with the painted end of the compass needle pointing away from magnetic north (i.e. pointing in the direction of the magnetic field). Conveniently, the magnetic south pole of the Earth is very close to its geographic north pole, so compasses tend to point North (Fig. 1b). Note that these compasses are cheap (though not necessary inexpensive) and sometimes either point in the direction opposite the way they should, or get completely stuck. Check them out before using them.

(a)

(b)

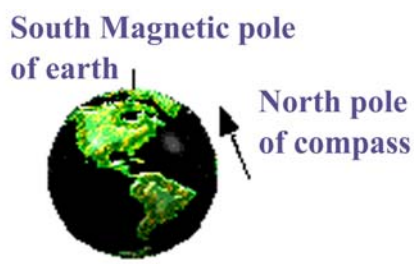


Figure 1 (a) A mini-compass like the ones we will be using in this lab. (b) The painted end of the compass points north because it points towards magnetic south.

2. Science Workshop 750 Interface

As always, we will use the Science Workshop 750 interface, this time for recording the magnetic field magnitude as measured by the magnetic field sensor (gauss meter).

3. Magnetic Field Sensor

The magnetic field sensor measures the strength of the magnetic field pointing into one of two white dots painted at its measurement end (far left in Fig. 2). Selecting "radial" mode records the strength of the field pointing into the dot on the side of the device, while "axial" records the strength of the field pointing into the dot on the end. There is also a tare button which sets the current field strength to zero (i.e. measures relative to it).

Figure 2 Magnetic field sensor, showing (from right to left) the range select switch, the tare button, and the radial/axial switch, which is set to radial.

4. Helmholtz Coil

Consider the Helmholtz Coil Apparatus shown in Fig. 3. It consists of two coaxial coils separated by a distance equal to their common radii. The coil can be operated in 3 modes. In the first, connections are made only to one set of banana plugs, pushing current through only one of the coils. In the second, a connection is made between the black plug from one coil to the red plug from the other. This sends current the same direction through both coils and is called "Helmholtz Mode." In the final configuration "Anti-Helmholtz Mode" a connection is made between the two black plugs, sending current in the opposite direction through the two coils.

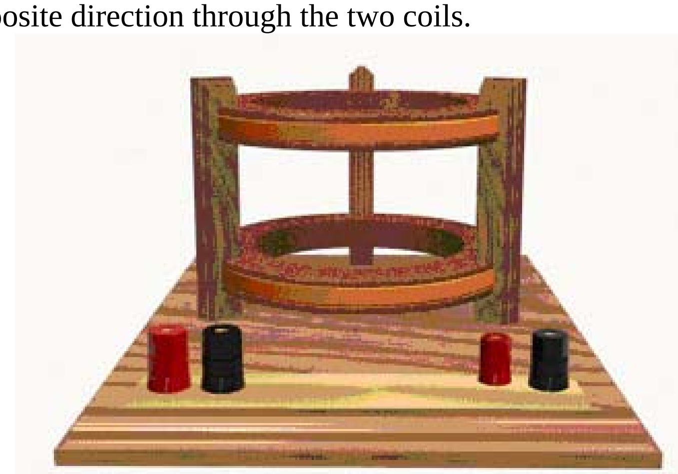


Figure 3 Helmholtz Coil Apparatus

4. Power Supply

Because the Helmholtz coils require a fairly large current in order to create a measurable field, we are unable to use the output of the 750 to drive them. For this reason, we will use an EZ dc power supply (Fig. 4). This supply limits both the voltage and the current, putting out the largest voltage possible consistent with both settings. That is, if the output is open (no leads connected, so no current) then the voltage output is completely determined by the voltage setting. On the other hand, if the output is shorted (a wire is placed between the two output plugs) then the voltage is completely determined by the current setting ($V = IR_{short}$).

Figure 4 Power Supply for Helmholtz Coil The power supply allows independent control of current (left knob) and voltage (right knob) with whichever limits the output the most in control. The green light next to the "CV" in this picture means that we are in "constant voltage" mode – the voltage setting is limiting the output (which makes sense since the output at the bottom right is not hooked up so there is currently no current flow).

GENERALIZED PROCEDURE

This lab consists of three main parts. In each you will measure the magnetic field generated either by bar magnets or by current carrying coils.

Part 1: Mapping Magnetic Field Lines Using Mini-Compasses
Using a compass you will follow a series of field lines originating near the north pole of a bar magnet.

Part 2: Constructing a Magnetic Field Diagram
A pair of bar magnets are placed so that either their opposite poles or same poles are facing each other and you will map out the field lines from these configurations.

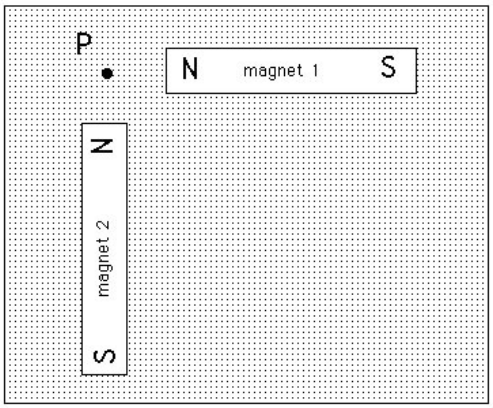
Part 3: Helmholtz Coil
In this part you will use the magnetic field sensor to measure the amplitude of the magnetic field generated from three different geometries of current carrying wire loops.

END OF PRE-LAB READING

Experiment 3: Magnetic Fields Pre-Lab Questions

Answer these questions on a separate sheet of paper and turn them in before the lab

1. Superposition



Consider two bar magnets placed at right angles to each other, as pictured at left.

(a) If a small compass is placed at point P, what direction does the painted end of the compass needle point?

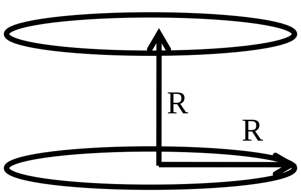
(b) If the compass needle instead pointed 15 degrees clockwise of where you predicted in (a), what could you *qualitatively* conclude about the relative strengths of the two magnets?

2. Helmholtz Coil

In class you calculated the magnetic field along the axis of a coil to be given by:

$$B_{axial} = \frac{N\,\mu_0\,I\,R^2}{2}\frac{1}{(z^2+R^2)^{3/2}}$$

where z is measured from the center of the coil.



As pictured at left, a Helmholtz coil is created by placing two such coils (each of radius *R* and *N* turns) a distance *R* apart.

(a) If the current in the two coils is parallel (Helmholtz configuration), what is the axial field strength at the center of the apparatus (midway between the two coils)? How does this compare to the field strength at the center of the single coil configuration (e.g. what is the ratio)?

(b) In the anti-Helmholtz configuration the current in the two coils is anti-parallel. What is field strength at the center of the apparatus in this situation?

(c) Our coils have a radius *R* = 7 cm and *N* = 168 turns, and we will run with *I* = 0.6 A in single coil and 0.3 A in Helmholtz and anti-Helmholtz mode. What, approximately, are the largest on-axis fields we should expect in these three configurations? Where (approximately) are the fields the strongest? ***Write the answer to this question in your notes. You will need it for the lab.***

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

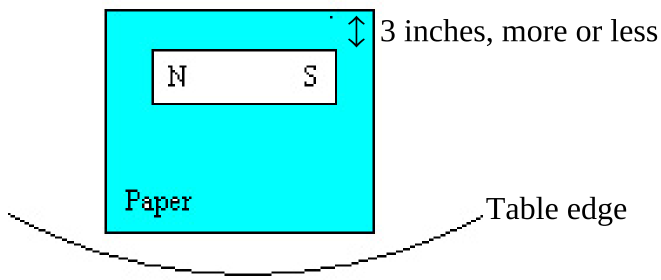
1. Download the LabView file and start up the program.
2. Connect the Magnetic Field Sensor to Analog Channel A on the 750 Interface
3. Without leads connected to the power supply, turn it on and set the voltage output to 2 V. Turn it off.

NOTE: When working with bar magnets, please do NOT force a north pole to touch a north pole (or force south poles to touch), as this will demagnetize the magnets.

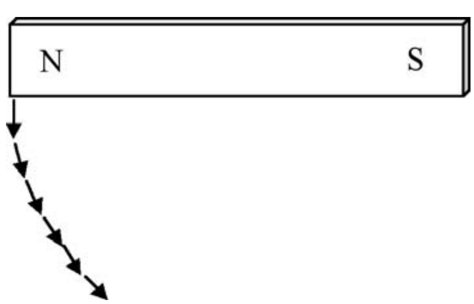
MEASUREMENTS

Part 1: Mapping Magnetic Field Lines Using Mini-Compasses

1. Tape a piece of brown paper (provided) onto your table.
2. Place a bar magnet about 3 inches from the far side of the paper, as shown below. Trace the outline of the magnet on the paper.



3. Place a compass near one end of the magnet. Make two dots on the paper, one at the end of the compass needle next to the magnet and the second at the other end of the compass needle. Now move the compass so that the end of the needle that was next to the magnet is directly over the second dot, and make a new dot at the other end of the needle. Continue this process until the compass comes back to the magnet or leaves the edge of the paper. Draw a line through the dots and indicate with an arrowhead the direction in which the North end of the needle pointed, as shown below



4. Repeat the process described above several more times (~4 field lines), starting at different locations on the magnet. Work fairly quickly – it is more important to get a feeling for the shape of the field lines than to map them precisely.

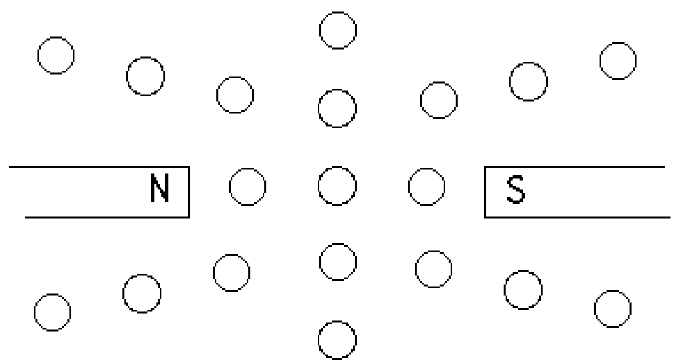
Question 1:

Mostly your field lines come back to the bar magnet, but some of them wander off and never come back to the bar magnet. Which part of your bar magnet do the ones that wander off never to return come from? Where are they going?

Part 2: Constructing a Magnetic Field Diagram

2A: Parallel Magnets

1. Arrange two bar magnets and a series of compasses as pictured here:



2. Sketch the compass needles' directions in the diagram. Based on these compass directions, sketch in some field lines.

Question 2:

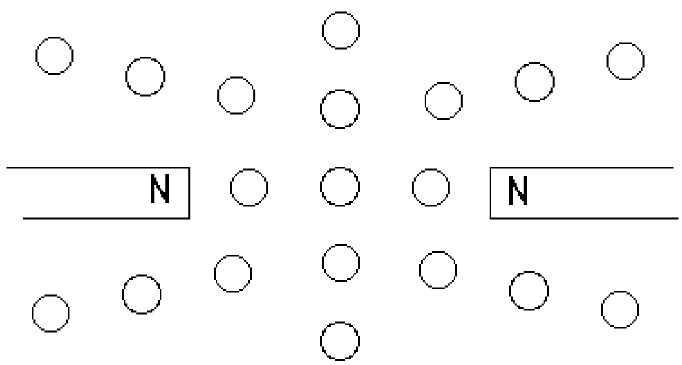
Is there any place in this region where the magnetic field is zero? If so, where? How can you tell?

Question 3:

Where is the magnetic field the strongest in this situation? How can you determine this from the field lines?

2B: Anti-Parallel Magnets

1. Arrange two bar magnets and a series of compasses as pictured here:



2. Sketch the compass needles' directions in the diagram. Based on these compass directions, sketch in some field lines.

Question 4:

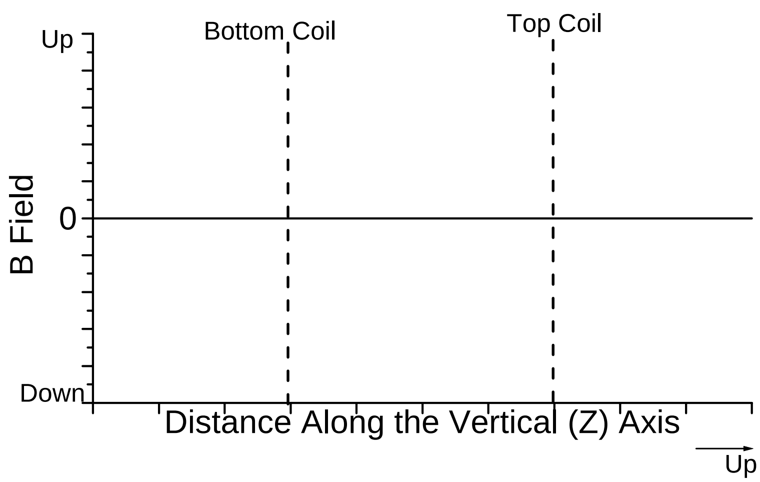
Is there any place in this region where the magnetic field is zero? If so, where? How can you tell?

Part 3: Helmholtz Coil

In this part we are going to measure the z-component of the field along the z-axis (central axis of the coils)

3A: Using a Single Coil

1. With the power supply off, connect the red lead from the power supply to the red plug of the top coil, and the black lead to the black plug of the top coil. Turn the current knob fully counter-clockwise (i.e. turn off the current) then turn on the power supply and slowly turn the current up to ~ 0.6 A.
2. Put the magnetic field sensor in axial mode, set its gain to 10x and place it along the central axis of the Helmholtz coil, pushing into the indentation at the center of the holder. Tare it to set the reading to zero.
3. Start recording magnetic field (press Go) and raise the magnetic field sensor smoothly along the z-axis until you are above the top coil. Try raising at different rates to convince yourself that this only changes the time axis, and not the measured magnitude of the field.
4. Sketch the results for field strength vs. position

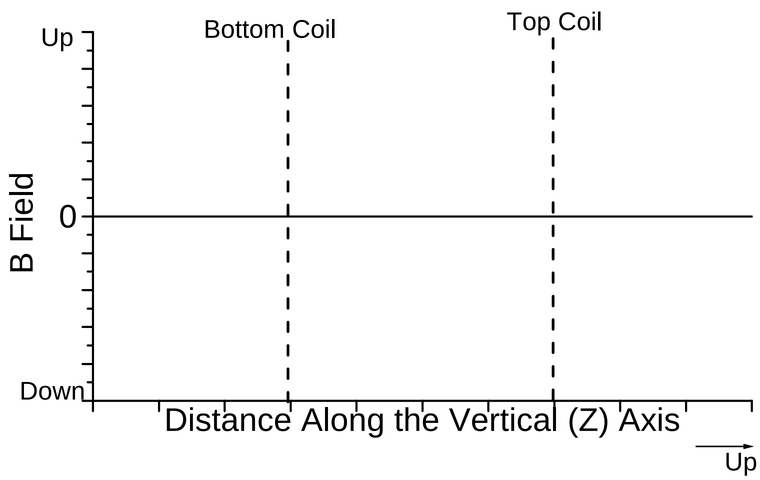


Question 5:

Where along the axis is the field from the single coil the strongest? What is its magnitude at this location? How does this compare to your pre-lab prediction?

3B: Helmholtz Configuration

1. Move the black lead to the black terminal of the lower coil, and connect a lead from the black terminal of the upper to the red terminal of the lower, sending current in the same direction through both coils. Set the current to ~ 0.3 A
2. Follow the procedure in 3A to again measure field strength along the z-axis, plotting on the below figure.

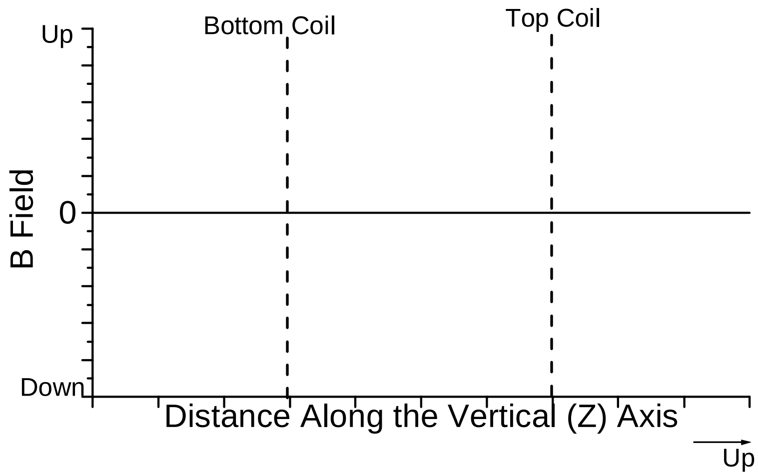


Question 6:

Where along the axis is the field from the strongest? What is its magnitude at this location? How does this compare to your pre-lab prediction. Aside from the location and strength of the maximum, is there a qualitative difference between the single coil and the Helmholtz coil field profile? If so, what is the difference?

3C: Anti-Helmholtz Configuration

1. Swap the leads to the lower coil, keeping the current at ~ 0.3 A, although now running in opposite directions in the top and bottom coil.
2. Follow the procedure in 3A to again measure field strength along the z-axis.



Question 7:

What are two main differences between the field profile in Anti-Helmholtz configuration and in Helmholtz configuration? Does the maximum field strength match your prediction from the pre-lab?

<u>Further Questions (for experiment, thought, future exam questions…)</u>

- What does the field profile look like if we place two bar magnets next to each other rather than collinear with each other (either parallel or anti-parallel to each other).
- What does the radial field profile (e.g. the x component of the field) look like along the z-axis of the Helmholtz coil?
- What do the radial and axial field profiles look like moving across the top of the Helmholtz coil rather than down its central axis?
- It looks as though there is a local maximum of magnetic field strength at some point on the axis for both the single coil and Helmholtz coil configurations (at least looking at them along the z-axis only). If we consider them three dimensionally are they still local maxima? That is, if we move off axis does the magnitude of the field also decrease as we move away from these maxima points?

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2006**

Experiment 4: Forces and Torques on Magnetic Dipoles

OBJECTIVES

1. To observe and measure the forces and torques acting on a magnetic dipole placed in an external magnetic field.

PRE-LAB READING

INTRODUCTION

In this lab you will suspend a magnetic dipole (a small but strong bar magnet) in the field of a Helmholtz coil (the same apparatus you used in Expt. 4). You will observe the force and torque on the dipole as a function of position, and hence external field.

The Details: Magnetic Dipoles in External Fields

As we have discussed in class, magnetic dipoles are characterized by their dipole moment $\boldsymbol{\mu}$, a vector that points in the direction of the B field generated by the dipole (at the center of the dipole). When placed in an external magnetic field B, they have a potential energy

$$U_{Dipole} = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$$

That is, they are at their lowest energy ("happiest") when aligned with a large external field

Torque

When in a non-zero external field the dipole will want to rotate to align with it. The magnitude of the torque which leads to this rotation is easily calculated:

$$\tau = \frac{dU}{d\theta} = -\frac{d}{d\theta}\mu B\cos(\theta) = \mu B\sin(\theta) = \left|\vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}\right|$$

Again, the direction of the torque is such that the dipole moment rotates to align with the field (perpendicular to the plane in which $\boldsymbol{\mu}$ and $\boldsymbol{B}$ lie, and obeying the right hand rule that if your thumb points in the direction of the torque, your fingers rotate from $\boldsymbol{\mu}$ to $\boldsymbol{B}$.

Force

In order to feel a force, the potential energy of the dipole must change with a change in its position. If the magnetic field **B** is constant, then this will not happen, and hence *the dipole feels no force in a uniform field*. However, if the field is non-uniform, such as is created by another dipole, then there can be a force. In general, the force is quite complex, but for a couple of special cases it is simple:

1) If the dipole is aligned with the external field it seeks higher field
2) If the dipole is anti-aligned it seeks lower field

These rules can be easily remembered just by remembering that the dipole always wants to reduce its potential energy. They can also be remembered by thinking about the way that the poles of bar magnets interact – opposites attract while likes repel.

In one dimension, when the dipole is aligned with the field, a rather straight forward mathematical expression may also be derived:

$$F = -\frac{dU}{dz} = \frac{d}{dz}\mu B = \mu \frac{dB}{dz}$$

Here it is important to note that the magnitude of the force depends not on the field but on the derivative of the field. Aligned dipoles climb uphill. The steeper the hill, the more force they feel.

APPARATUS

1. Teach Spin Apparatus

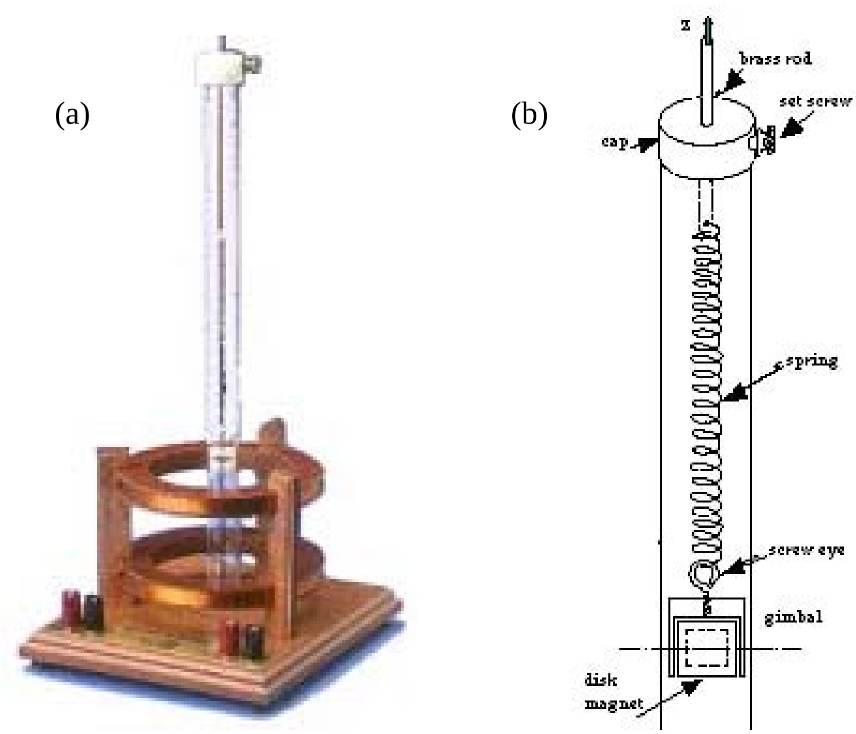


Figure 1 The Teach Spin Apparatus (a) The Helmholtz apparatus has a tower assembly (b) placed along its central axis. The tower contains a disk magnet which is free to rotate (on a gimbal) about an axis perpendicular to the tube and constrained to move vertically. The amount of motion can be converted into a force knowing the spring constant of the spring.

The central piece of equipment used in this lab is the Teach Spin apparatus (Fig. 1). It consists of the Helmholtz coil that you used in experiment 3, along with a Plexiglas tube containing a magnet on a spring. The magnet can both rotate and move vertically, allowing you to visualize both torques and the forces on dipoles.

It will be useful to recall some results from experiment 3 involving the Helmholtz coil. There are three different modes of operation – you can energize just a single coil, both coils in parallel (Helmholtz configuration) or both coils anti-parallel (anti-Helmholtz). The field profiles (as well as the derivatives of those profiles – necessary for thinking about force) look like the following:

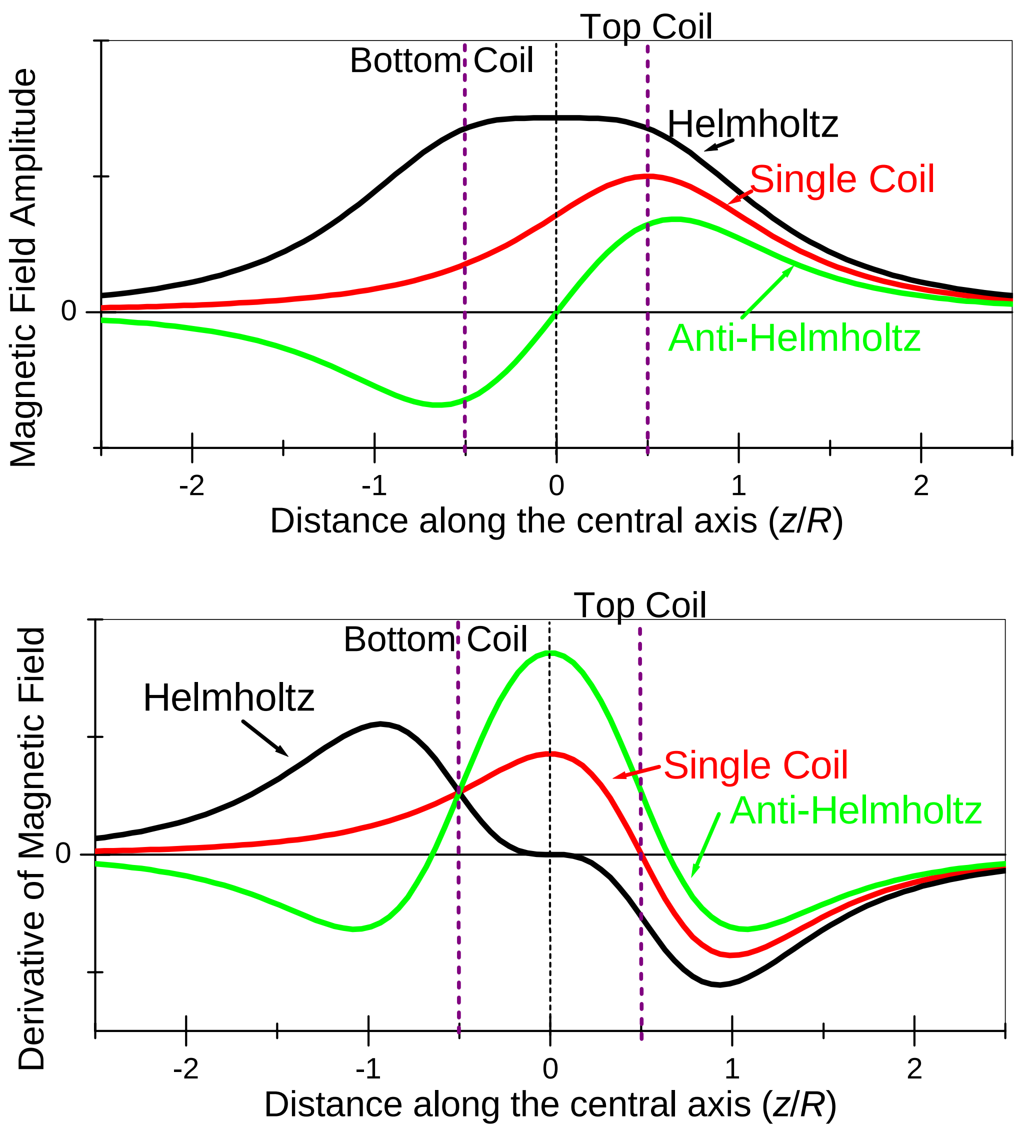


Figure 2: The *z*-component of the magnetic field and its derivative for the three modes of operation of the Helmholtz coil. See page the last page of this write-up for an "iron-filings" representation of these three field configurations.

2. Power Supply

We will also use the same power supply as in experiment 4 in order to create large enough fields in the Helmholtz apparatus to exert a measurable force on the magnet.

GENERALIZED PROCEDURE

This lab consists of five main parts. In each you will observe the effects (torque & force) of different magnetic field configurations on the disk magnet (a dipole).

Part 1: Dipole at center of Helmholtz Coil
You will move the disk magnet to the center of the Helmholtz apparatus and randomly align it and then see what happens when the coil is energized.

Part 2: Reversing the field
You will reverse the direction of the field and see what happens.

Part 3: Moving Through the Helmholtz Apparatus
Here you slowly pull the disk magnet up from the bottom of the Helmholtz apparatus (in Helmholtz mode) and out through the top, observing any torques or forces on the magnet.

Part 4: Dipole at center of Anti-Helmholtz Coil
Here you repeat part 1 in anti-Helmholtz configuration

Part 5: Moving Through the Anti-Helmholtz Apparatus
Here you slowly pull the disk magnet up from the bottom of the Helmholtz apparatus (in anti-Helmholtz mode) and out through the top, observing any torques or forces on the magnet.

END OF PRE-LAB READING

Experiment 4: Forces and Torques on Magnetic Dipoles

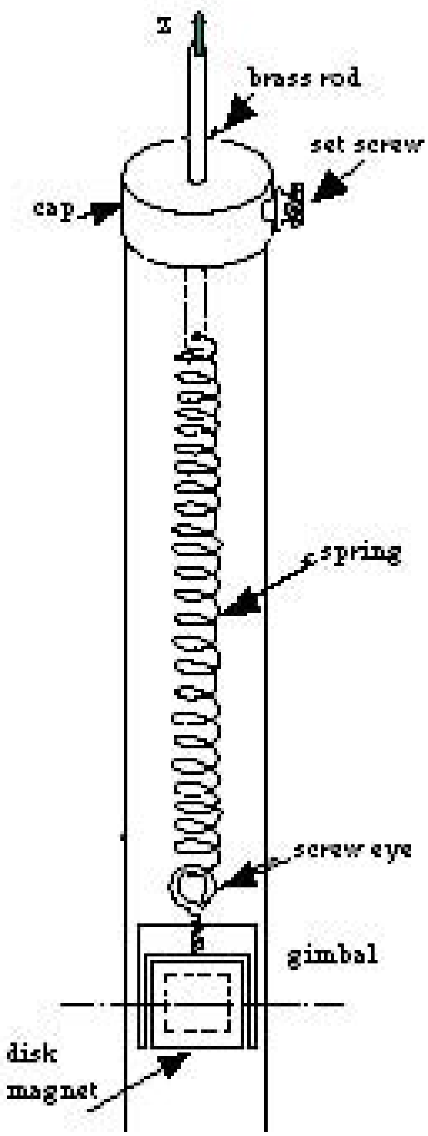
Answer these questions on a separate sheet of paper and turn them in before the lab

1. Force on a Dipole in the Helmholtz Apparatus

In class you calculated the magnetic field along the axis of a coil to be given by:

$$B_{axial} = \frac{N\,\mu_0\,I\,R^2}{2}\frac{1}{(z^2+R^2)^{3/2}}$$

where z is measured from the center of the coil.



In this lab we will have a disk magnet (a dipole) suspended on a spring, which we will use to observe forces on dipoles due to different magnetic field configurations.

(a) Assuming we energize only the top coil (current running counter-clockwise in the coil, creating the field quoted above), and assuming that the dipole is always well aligned with the field and on axis, what is the force on the dipole as a function of position? (HINT: In this situation $F = \mu \; dB/dz$)

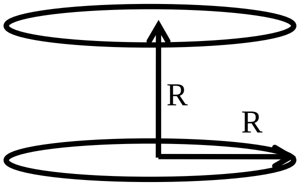
(b) The disk magnet (together with its support) has mass *m*, the spring has spring constant *k* and the magnet has magnetic moment μ. With the current on, we lift the brass rod until the disk magnet is sitting a distance z_0 above the top of the coil. Now the current is turned off. How does the magnet move once the field is off (give both direction and distance)?

(c) At what height(s) is the force on the dipole the largest?

(d) What is the force where the field is the largest?

(e) Our coils have a radius R = 7 cm and N = 168 turns, and the experiment is done with I = 1 A in the coil. The spring constant k ~ 1 N/m, and μ ~ 0.5 A m^2. The mass m ~ 5 g is in the shape of a cylinder ~ 0.5 cm in diameter and ~ 1 cm long. If we place the magnet at the location where the spring is stretched the furthest when the field is on, at about what height will the magnet sit after the field is turned off?

2. Motion of a Dipole in a Helmholtz Field



In Part I of this experiment we will place the disk magnet (a dipole with moment μ) at the center of the Helmholtz Apparatus (in Helmholtz mode). We will start with the disk magnet aligned along the x-axis (perpendicular to the central z-axis of the coils), and then energize the coils with a current of 1 A.

Recall that a Helmholtz coil consists of two coils of radius R and N turns each, separated by a distance R, as pictured above. The field from each coil is given at the beginning of the previous problem.

(a) Will the disk magnet experience a torque, a force or both?

(b) If the magnet experiences a torque:

Approximately how much time will it take for the magnet to rotate 90º, so that it is aligned with the external field? Give your answer first in terms of an approximate expression using R, N, I, and μ, and then numerically, using the values given in problem 1e above.

(c) If the magnet experiences a force:

Approximately how much time will it take for the magnet to move to its new equilibrium position? Give your answer first in terms of an approximate expression using R, N, I, k and μ, and then numerically, using the values given in problem 1e above.

Record the answers to this question in your notes as you will be asked to compare them to what you observe in the lab.

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file and start up the program.
2. Without leads connected to the power supply, turn it on and set the voltage output to 12 V. Turn the current knob fully counter-clockwise (off).
3. Connect the leads to the Helmholtz apparatus, in Helmholtz mode.
4. Increase the current to approximately 1 A, then turn off the power supply (with the push button – do not change the voltage or current settings).

MEASUREMENTS

Part 1: Dipole in Helmholtz Mode

1. Slide the disk magnet to the center of the Helmholtz apparatus (0 on scale)
2. Randomly align the disk magnet using a bar magnet (try to make off axis)
3. Turn on the power supply, carefully watching the disk magnet

Question 1:

Did the disk magnet rotate? (Was there a torque on the magnet?)

Question 2:

Did the spring stretch or compress? (Was there a force on the magnet?)

Part 2: Reversing the Leads

1. Without touching the apparatus (or even bumping the table – be VERY careful) disconnect the leads from the power supply and insert them in the opposite direction (flip the current direction).
2. Carefully watch the dipole as you do this. Repeat the experiment several times.

Question 3:

What happened to the orientation of the disk magnet when you change the current direction in the coils in the Helmholtz configuration? Is this what you expect? Why?

Part 3: Moving a Dipole Along the Axis of the Helmholtz Apparatus

1. Now lower the disk magnet to bottom of the tube
2. Slowly pull the disk magnet up through the apparatus, until it is out the top. While pulling watch both the orientation of the magnet and the stretch or compression of the spring.

Question 4:

Starting from the bottom, describe the direction of the force (up or down) and the orientation of the disk magnet, paying careful attention to locations where they change.

Question 5:

Where does the force appear to be the largest? The smallest? How should you know this?

Part 4: Dipole in Anti-Helmholtz

1. Switch the apparatus to Anti-Helmholtz mode and increase the current to 2 A. Then turn off the power supply.
2. Move the disk magnet to the center (0 on scale) and randomly align it (off axis)
3. Turn on the power supply, carefully watching the disk magnet

Question 6:

Did the disk magnet rotate? (Was there a torque on the magnet?)

Question 7:

Did the spring stretch or compress? (Was there a force on the magnet?)

Part 5: Moving a Dipole Along the Axis of an Anti-Helmholtz Coil

1. Now lower the disk magnet to bottom of the tube
2. Slowly pull the disk magnet up through the apparatus, until it is out the top. While pulling watch both the orientation of the magnet and the stretch or compression of the spring.

Question 8:

Starting from the bottom, describe the direction of the force (up or down) and the orientation of the disk magnet, paying careful attention to locations where they change.

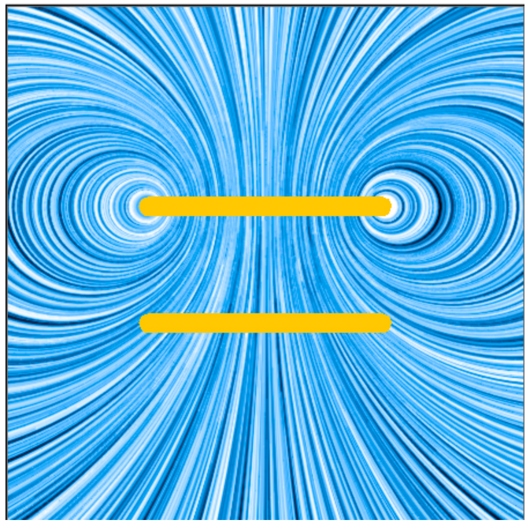
Question 9:

Where does the force appear to be the largest? The smallest? How should you know this?

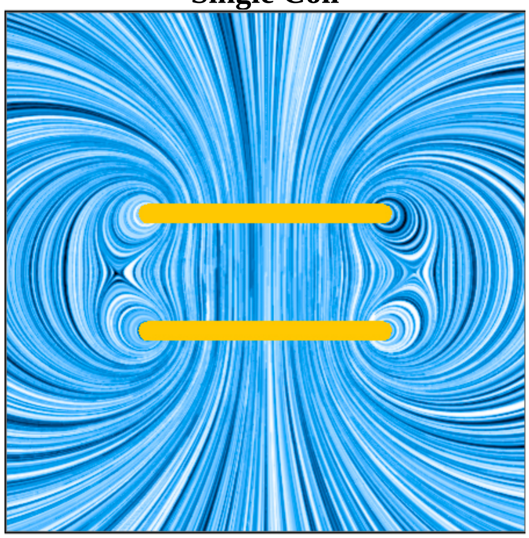
Further Questions (for experiment, thought, future exam questions…)

- What happens as we move through with just a single coil energized? Is it similar to the Helmholtz or anti-Helmholtz? How is it different?
- Are there places where we can put the disk magnet and then randomly orient it without either changing the force on it or having a torque rotate it back to alignment (in any of the 3 field configurations)?
- If you were to align the disk magnet with the x-axis (perpendicular the coil axis) and then center it in anti-Helmholtz mode, would there be a torque or force on it?

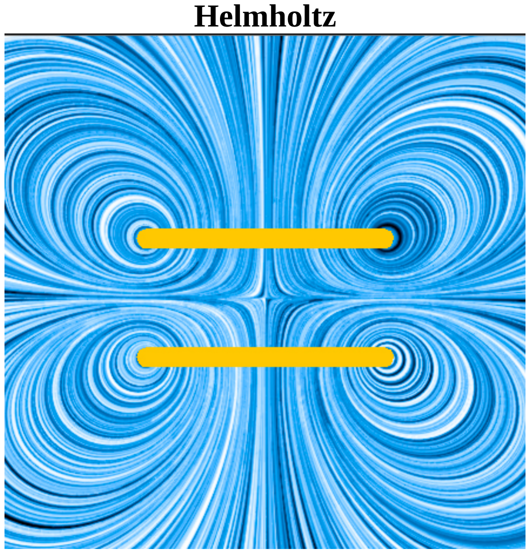
Iron Filings Patterns for Fields in the Helmholtz Apparatus



Single Coil



Helmholtz



Anti-Helmholtz

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2006**

Experiment 5: Faraday's Law

OBJECTIVES

1. To become familiar with the concepts of changing magnetic flux and induced current associated with Faraday's Law of Induction.

2. To see how and why the direction of the magnetic force on a conductor carrying an induced current is consistent with Lenz's Law. Lenz's Law says that the system always responds so as to try to keep things the same.

PRE-LAB READING

INTRODUCTION

In this lab you will develop an intuition for Faraday's and Lenz's Laws. By moving a coil of wire over a magnet you will change the magnetic flux through the coil, generating and EMF and hence current in the loop which you will measure using the 750.

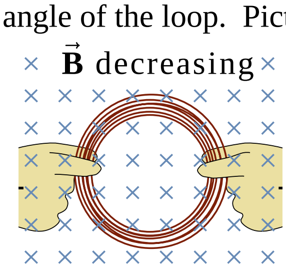
The Details: Faraday's Law

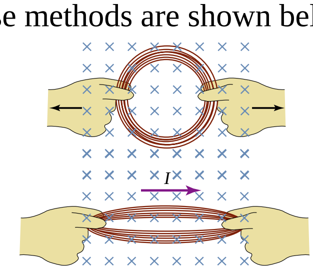
Faraday's Law states that a changing magnetic flux generates an EMF (electromotive force). Mathematically:

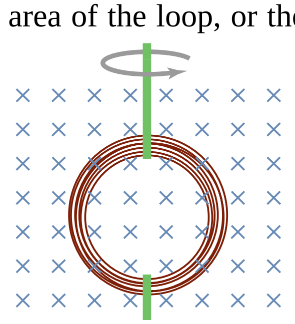
$$\mathcal{E} = -\frac{d\Phi_B}{dt}, \text{ where } \Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} \text{ is the magnetic flux, and } \mathcal{E} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} \text{ is the EMF}$$

In the formula above, $\vec{\mathbf{E}}$ is the electric field measured in the rest frame of the circuit, if the circuit is moving.

Changing Magnetic Flux: How do we get the magnetic flux Φ_B to change? Looking at the integral in the case of a uniform magnetic field, $\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = BA\cos(\theta)$, hints at three distinct methods: by changing the strength of the field, the area of the loop, or the angle of the loop. Pictures of these methods are shown below.







In each of the cases pictured above, the magnetic flux into the page is decreasing with time (because the (1) B field, (2) loop area or (3) projected area are decreasing with time). This decreasing flux creates an EMF. In which direction? We can use Lenz's Law to find out.

Lenz's Law

Lenz's Law is a non-mathematical statement of Faraday's Law. It says that systems will always act to *oppose* changes in magnetic flux. For example, in each of the above cases the flux into the page is decreasing with time. The loop doesn't want a decreased flux, so it will generate a clockwise EMF, which will drive a clockwise current, creating a B field into the page (inside the loop) to make up for the lost flux. This, by the way, is the meaning of the minus sign in Faraday's law. I recommend that you use Lenz's Law to determine the direction of the EMF and then use Faraday's Law to calculate the amplitude. By the way, just as with Faraday's Law, you don't need a physical circuit to use Lenz's Law. Just pretend that there is a wire in which current could flow and ask in what direction it would need to flow to *oppose* the changing flux. In general, *opposing* a change in flux means *opposing* what is happening to change the flux (e.g. forces or torques *oppose* the change).

APPARATUS

1. Magnet Stand

The magnetic flux of Faraday's Law will be generated by a high field permanent magnet, sitting on a support beam so that you may move a coil from above to below and back.

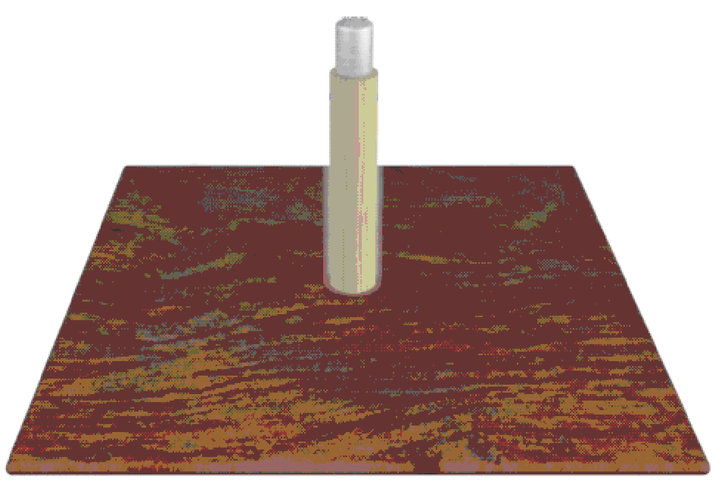


Figure 1 The Magnet Stand

2. Wire Loop, Current Sensor and Science Workshop 750 Interface

The magnetic field will penetrate a loop of wire, which you will plug into the current sensor, which is in turn plugged into channel A of the 750. In this lab we will use the convention that positive current flows counter-clockwise when observed from above. The current sensor records current that flows into its red terminal and out its negative terminal as positive, so make sure that you hook up the wire to the current sensor so that these two conventions are compatible with each other.

Figure 2 The Current Sensor

GENERALIZED PROCEDURE

This lab consists of two parts. In each you will observe the effects (current & force) of moving a loop around a dipole.

Part 1: Current and Flux through a Loop Moving Past a Dipole

You will move a wire loop from above to below a magnetic dipole, and observe plots of the current flowing through the loop (measured) and the flux through the loop (calculated).

Part 2: Feeling the Force

In this part you will repeat the motion, using a hollow aluminum cylinder instead of the wire loop. In doing so you will be able to feel the force on the cylinder due to Lenz's Law.

END OF PRE-LAB READING

Experiment 5: Faraday's Law

Answer these questions on a separate sheet of paper and turn them in before the lab

1. Calculating Flux from Current and Faraday's Law

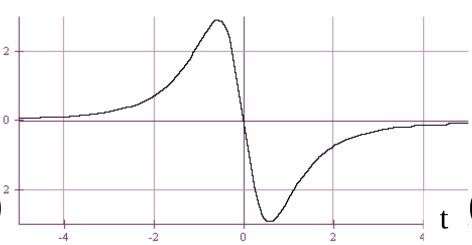
In part 1 of this lab you will move a coil from well above to well below a strong permanent magnet. You will measure the current in the loop during this motion using a current sensor. The program will also display the flux "measured" through the loop, even though this value is never directly measured. In this problem you will understand how.

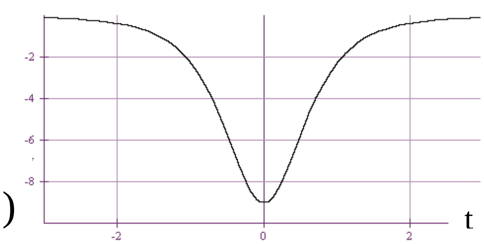
(a) Starting from Faraday's Law and Ohm's law, write an equation relating the current in the loop to the time derivative of the flux through the loop.

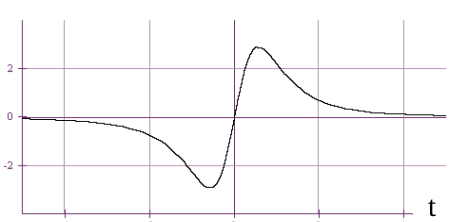
(b) Now integrate that expression to get the time dependence of the flux through the loop $\Phi(t)$ as a function of current $I(t)$. What assumption must the software make (what value must it arbitrarily set) before it can plot flux vs. time?

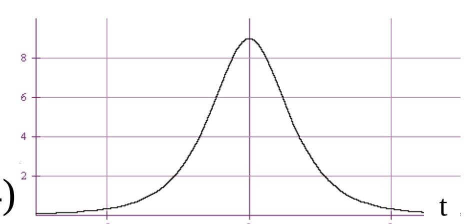
2. Predictions: Coil Moving Past Magnetic Dipole

In moving the coil over the magnet, measurements of current and flux for each of several motions will look like one of the below plots. For current, counter-clockwise when viewed from above is positive. For flux, upwards is positive. The north pole of the magnet is pointing up.

(1)


(2)


(3)


(4)


Suppose you move the loop from well *above* the magnet to well *below* the magnet at a constant speed. Which graph most closely resembles the graph of:

(a) *magnetic flux through the loop* as a function of time?

(b) *current through the loop* as a function of time?

2. Predictions: Coil Moving Past Magnetic Dipole *continued*

Suppose you now move the loop from well *below* the magnet to well *above* the magnet at a constant speed. Which graph most closely resembles the graph of:
(c) *magnetic flux through the loop* as a function of time?
(d) *current through the loop* as a function of time?

3. Predictions: Force on Coil Moving Past Magnetic Dipole

In part 2 of this lab you will feel the force on a conducting loop as it moves past the magnet. For the following conditions, in what direction should the magnetic force point?

As you are moving the loop from well *above* the magnet to well *below* the magnet at a constant speed…
(a) … and the loop is *above* the magnet.
(b) … and the loop is *below* the magnet

As you are moving the loop from well *below* the magnet to well *above* the magnet at a constant speed…
(c) … and the loop is *below* the magnet.
(d) … and the loop is *above* the magnet

Make sure that you record your answers to the previous two problems asking for predictions as you will need them in the lab.

4. Feeling the Force

In part 2, rather than using the same coil we use in part 1, we will instead use an aluminum cylinder to "better feel" the force. What possible differences could make the cylinder work better than the coil? If we were to double the number of turns in the coil would we be more likely to be able to feel the force, less likely, or would there be no difference? In case you are interested, the wire is copper, and of roughly the same diameter as the thickness of the aluminum cylinder, although this information won't necessarily help you in answering the question. (HINT: Write down the equation for force on a current in a magnetic field)

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file and start up the program.
2. Connect the current sensor to channel A of the 750.
3. Connect the wire loop to the current sensor so that, starting at the black terminal, the wire loops counterclockwise (when viewed from above) and then enters the red terminal of the current sensor

MEASUREMENTS

Part 1: Current and Flux through a Loop Moving Past a Dipole

1. Press 'Go' to start recording current and flux
2. Move the wire loop from well above to well below the magnet and back again. Try to make the motion as smooth as possible and at a constant velocity.

Question 1:

Did your measurements agree with your predictions? If not, which predictions did you miss and why?

Part 2: Feeling the Force

Although we could do this part of the lab with the same coil we just used, in order to better feel the force we will instead use an aluminum tube.

1. First hold the aluminum tube near the side of the magnet to convince yourself that Al is non-magnetic.
2. Place the tube over the Plexiglas and then push the tube downwards.
3. When you get to the bottom, pull the tube back up.

Question 2:

Did your measurements agree with your predictions? If not, which predictions did you miss and why?

Further Questions (for experiment, thought, future exam questions…)

- What happens if you move the coil more quickly? Does the magnitude of the current change? Does the magnitude of the flux change? In part 2, does the force change?
- If the current, flux or force do not change in this situation, is there anything we could do to make them change? If they do change, what other changes could we make that would counter-act the change of moving more quickly?
- What happens to the force when the tube is exactly centered on the magnet? Why?
- Do the effects depend on history? In other words, is moving from the middle to the bottom any different if the motion started at the top than if it started at the bottom and reversed at the middle?
- What happens if we define the direction of positive current to be clockwise (in other words, if we flip the coil over)? Does this change have any affect on our definition of flux?

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2006**

Experiment 6: Ohm's Law, RC and RL Circuits

OBJECTIVES

1. To explore the measurement of voltage & current in circuits
2. To see Ohm's law in action for resistors
3. To explore the time dependent behavior of RC and RL Circuits

PRE-LAB READING

INTRODUCTION

When a battery is connected to a circuit consisting of wires and other circuit elements like resistors and capacitors, voltages can develop across those elements and currents can flow through them. In this lab we will investigate three types of circuits: those with only resistors in them and those with resistors and either capacitors (RC circuits) or inductors (RL circuits). We will confirm that there is a linear relationship between current through and potential difference across resistors (Ohm's law: $V = IR$). We will also measure the very different relationship between current and voltage in a capacitor and an inductor, and study the time dependent behavior of RC and RL circuits.

The Details: Measuring Voltage and Current

Imagine you wish to measure the voltage drop across and current through a resistor in a circuit. To do so, you would use a voltmeter and an ammeter – similar devices that measure the amount of current flowing in one lead, through the device, and out the other lead. But they have an important difference. An ammeter has a very low resistance, so when placed in series with the resistor, the current measured is not significantly affected (Fig. 1a). A voltmeter, on the other hand, has a very high resistance, so when placed in parallel with the resistor (thus seeing the same voltage drop) it will draw only a very small amount of current (which it can convert to voltage using Ohm's Law $V_R = V_{meter} = I_{meter}R_{\text{meter}}$), and again will not appreciably change the circuit (Fig. 1b).

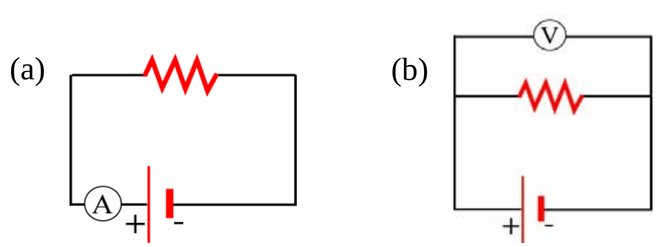


Figure 1: Measuring current and voltage in a simple circuit. To measure current *through* the resistor (a) the ammeter is placed in series with it. To measure the voltage drop *across* the resistor (b) the voltmeter is placed in parallel with it.

The Details: Capacitors

Capacitors store charge, and develop a voltage drop V across them proportional to the amount of charge Q that they have stored: $V = Q/C$. The constant of proportionality C is the capacitance (measured in Farads = Coulombs/Volt), and determines how easily the capacitor can store charge. Typical circuit capacitors range from picofarads (1 pF = 10^{-12} F) to millifarads (1 mF = 10^{-3} F). In this lab we will use microfarad capacitors (1 μF = 10^{-6} F).

RC Circuits

Consider the circuit shown in Figure 2. The capacitor (initially uncharged) is connected to a voltage source of constant emf $\mathcal{E}$. At $t = 0$, the switch S is closed.

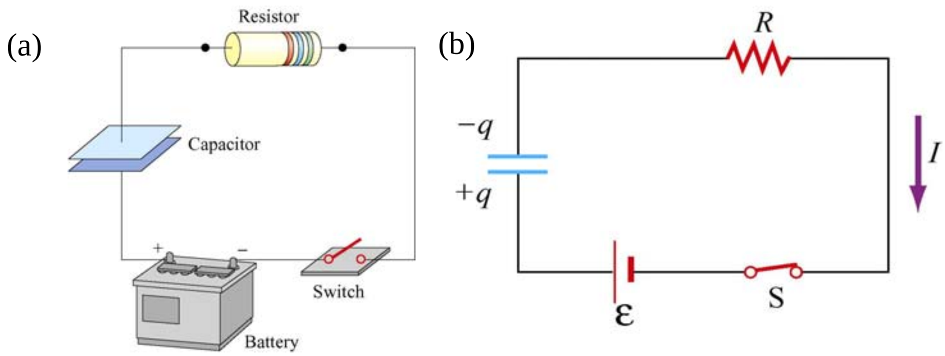


Figure 2 (a) *RC* circuit (b) Circuit diagram for $t > 0$

In class we derived expressions for the time-dependent charge on, voltage across, and current through the capacitor, but even without solving differential equations a little thought should allow us to get a good idea of what happens. Initially the capacitor is uncharged and hence has no voltage drop across it (it acts like a wire or "short circuit"). This means that the full voltage rise of the battery is dropped across the resistor, and hence current must be flowing in the circuit ($V_R = IR$). As time goes on, this current will "charge up" the capacitor – the charge on it and the voltage drop across it will increase, and hence the voltage drop across the resistor and the current in the circuit will decrease. This idea is captured in the graphs of Fig. 3.

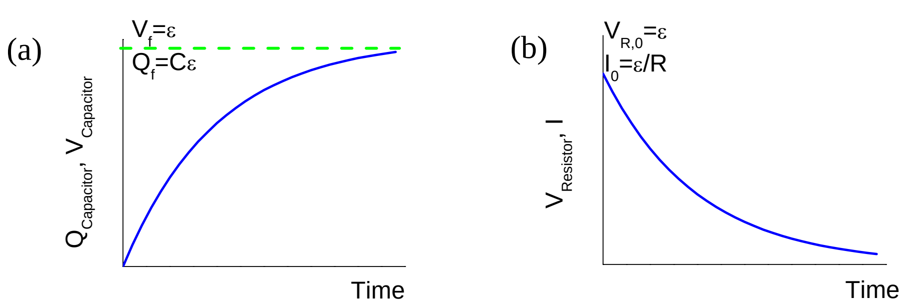


Figure 3 (a) Voltage across and charge on the capacitor increase as a function of time while (b) the voltage across the resistor and hence current in the circuit decrease.

After the capacitor is "fully charged," with its voltage essentially equal to the voltage of the battery, the capacitor acts like a break in the wire or "open circuit," and the current is essentially zero. Now we "shut off" the battery (replace it with a wire). The capacitor will then release its charge, driving current through the circuit. In this case, the voltage across the capacitor and across the resistor are equal, and hence charge, voltage and current all do the same thing, decreasing with time. As you saw in class, this decay is exponential, characterized by a time constant t, as pictured in fig. 4.

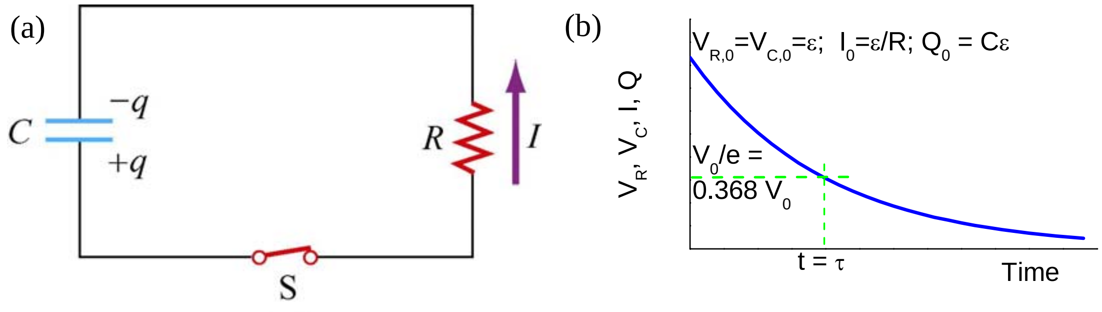


Figure 4 Once (a) the battery is "turned off," the voltages across the capacitor and resistor, and hence the charge on the capacitor and current in the circuit all (b) decay exponentially. The time constant τ is how long it takes for a value to drop by e.

The Details: Inductors

Inductors store energy in the form of an internal magnetic field, and find their behavior dominated by Faraday's Law. In any circuit in which they are placed they create an EMF ε proportional to the time rate of change of current I through them: $\varepsilon = L\, dI/dt$. The constant of proportionality L is the inductance (measured in Henries = Ohm s), and determines how strongly the inductor reacts to current changes (and how large a self energy it contains for a given current). Typical circuit inductors range from nanohenries to hundreds of millihenries. The direction of the induced EMF can be determined by Lenz's Law: it will always oppose the change (inductors try to keep the current constant)

RL Circuits

If we replace the capacitor of figure 2 with an inductor we arrive at figure 5. The inductor is connected to a voltage source of constant emf $\mathcal{E}$. At $t = 0$, the switch S is closed.

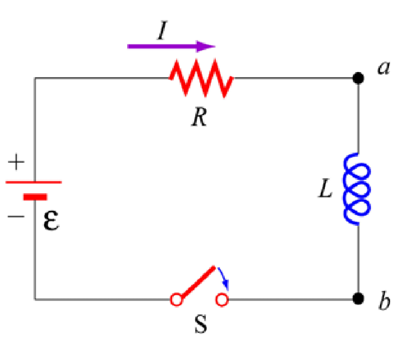


Figure 5 RL circuit. For t<0 the switch S is open and no current flows in the circuit. At t=0 the switch is closed and current I can begin to flow, as indicated by the arrow.
.

As we saw in class, before the switch is closed there is no current in the circuit. When the switch is closed the inductor wants to keep the same current as an instant ago – none. Thus it will set up an EMF that opposes the current flow. At first the EMF is identical to that of the battery (but in the opposite direction) and no current will flow. Then, as time passes, the inductor will gradually relent and current will begin to flow. After a long time a constant current ($I = V/R$) will flow through the inductor, and it will be content (no changing current means no changing B field means no changing magnetic flux means no EMF). The resulting EMF and current are pictured in Fig. 6.

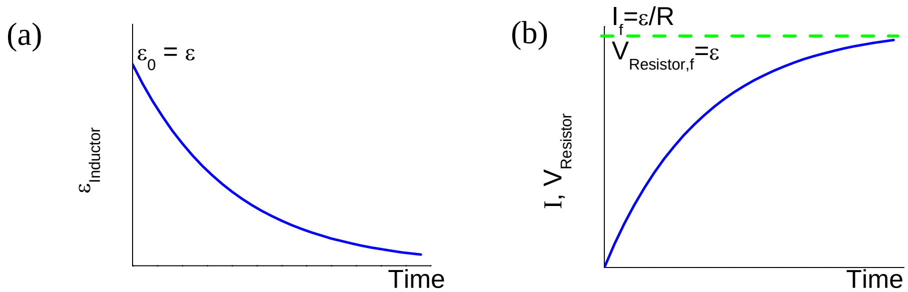


Figure 6 (a) "EMF generated by the inductor" decreases with time (this is what a voltmeter hooked in parallel with the inductor would show) (b) the current and hence the voltage across the resistor increase with time, as the inductor 'relaxes.'

After the inductor is "fully charged," with the current essentially constant, we can shut off the battery (replace it with a wire). Without an inductor in the circuit the current would instantly drop to zero, but the inductor does not want this rapid change, and hence generates an EMF that will, for a moment, keep the current exactly the same as it was before the battery was shut off. In this case, the EMF generated by the inductor and voltage across the resistor are equal, and hence EMF, voltage and current all do the same thing, decreasing exponentially with time as pictured in fig. 7.

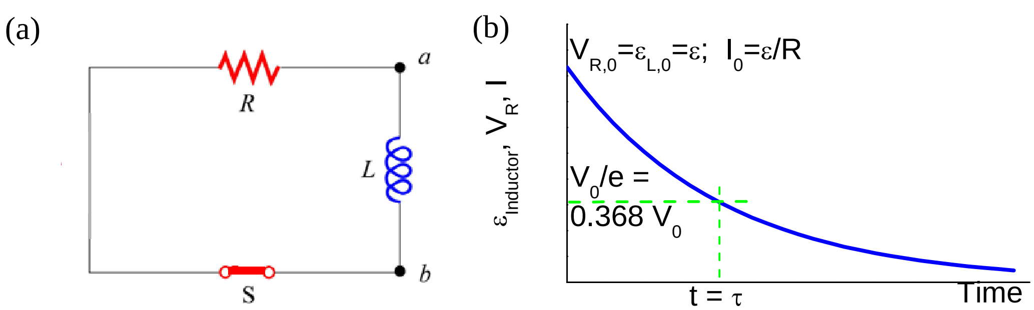


Figure 7 Once (a) the battery is turned off, the EMF induced by the inductor and hence the voltage across the resistor and current in the circuit all (b) decay exponentially. The time constant τ is how long it takes for a value to drop by e.

The Details: Non-Ideal Inductors
So far we have always assumed that circuit elements are ideal, for example, that inductors only have inductance and not capacitance or resistance. This is generally a decent assumption, but in reality no circuit element is truly ideal, and today we will need to consider this. In particular, today's "inductor" has both inductance and resistance (real inductor = ideal inductor in series with resistor). Although there is no way to physically separate the inductor from the resistor in this circuit element, with a little thought (which you will do in the pre-lab) you will be able to measure both the resistance and inductance.

APPARATUS

1. Science Workshop 750 Interface

In this lab we will again use the Science Workshop 750 interface to create a "variable battery" which we can turn on and off, whose voltage we can change and whose current we can measure.

2. AC/DC Electronics Lab Circuit Board

We will also use, for the first of several times, the circuit board pictured in Fig. 8. This is a general purpose board, with (A) battery holders, (B) light bulbs, (C) a push button switch, (D) a variable resistor called a potentiometer, and (E) an inductor. It also has (F) a set of 8 isolated pads with spring connectors that circuit components like resistors and capacitors can easily be pushed into. Each pad has two spring connectors connected by a wire (as indicated by the white lines). The right-most pads also have banana plug receptacles, which we will use to connect to the output of the 750.

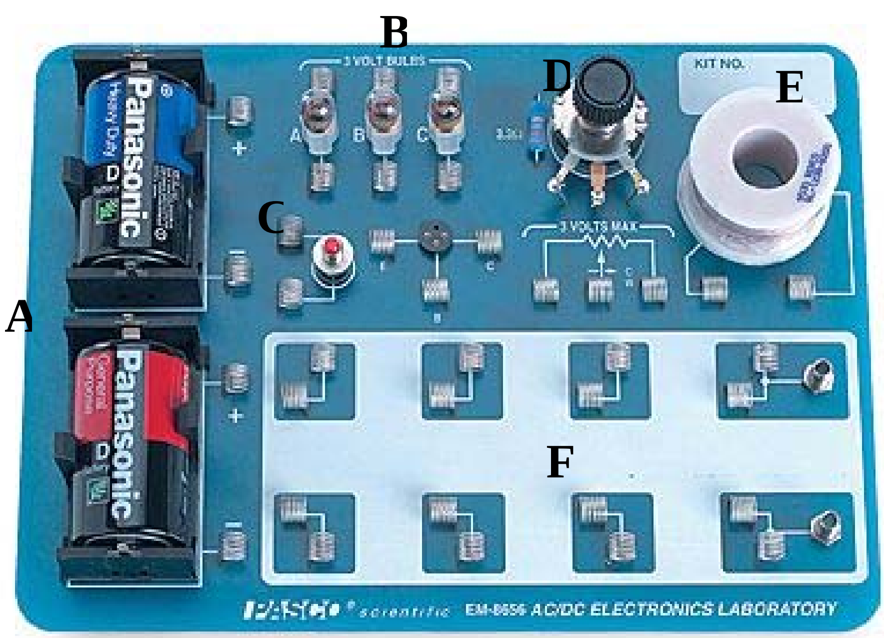


Figure 8 The AC/DC Electronics Lab Circuit Board, with (A) Battery holders, (B) light bulbs, (C) push button switch, (D) potentiometer, (E) inductor and (F) connector pads

3. Current & Voltage Sensors

Recall that both current and voltage sensors follow the convention that red is "positive" and black "negative." That is, the current sensor records currents flowing in the red lead and out the black as positive. The voltage sensor measures the potential at the red lead minus that at the black lead.

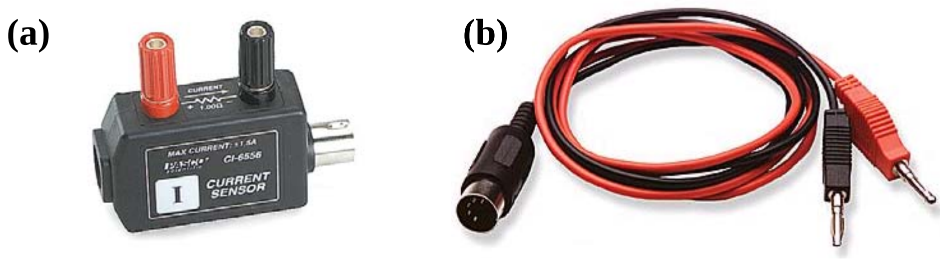


Figure 9 (a) Current and (b) Voltage Sensors

4. Resistors & Capacitors

We will work with resistors and capacitors in this lab. Resistors (Fig. 8a) have color bands that indicate their value (see appendix A if you are interested in learning to read this code), whereas capacitors (Fig. 8b) are typically stamped with a numerical value.

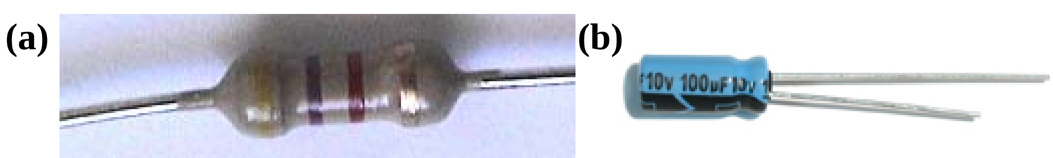


Figure 10 Examples of a (a) resistor and (b) capacitor. Aside from their size, most resistors look the same, with 4 or 5 colored bands indicating the resistance. Capacitors on the other hand come in a wide variety of packages and are typically stamped both with their capacitance and with a maximum working voltage.

GENERALIZED PROCEDURE

This lab consists of five main parts. In each you will set up a circuit and measure voltage and current while the battery periodically turns on and off. In the last two parts you are encouraged to develop your own methodology for measuring the resistance and inductance of the coil on the AC/DC Electronics Lab Circuit Board both with and without a core inserted. The core is a metal cylinder which is designed to slide into the coil and affect its properties in some way that you will measure.

Part 1: Measure Voltage Across & Current Through a Resistor

Here you will measure the voltage drop across and current through a single resistor attached to the output of the 750.

Part 2: Resistors in Parallel

Now attach a second resistor in parallel to the first and see what happens to the voltage drop across and current through the first.

Part 3: Measuring Voltage and Current in an *RC* Circuit

In this part you will create a series RC (resistor/capacitor) circuit with the battery turning on and off so that the capacitor charges then discharges. You will measure the time constant in two different ways (see Pre-Lab #5) and use this measurement to determine the capacitance of the capacitor.

Part 4: Measure Resistance and Inductance Without a Core

The battery will alternately turn on and turn off. You will need to hook up this source to the coil and, by measuring the voltage supplied by and current through the battery, determine the resistance and inductance of the coil.

Part 5: Measure Resistance and Inductance With a Core

In this section you will insert a core into the coil and repeat your measurements from part 1 (or choose a different way to make the measurements).

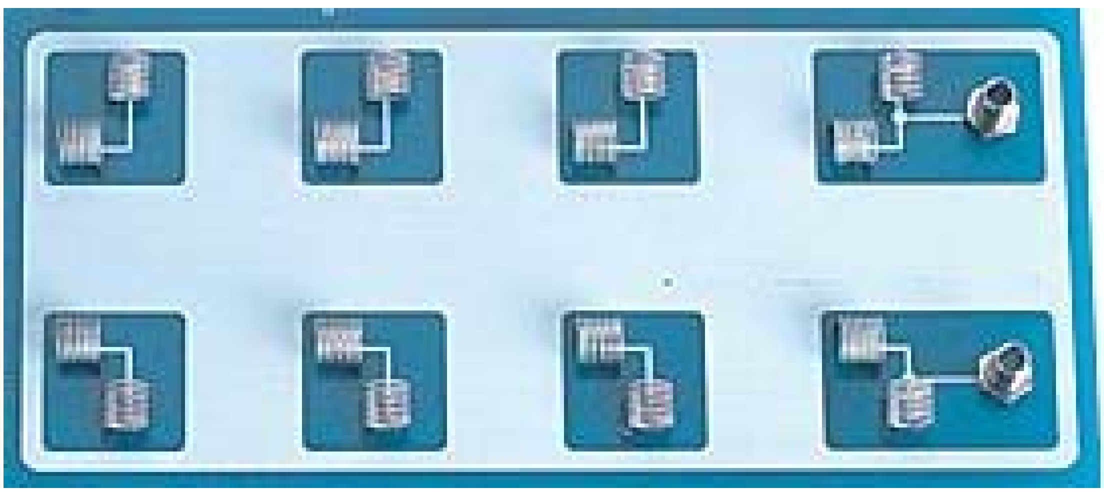
END OF PRE-LAB READING

Experiment 6: Ohm's Law, RC and RL Circuits

Answer these questions on a separate sheet of paper and turn them in before the lab

1. Measuring Voltage and Current

In Part 1 of this experiment you will measure the potential drop across and current through a single resistor attached to the "variable battery." On a diagram similar to the one below, indicate where you will attach the leads to the resistor, the battery, the voltage sensor (V), and the current sensor (A). For the battery and sensors make sure that you indicate which color lead goes where, using the convention that red is "high" (or the positive input) and black is "ground." Reread the pre-lab description of this board carefully to understand the various parts. When you draw a resistor or other circuit element it should go between two pads (dark green areas) with each end touching one of the spring clips (the metal coils). Do NOT just draw a typical circuit diagram. You need to think about how you will actually wire this board during the lab. RECALL: ammeters must be in series with the element they are measuring current through, while voltmeters must be in parallel.



2. Resistors in Parallel

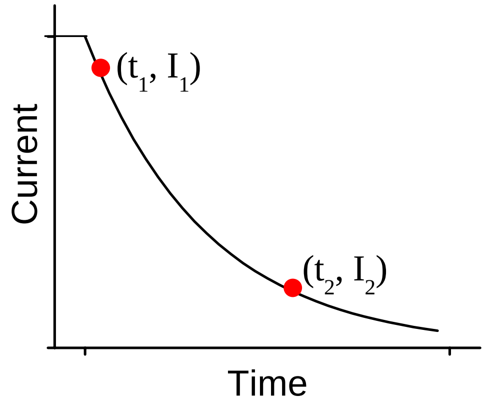
In Part 2 you will add a second resistor in parallel with the first. Show where you would attach this second resistor in the diagram you drew for question 1, making sure that the ammeter continues to measure the current through the first resistor and the voltmeter measures the voltage across the first resistor.

3. Measuring the Time Constant τ

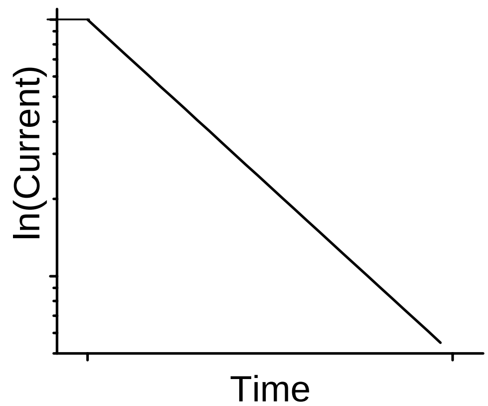
As you have seen, current always decays exponentially in RC circuits with a time constant τ: $I = I_0 \exp(-t/\tau)$.

We will measure this time constant in two different ways.

(a) After measuring the current as a function of time we choose two points on the curve (t_1,I_1) and (t_2, I_2). What relationship must we choose between I_2 and I_1 in order to determine the time constant by subtraction: $\tau = t_2 - t_1$? Should we be able to find a t_2 that satisfies this for any choice of t_1?



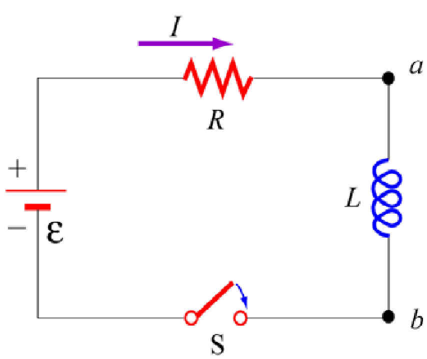
(b) We can also plot the natural log of the current vs. time, as shown at right. If we fit a line to this curve we will obtain a slope *m* and a y-intercept *b*. From these fitting parameters, how can we calculate the time constant?



(c) Which of these two methods is more likely to help us obtain an accurate measurement of the time constant? Why?

Make sure that you record your answers to question 3 in your notes as you will need them for the lab.

4. RL Circuits



Consider the circuit at left, consisting of a battery (emf ε), an inductor *L*, resistor *R* and switch *S*.

For times $t<0$ the switch is open and there is no current in the circuit. At $t=0$ the switch is closed.

(a) Using Kirchhoff's loop rules (really Faraday's law now), write an equation relating the emf on the battery, the current in the circuit and the time derivative of the current in the circuit.

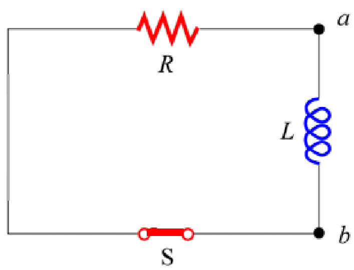
In class we stated that this equation was solved by an exponential. In other words:

$$I = A(X - \exp(-t/\tau))$$

(b) Plug this expression into the differential equation you obtained in (a) in order to confirm that it indeed is a solution and to determine what the time constant τ and the constants A and X are. What would be a better label for A? (HINT: You will also need to use the initial condition for current. What is $I(t=0)$?).

(c) Now that you know the time dependence for the current I in the circuit you can also determine the voltage drop V_R across resistor and the EMF generated by the inductor. Do so, and confirm that your expressions match the plots in Fig. 6a or 2b.

5. 'Discharging' an Inductor



After a long time T the current will reach an equilibrium value and inductor will be "fully charged." At this point we turn off the battery (ε=0), allowing the inductor to 'discharge,' as pictured at left. Repeat each of the steps a-c in problem 4, noting that instead of $\exp(-t/\tau)$, our expression for current will now contain $\exp(-(t-T)/\tau)$.

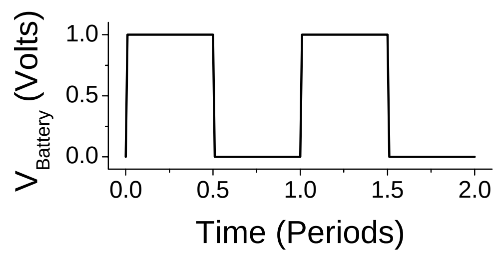
6. The Coil

The coil you will be measuring has is made of thin copper wire (radius ~ 0.25 mm) and has about 600 turns of average diameter 25 mm over a length of 25 mm. What approximately should the resistance and inductance of the coil be? The resistivity of copper at room temperature is around 20 nΩ-m. Note that your calculations can only be approximate because this is not at all an ideal solenoid (where length >> diameter).

7. A Real Inductor

As mentioned above, in this lab you will work with a coil that does not behave as an ideal inductor, but rather as an ideal inductor in series with a resistor. For this reason you have no way to independently measure the voltage drop across the resistor or the EMF induced by the inductor, but instead must measure them together. None-the-less, you want to get information about both. In this problem you will figure out how.

(a) In the lab you will hook up the circuit of problem 4 with the ideal inductor *L* of that problem now replaced by a coil that is a non-ideal inductor – an inductor *L* and resistor *r* in series. The battery will periodically turn on and off, displaying a voltage as shown here:



Sketch the current through the battery as well as what a voltmeter hooked across the coil would show versus time for the two periods shown above. Assume that the period of the battery turning off and on is comparable to but longer than several time constants of the circuit.

(b) How can you tell from your plot of the voltmeter across the coil that the coil is not an ideal inductor? Indicate the relevant feature clearly on the plot. Can you determine the resistance of the coil, *r*, from this feature?

(c) In the lab you will find it easier to make measurements if you do NOT use an additional resistor *R*, but instead simply hook the battery directly to the coil. (Why? Because the time constant is difficult to measure with extra resistance in the circuit). Plot the current through the battery and the reading on a voltmeter across the coil for this case. We will only bother to measure the current. Why?

(d) For this case (only a battery & coil) how will you determine the resistance of the coil, *r*? How will you determine its inductance *L*?

Make sure that you record your answer to 7d in your notes as you will need it for the lab.

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop (right click on the link and choose "Save Target As" to the desktop. Overwrite any file by this name that is already there). Start LabView by double clicking on this file.
2. Connect the Voltage Sensor to Analog Channel A on the 750 Interface and the Current Sensor to Analog Channel B.
3. Connect cables from the output of the 750 to the banana plug receptacles on the lower right side of the circuit board (red to the sin wave marked output, black to ground).

MEASUREMENTS

Part 1: Measuring the Resistance of a Single Resistor

1. Hook up the circuit as you determined it should be set up in Pre-Lab #1 (to measure the voltage across and current through a single resistor driven by the "variable battery.")
2. Record V and I for 1 second. (Press the green "Go" button above the graph). During this time the battery will switch between putting out 1 Volt and 0 Volts.

Question 1:

When the battery is "on" what is the voltage drop across the resistor and what is the current through it? What is the resistance of the resistor (calculate it from what you just measured, do NOT figure it out from the color code).

Part 2: Resistors in Parallel

1. Hook up the circuit as you determined it should be set up in Pre-Lab #2 (to measure the voltage across and current through the first resistor connected in parallel to a second resistor)
2. Record V and I for 1 second. (Press the green "Go" button above the graph). During this time the battery will switch between putting out 1 Volt and 0 Volts.

Question 2:

When the battery is "on" what is the voltage drop across the resistor and what is the current through it? Did these values change from Part 1? Why or why not?

Part 3: Measuring Voltage and Current in an *RC* Circuit

3A: Using a Single Resistor

1. Create a circuit with the first resistor and the capacitor in series with the battery
2. Connect the voltage sensor (still in channel A) across the capacitor
3. Record the voltage across the capacitor *V* and the current sourced by the battery *I* (Press the green "Go" button above the graph). During this time the battery will switch between putting out 1 Volt and 0 Volts.

Question 4:

Using the two-point method (which you calculated in Pre-Lab #3a), what is the time constant of this circuit? Using this time constant, the resistance you measured in Question 1 and the typical expression for an RC time constant, what is the capacitance of the capacitor?

Question 5:

Using the logarithmic method (which you calculated in Pre-Lab #3b), what is the time constant of this circuit? Using this time constant, what is the capacitance of the capacitor?

3B: Using Two Resistors in Series

1. Put the second resistor in series with the first resistor and capacitor
2. Connect the voltage sensor (still in channel A) across the capacitor
3. Measure the current sourced by the battery (Press the green "Go" button above the graph).

Question 6:

Using one of the two methods used above, what is the time constant of this new circuit? What must the resistance of the second resistor be?

Part 4: Measure Resistance and Inductance Without a Core

1. Connect cables from the output of the 750 to either side of the coil (using the clips)
2. Make sure that the core is removed from the coil
3. Record the current through and voltage across the battery for a fraction of a second. (Press the green "Go" button above the graph).

Question 7:

What is the maximum current during the cycle? What is the EMF generated by the inductor at the time this current is reached?

Question 8:

What is the time constant τ of the circuit?

Question 9:

What are the resistance r and inductance L of the coil? Calculate this using your answer to Pre-Lab #7d.

Part 5: Measure Resistance and Inductance With a Core

1. Insert the core into the center of the coil
2. Record the current through and voltage across the battery for a fraction of a second. (Press the green "Go" button above the graph).

Question 10:

Does the maximum current in the circuit change due to the introduction of the core? If it does, try to explain as clearly as possible why this happens (including why the change to bigger or smaller makes sense)

Question 11:

Does the time constant τ of the circuit change due to the introduction of the core? If it does, try to explain as clearly as possible why this happens (including why the change to longer or shorter makes sense)

Question 12:

What are the new resistance r and inductance L of the coil?

Further Questions (for experiment, thought, future exam questions…)

- What happens if we instead put the second resistor in parallel with the first?
- What if we instead put the second resistor in parallel with the capacitor? Does the initial current change? The final current? The final voltage (and hence charge) on the capacitor?
- What if we change the order of the elements in the circuit (e.g. put the capacitor between the two resistors, or switch the leads from the battery)?
- The ammeter is marked as having a 1 ohm resistance, small, but not tiny. Can you see the effects of the ammeter resistance in the circuits of part 1 and 2? Can you measure the voltage drop across the ammeter? Does this make the measurement of the current through the resistor inaccurate?
- What happens if we put a resistor R in series with the coil? In parallel with the coil?
- What happens if you make the battery switch on and off with a period shorter than the time constant of the circuit? Would you still be able to determine the inductance L and resistance r of the coil using the same method?
- What happens if you only partially insert the core into the coil? Can you continuously adjust the core's effects or there an abrupt jump from one behavior to another? Would another core (like your finger) have the same effects?
- If the coil were made of some superconducting material, what would its resistance be? Would the EMF you measure be any different? Would the potential difference from one side of the inductor to the other $\left(\Delta V = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}\right)$ be any different?

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2006**

Experiment 7: Undriven & Driven RLC Circuits

OBJECTIVES

1. To explore the time dependent behavior of RLC Circuits, both driven (with an AC function generator) and undriven
2. To understand the idea of resonance, and to determine the behavior of current and voltage in a driven RLC circuit above, below and at the resonant frequency

PRE-LAB READING

INTRODUCTION

As most children know, if you get a push on a swing and just sit still on it, you will go back and forth, gradually slowing down to a stop. If, on the other hand, you move your body back and forth you can drive the swing, making it swing higher and higher. This only works if you move at the correct rate though – too fast or too slow and the swing will do nothing.

This is an example of resonance in a mechanical system. In this lab we will explore its electrical analog – the RLC (resistor, inductor, capacitor) circuit – and better understand what happens when it is undriven and when it is driven above, below and at the resonant frequency.

The Details: Oscillations

In this lab you will be investigating current and voltages (EMFs) in RLC circuits. These oscillate as a function of time, either continuously (Fig. 1a, for driven circuits) or in a decaying fashion (Fig. 1b, for undriven circuits).

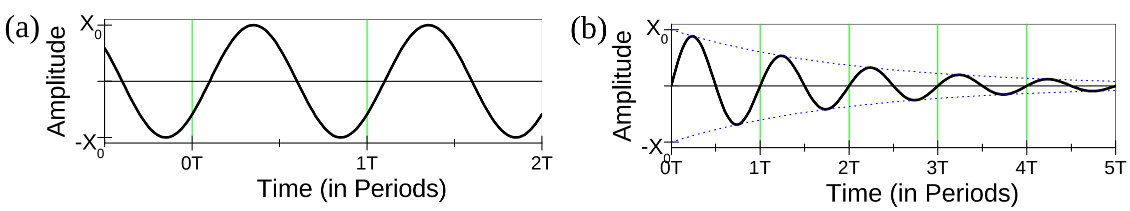


Figure 1 Oscillating Functions. (a) A purely oscillating function $x = x_0 \sin(\omega t + \varphi)$ has fixed amplitude x_0, angular frequency ω (period $T = 2\pi/\omega$ and frequency $f = \omega/2\pi$), and phase ϕ (in this case $\phi = -0.2\pi$). (b) The amplitude of a damped oscillating function decays exponentially (amplitude *envelope* indicated by dotted lines)

Undriven Circuits: Thinking about Oscillations

Consider the RLC circuit of fig. 2 below. The capacitor has an initial charge Q_0 (it was charged by a battery no longer in the circuit), but it can't go anywhere because the switch is open. When the switch is closed, the positive charge will flow off the top plate of the capacitor, through the resistor and inductor, and on to the bottom plate of the capacitor. This is the same behavior that we saw in RC circuits. In those circuits, however, the current flow stops as soon as all the positive charge has flowed to the negatively charged plate, leaving both plates with zero charge. The addition of an inductor, however, introduces inertia into the circuit, keeping the current flowing even when the capacitor is completely discharged, and forcing it to charge in the opposite polarity (Fig 2b).

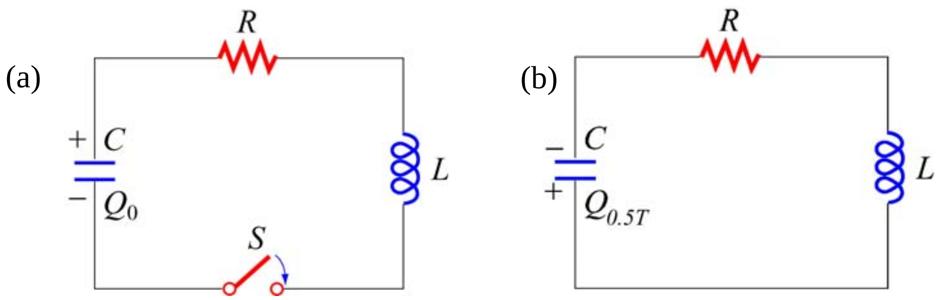


Figure 2 Undriven RLC circuit. (a) For t<0 the switch S is open and although the capacitor is charged ($Q = Q_0$) no current flows in the circuit. (b) A half period after closing the switch the capacitor again comes to a maximum charge, this time with the positive charge on the lower plate.

This oscillation of positive charge from the upper to lower plate of the capacitor is only one of the oscillations occurring in the circuit. For the two times pictured above (t=0 and t=0.5 T) the charge on the capacitor is a maximum and no current flows in the circuit. At intermediate times current is flowing, and, for example, at t = 0.25 T the current is a maximum and the charge on the capacitor is zero. Thus another oscillation in the circuit is between charge on the capacitor and current in the circuit. This corresponds to yet another oscillation in the circuit, that of energy between the capacitor and the inductor. When the capacitor is fully charged and the current is zero, the capacitor stores energy but the inductor doesn't ($U_C = Q^2/2C;\ U_L = \frac{1}{2}LI^2 = 0$). A quarter period later the current I is a maximum, charge Q = 0, and all the energy is in the inductor: $U_C = Q^2/2C = 0;\ U_L = \frac{1}{2}LI^2$. If there were no resistance in the circuit this swapping of energy between the capacitor and inductor would be perfect and the current (and voltage across the capacitor and EMF induced by the inductor) would oscillate as in Fig. 1a. A resistor, however, damps the circuit, removing energy by dissipating power through Joule heating ($P=I^2R$), and eventually ringing the current down to zero, as in Fig. 1b. Note that only the resistor dissipates power. The capacitor and inductor both store energy during half the cycle and then completely release it during the other half.

Driven Circuits: Resonance

Instead of simply charging the capacitor and then letting the system go, we could instead add a battery that periodically pushed current through the system. Such a battery is called an *AC* (alternating current) *function generator*, and the voltage it generates can oscillate with a given amplitude, frequency and shape (in this lab we will use a sine wave). When hooked up to an RLC circuit we get a driven RLC circuit (Fig. 3a) where the current oscillates at the same frequency as, but not necessarily in phase with, the driving voltage. The amplitude of the current depends on the driving frequency, reaching a maximum when the function generator drives at the resonant frequency, just like a swing (Fig. 3b)

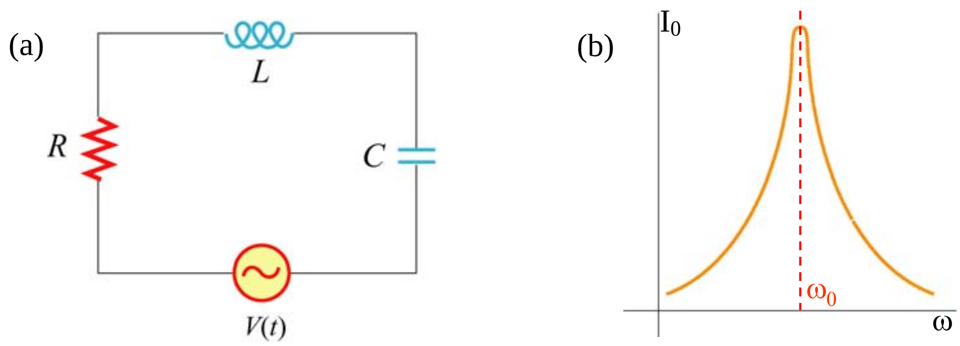


Figure 3 Driven *RLC* Circuit. (a) The circuit (b) The magnitude of the oscillating current I_0 reaches a maximum when the circuit is driven at its resonant frequency

One Element at a Time

In order to understand how this resonance happens in an RLC circuit, its easiest to build up an intuition of how each individual circuit element responds to oscillating currents. A resistor obeys Ohm's law: $V = IR$. It doesn't care whether the current is constant or oscillating – the amplitude of voltage doesn't depend on the frequency and neither does the phase (the response voltage is always in phase with the current).

A capacitor is different. Here if you drive current at a low frequency the capacitor will fill up and have a large voltage across it, whereas if you drive current at a high frequency the capacitor will begin discharging before it has a chance to completely charge, and hence it won't build up as large a voltage. We see that the voltage is frequency dependent and that the current *leads* the voltage (with an uncharged capacitor you see the current flow and then the charge/potential on the capacitor build up).

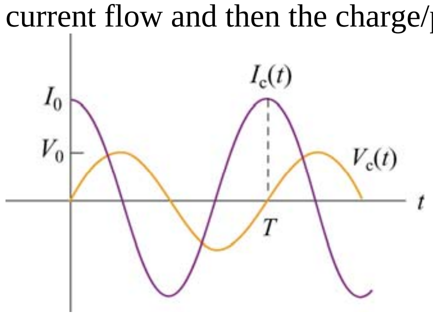


Figure 4 Current and Voltage for a Capacitor A capacitor driven with a sinusoidal current will develop a voltage that lags the current by 90º (the voltage peak comes ¼ period later than the current peak).

An inductor is similar to a capacitor but the opposite. The voltage is still frequency dependent but the inductor will have a larger voltage when the frequency is high (it doesn't like change and high frequency means lots of change). Now the current *lags* the voltage – if you try to drive a current through an inductor with no current in it, the inductor will immediately put up a fight (create an EMF) and then later allow current to flow.

When we put these elements together we will see that at low frequencies the capacitor will "dominate" (it fills up limiting the current) and current will lead whereas at high frequencies the inductor will dominate (it fights the rapid changes) and current will lag. At resonance the frequency is such that these two effects balance and the current will be largest in the circuit. Also at this frequency the current is in phase with the driving voltage (the AC function generator).

Resistance, Reactance and Impedance

We can make the relationship between the magnitude of the current through a circuit element and magnitude of the voltage drop across it (or EMF generated by it for an inductor) more concrete by introducing the idea of impedance. Impedance (usually denoted by Z) is a generalized resistance, and is composed of two parts – resistance (R) and reactance (X). All of these terms refer to a constant of proportionality between the magnitude of current through and voltage across (EMF generated by) a circuit element: $V_0 = I_0 Z$, $V = IR$, $V_0 = I_0 X$. The difference is in the phase between the current and voltage. In an element with only resistance (a resistor) the current through it is in phase with the voltage across it. In an element with only reactance (capacitor, inductor) the current leads or lags the voltage by 90º. A combination of these elements in series or parallel will lead to a circuit with impedance $Z = \sqrt{R^2 + X^2}$ and a phase that depends on the ratio of the reactance and resistance: $\tan\varphi = X/R$ (note that the phase ϕ has the correct behavior as $X \to 0$ or $R \to 0$).

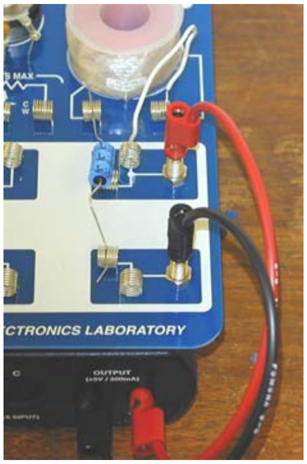
The reactance of an inductor $X_L = \omega L$ and of a capacitor $X_C = -1/\omega C$. First of all, note that these have the correct frequency dependence. An inductor has a high reactance at high frequencies (it takes a lot of effort to change the current through an inductor at high frequencies) whereas a capacitor has a high reactance at low frequencies (it "fills up" to have a large potential across it). The sign on the capacitive reactance is a convention, indicating that it leads to the current leading rather than lagging ($V(t) = V_0 \sin(\omega t + \phi)$ & $I(t) = I_0 \sin \omega t$, so phase ϕ is negative for capacitors). Some people instead write $X = X_L - X_C$ and keep all reactances positive – feel free to use whichever convention you prefer.

APPARATUS

1. Science Workshop 750 Interface

In this lab we will again use the Science Workshop 750 interface as an AC function generator, whose voltage we can set and current we can measure. We will also use it to measure the voltage across the capacitor using a voltage probe.

2. AC/DC Electronics Lab Circuit Board



We will also again use the circuit board, set up with a 100 μF capacitor in series with the coil (which serves both as the resistor and inductor in the circuit), as pictured at left.

Figure 5 Setup of the AC/DC Electronics Lab Circuit Board.

In addition, we will connect a voltage probe in parallel with the capacitor (not pictured).

GENERALIZED PROCEDURE

In this lab you will measure the behavior of a series RLC circuit, both when driven sinusoidally by a function generator and when undriven.

Part 1: Free Oscillations in an Undriven RLC Circuit
The capacitor is charged with a DC battery which is then turned off, allowing the circuit to ring down.

Part 2: Energy Ringdown in an Undriven RLC Circuit
Part 1 is repeated, except that the energy is reported instead of current and voltage.

Part 3: Driving the RLC Circuit on Resonance
Now the circuit is driven with a sinusoidal voltage and you will adjust to frequency while monitoring plots of $I(t)$ and $V(t)$ as well as V vs. I.

Part 4: What's The Frequency?
The circuit is driven with an unknown frequency and you must determine if its above or below resonance.

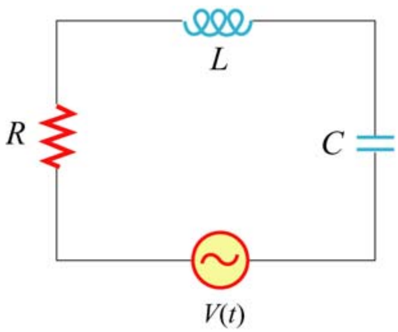
Part 5: What's That Trace?
Current and voltage across the function generator and capacitor are recorded, but you must determine which trace is which.

END OF PRE-LAB READING

Experiment 8: Undriven & Driven RLC Circuits

Answer these questions on a separate sheet of paper and turn them in before the lab

1. RLC Circuits



Consider the circuit at left, consisting of an AC function generator ($V(t) = V_0 \sin(\omega t)$, with V_0 = 5 V), an inductor L = 8.5 mH, resistor R = 5 Ω, capacitor C = 47 μF and switch S.

The circuit has been running in equilibrium for a long time. We are now going to shut off the function generator (instantaneously replace it with a wire).

(a) Assuming that our driving frequency ω is not necessarily on resonance, what is the frequency with which the system will ring down (in other words, that current will oscillate at after turning off the function generator)? Feel free to use an approximation if you wish, just make sure you know you are.

(b) At what (numerical) frequency f should we drive to maximize the peak magnetic energy in the inductor?

(c) In this case, if we time the shut off to occur when the magnetic energy in the inductor peaks, after how long will the electric energy in the capacitor peak?

(d) Approximately how much energy will the resistor have dissipated during that time?

2. Phase in an RLC Circuit

Using the same circuit as in problem 1, only this time leaving the function generator on and driving below resonance, which in the following pairs leads (if either):

(a) Voltage across the capacitor or voltage across the resistor

(b) Voltage across the function generator or voltage measured across the inductor

(c) Current or voltage across the resistor

(d) Current or voltage across the function generator

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop. Start LabView by double clicking on this file.
2. Set up the circuit pictured in Fig. 5 of the pre-lab reading (no core in the inductor!)
3. Connect a voltage probe to channel A of the 750 and connect it across the capacitor.

MEASUREMENTS

Part 1: Free Oscillations in an Undriven RLC Circuit

In this part we turn on a battery long enough to charge the capacitor and then turn it off and watch the current oscillate and decay away.

1. Press the green "Go" button above the graph to perform this process.

Question 1:

What is the period of the oscillations (measure the time between distant zeroes of the current and divide by the number of periods between those zeroes)? What is the frequency?

Question 2:

Is this frequency the same as, larger than or smaller than what you calculated it should be? If it is not the same, why not?

Part 2: Energy Ringdown in an Undriven RLC Circuit

1. Repeat the process of part 1, this time recording the energy stored in the capacitor $\left(U_C = \frac{1}{2}CV^2\right)$ and inductor $\left(U_L = \frac{1}{2}LI^2\right)$, and the sum of the two.

Question 3:

The circuit is losing energy most rapidly at times when the slope of total energy is steepest. Is the electric (capacitor) or magnetic (inductor) energy a local maximum at those times? Briefly explain why.

Part 3: Driving the RLC Circuit on Resonance

Now we will use the function generator to drive the circuit with a sinusoidal voltage.

1. Enter the frequency that you measured in part 1 of the lab as a starting point to find the resonant frequency.
2. Press GO to start recording the function generator current and voltage vs. time, as well as a "phase plot" of voltage vs. current.
3. Adjust the frequency up and down to find the resonant frequency and observe what happens when driving above and below resonance.

Question 4:

What is the resonant frequency? What are two ways in which you know?

Question 5:

What is the impedance of the circuit when driven on resonance (hint: use the phase plot)?

Question 6:

When driving on resonance, insert the core into the inductor. Are you now driving at, above or below the new resonant frequency of the circuit? How can you tell? Why?

Part 4: What's The Frequency?

For the remainder of the lab you will make some measurements where you are given incomplete information (for example, you won't be shown the frequency or won't be told what is being plotted). From the results you must determine the missing information. If you find this difficult, play with the circuit using the "further questions" tab to get a better feeling for how the circuit behaves.

1. Remove the core from the inductor
2. Press GO to record the function generator current and voltage

Question 7:

At this frequency is the circuit capacitor- or inductor-like? Are we above or below resonance?

Part 5: What's That Trace?

1. Press GO to record the function generator current and voltage as well as the voltage across the capacitor. Note that you are not told which trace corresponds to which value.

Question 8:

What value is recorded in each of the three traces (I, V_{FG} or V_C)? How do you know?

Question 9:

Are we above, below or on resonance? How do you know?

Further Questions (for experiment, thought, future exam questions…)

- What happens if we insert the coil core in the undriven circuit?
- For a random frequency can you bring the circuit into resonance by slowly inserting the core into the coil? Are there any conditions on the frequency (e.g. does it need to be above or below the resonant frequency of the circuit with the empty coil)?
- Could you do part 5 if you were given only two traces instead of three? Would it matter which two you were given?
- What is the energy doing in the driven case? Is the resistor still dissipating power? If so, where is this power coming from?
- What happens to the resonant frequency of the circuit if a resistor is placed in series with the capacitor and coil? In parallel? NOTE: You can use the variable resistor, called a potentiometer or "pot" (just to the left of the coil, connect to the center and right most contacts, allowing you to adjust the extra resistance from 0 Ω to 3.3Ω by simply turning the knob).
- With the resistor in series with the coil and capacitor, at what frequency is the energy dissipation a maximum? How could you verify this experimentally?

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2006**

Experiment 8: Microwaves

OBJECTIVES

1. To observe the polarization and angular dependence of radiation from a microwave generator
2. To measure the wavelength of the microwave radiation by analyzing an interference pattern similar to a standing wave

PRE-LAB READING

INTRODUCTION

Heinrich Hertz first generated electromagnetic waves in 1888, and we replicate Hertz's original experiment here. The method he used was to charge and discharge a capacitor connected to a spark gap and a quarter-wave antenna. When the spark "jumps" across the gap the antenna is excited by this discharge current, and charges oscillate back and forth in the antenna at the antenna's natural resonance frequency. For a brief period around the breakdown ("spark"), the antenna radiates electromagnetic waves at this high frequency. We will detect and measure the wavelength λ of these bursts of radiation. Using the relation $f\lambda = c = 3\times10^{10}\,\text{cm/s}$, we will then deduce the natural resonance frequency of the antenna, and show that this frequency is what we expect on the basis of the very simple considerations given below.

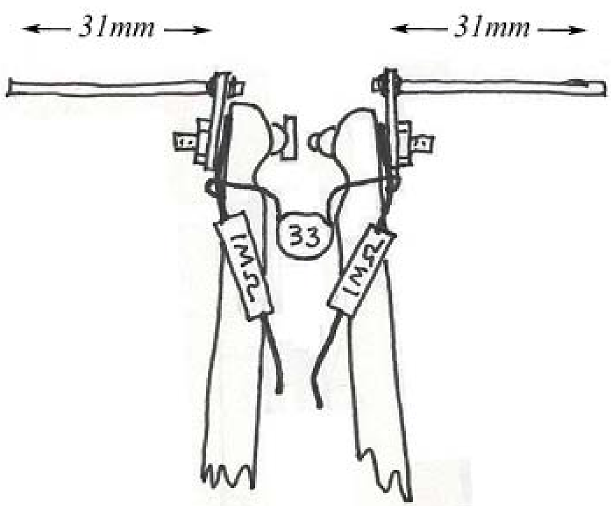


Figure 1 Spark-gap transmitter. The "33" is a 33 pF capacitor. It is responsible for storing energy to be rapidly discharged across a "spark gap," formed by two tungsten cylinders pictured directly above it (one with a vertical axis, one horizontal). Two MΩ resistors limit current off of the capacitor and back out the leads, protecting the user from shocks from the 800 V to which the capacitor will be charged. They also limit radiation at incorrect frequencies.

The 33-pF capacitor shown in fig. 1 is charged by a high-voltage power supply on the circuit board provided. This HVPS voltage is typically 800 V, but this is safe because the current from the supply is limited to a very small value. When the electric field that this voltage generates in the "spark gap" between the tungsten rods is high enough (when it exceeds the breakdown field of air of about 1000 V/mm) the capacitor discharges across the gap (fig. 2a). The voltage on the capacitor then rebuilds, until high enough to cause another spark, resulting in a continuous series of charges followed by rapid bursts of discharge (fig. 2b).

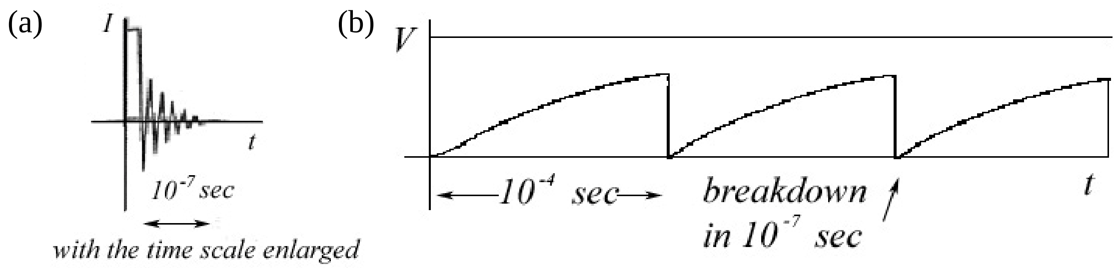


Figure 2 Charging and Discharging the Capacitor. The capacitor is slowly charged (limited by the RC time constant, with R = 4.5 MΩ) and then (a) rapidly discharges across the spark gap, resulting in (b) a series of slow charge/rapid discharge bursts. This is an example of a "relaxation oscillator."

The radiation we are seeking is generated in this discharge.

Resonant Frequency of the Antenna

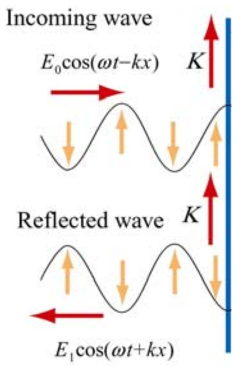
The frequency of the radiation is determined by the time it takes charge to flow along the antenna. Just before breakdown, the two halves of the antenna are charged positive and negative $(+,-)$ forming an electric dipole. There is an electric field in the vicinity of this dipole. During the short time during which the capacitor discharges, the electric field decays and large currents flow, producing magnetic fields. The currents flow through the spark gap and charge the antenna with the opposite polarity. This process continues on and on for many cycles at the resonance frequency of the antenna. The oscillations damp out as energy is dissipated and some of the energy is radiated away until the antenna is finally discharged.

How fast do these oscillations take place – that is, what is the resulting frequency of the radiated energy? An estimate can be made by thinking about the charge flow in the antenna once a spark in the gap allows charge to flow from one side to the other. If l is the length of one of the halves of the antenna (about $l = 31\text{ mm}$ in our case), then the distance that the charge oscillation travels going from the $(+,-)$ polarity to the $(-,+)$ polarity and back again to the original $(+,-)$ polarity is $4l$ (from one tip of the antenna to the other tip and back again). The time T it takes for this to happen, assuming that information flows at the speed of light c, is $T = 4l / c$, leading to electromagnetic radiation at a frequency of $1/T$.

Detecting (Receiving) the Radiation

In addition to generating EM radiation we will want to detect it. For this purpose we will use a receiving antenna through which charge will be driven by the incoming EM radiation. This current is rectified and amplified, and you will read its average value on a multimeter (although the fields come in bursts, the multimeter will show a roughly constant amplitude because the time between bursts is very short, as you will calculate in pre-lab #1).

Creating Standing Waves (sort of) Using a Reflector



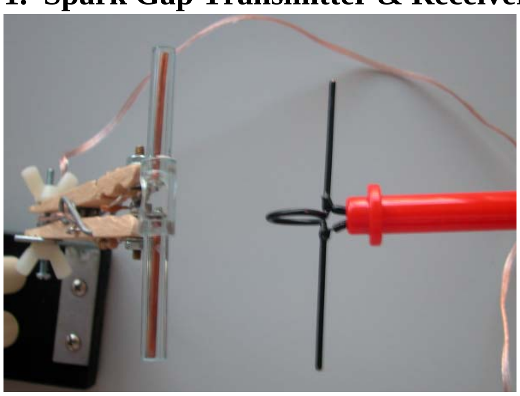
If a conducting sheet (blue in the figure at left) is placed in the path of an electromagnetic wave, surface currents will be generated in the conductor that reradiate the wave backwards, as a reflected wave. The phase of the reflected wave is set so as to guarantee that the electric field at the surface of the conductor is always zero. The reflected and incoming waves interfere, summing to create a new standing wave, which has nodes every half wavelength. In the figure at left at the time pictured the waves cancel everywhere. A small time later this will no longer be the case, and the cancellation will only be perfect at those nodes (as you will calculate in pre-lab #3).

Thus in order to measure the wavelength of radiation we can simply move the receiver between the reflector and the antenna and look for nodes. Using the distance between nodes we can then calculate the wavelength.

This ideal picture doesn't perfectly hold in our case. The antenna creates dipole radiation rather than plane waves, so the amplitude of the radiation is not spatially constant, but instead falls with distance from the transmitter. Thus we do not create a perfect standing wave. However, minima in the radiation intensity will still be created where standing wave nodes occur in the ideal case, and you will still be able to measure the wavelength.

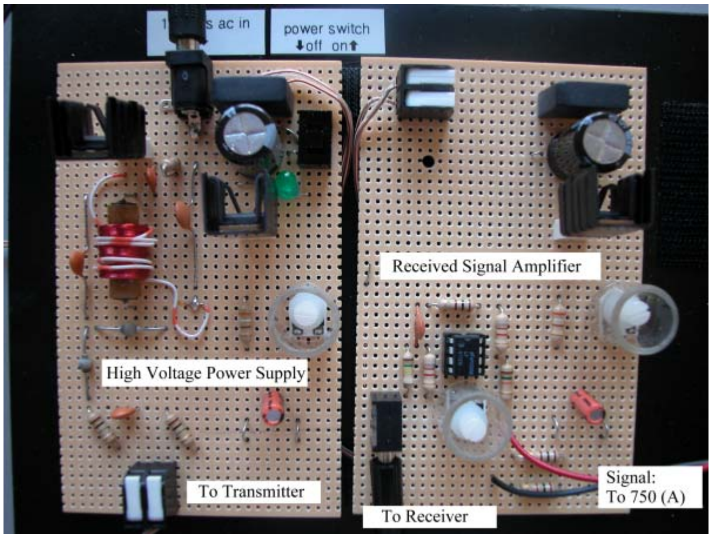
APPARATUS

1. Spark Gap Transmitter & Receiver



These have been described in detail above. The spark gap of the transmitter (pictured left) can be adjusted by turning the plastic wing nut (top). It is permanently wired in to the high voltage power supply on the circuit board. The receiver (pictured right) must be plugged in to the circuit board.

2. Circuit Board



This board contains a high voltage power supply for charging the transmitter, as well as an amplifier for boosting the signal from the receiver. It is powered by a small DC transformer that must be plugged in (AC in). When power is on, the green LED (top center) will glow.

3. Science Workshop 750 Interface and Voltage Probe

We read the signal strength from the receiver – proportional to the radiation intensity at the receiver – by connecting the output (lower right of circuit board) to a voltage probe plugged in to channel A of the 750.

GENERALIZED PROCEDURE

In this lab you will turn on the transmitter, and then, using the receiver, measure the intensity of the radiation at various locations and orientations. It consists of three main parts.

Part 1: Polarization of the Emitted Radiation
In this part you will measure to see if the produced radiation is polarized, and if so, along what axis.

Part 2: Angular Dependence of the Emitted Radiation
Next, you will measure the angular dependence of the radiation, determining if your position relative to the transmitter matters.

Part 3: Wavelength of the Emitted Radiation
Finally, you will measure the wavelength (and hence frequency) of the emitted radiation by placing a reflector near the transmitter and creating a standing-wave like pattern. By measuring the distances between minima, you can determine the wavelength (as calculated in pre-lab #3).

END OF PRE-LAB READING

Experiment 8: Microwaves

Answer these questions on a separate sheet of paper and turn them in before the lab

1. Spark Gap Distance and Timing

The time to charge the transmitter capacitor until it discharges depends on the resistance in the charging circuit (R = 4.5 MΩ), the capacitance (C = 33 pF) and the voltage required to initiate breakdown. Assume that the power supply supplies 800 V but that breakdown typically occurs at a voltage of about 500 V on the capacitor.

(a) Thinking of the tungsten electrodes as parallel plates, how far apart must they be in order generate a spark at 500 V?

(b) In reality, the electrodes aren't parallel plates, but rather cylinders with a fairly small radius of curvature. Given this, will the distance needed between the electrodes to generate sparking be smaller or larger than you calculated in (a)? Why?

(c) About how much time will it take for the power supply to charge the capacitor from empty to discharge?

2. Wavelength and Frequency of the Radiation

The spark-gap antenna is a quarter-wavelength antenna, radiating as described above. Using l = 31 mm for the length of one of the arms of the antenna, what is

(a) the wavelength of the emitted radiation?

(b) the frequency of the emitted radiation?

3. Reflections

Now place the transmitter some distance in front of a perfectly conducting sheet, oriented so that the propagation direction of the waves hitting the reflector is perpendicular to the plane of the reflector (so that they'll reflect straight back out towards the transmitter). For example, place the transmitter at $z = -D$ with the antenna parallel to the x-axis, and have the reflector fill the z=0 (xy-) plane.

(a) Write an equation for the electric field component of the radiation from the transmitter (the *incident* wave). Treat the field as plane wave, with a constant amplitude E_0 and angular frequency ω_0.

(b) What condition must the total electric field satisfy at the surface of the conductor (z=0)?

(c) What is the direction of propagation of the reflected wave?

(d) Write an equation for the time dependent amplitude & direction of the reflected wave, making the same assumptions as above.

(e) Write an equation for the total amplitude of the electric field as a function of position, by adding (c) and (d).

(f) Nodes are locations (in this case planes) where the electric field is zero at all times. What is the distance between nodes along the z-axis?

(g) What is the numerical distance you thus expect for our transmitter (i.e. use 2a)?

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

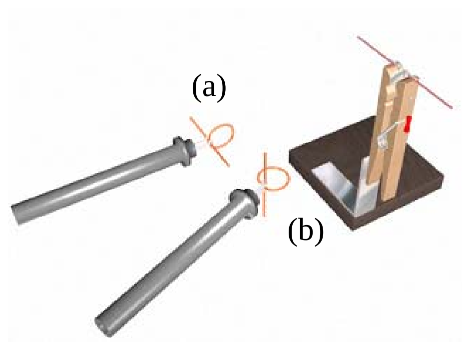
1. Download the LabView file from the web and save the file to your desktop. Start LabView by double clicking on this file.
2. Plug the power supply into the circuit board
3. Plug the receiver into the input jack on the circuit board
4. Turn on the transmitter – a LED will light indicating it is on
5. Adjust the spark gap using the wing nut on the clothespin antenna. Start with a large gap, and close the gap until a steady spark is observed. You should observe a small, steady bright blue light and hear the hum of sparking.
6. Use the receiver to measure the intensity of the radiation as described below

MEASUREMENTS

<u>Part 1: Polarization of the Emitted Radiation</u>

In this part we will measure the polarization of the emitted radiation.

1. Press the green "Go" button above the graph to perform this process.
2. Rotate the receiver between the two orientations (a & b) pictured at right



Question 1:

Which orientation, if either, results in a larger signal in the receiver?

Question 2:

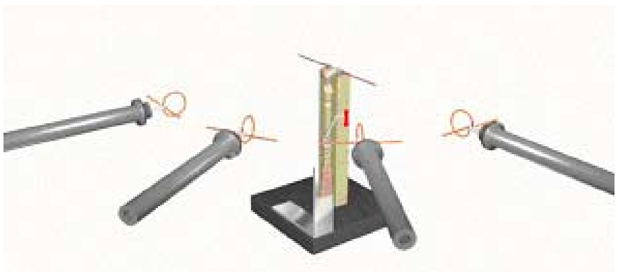
Is the electric field polarized? That is, is it oscillating along a certain direction, as opposed to being unpolarized in which case it points along a wide variety of directions? If it is polarized, along which axis?

Question 3:

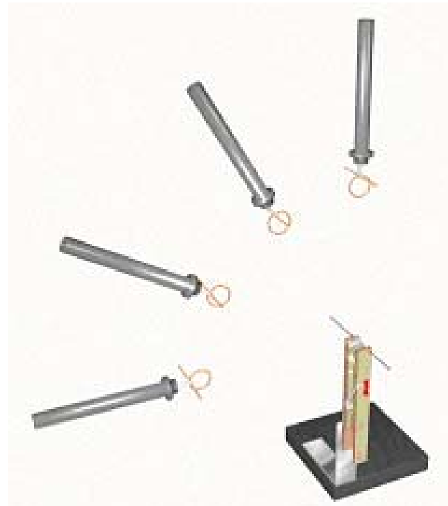
Is the magnetic field polarized? If so, along which axis? How do you know?

Part 2: Angular Dependence of the Emitted Radiation

1. Now measure the angular dependence of the radiation intensity by moving the receiver along the two paths indicated in the below figures.



Angular dependence - Horizontal



Angular dependence - Vertical

Question 4:

Which kind of motion, horizontal or vertical, shows a larger change in radiation intensity over the range of motion?

Part 3: Wavelength of the Emitted Radiation

Now we will use the reflector to create an interference pattern that we can use to measure the wavelength of the radiation. We will do this in two ways.

1. Position the reflector so that it is about 30-40 cm from the spark gap transmitter (note that you can insert the tube into notch on the bottom of the reflector in order to make a stable base for it to stand on). You may need to change this distance slightly to get good results. Experiment!
2. Orient the transmitter so that the propagation direction of the waves hitting the reflector is perpendicular to the plane of the reflector (so that they'll reflect straight back out towards the transmitter).
3. Place the receiver between the transmitter and the reflector and then on the other side of the reflector to see that the waves are reflected (not at all transmitted).
4. Place the receiver, in the best orientation, between the reflector and the transmitter very near the reflector.

Part 3A: Imprecise Method

1. Slowly move the receiver towards the transmitter, looking for a minimum in the radiation intensity. Be careful to keep the receiver at a constant height and in line between the transmitter and the reflector.
2. Measure the distance between the first and second minimum that you find.

Question 5:

What is the distance between intensity minima?

Part 3B: More Precise Method

1. Slowly move the receiver towards the transmitter, looking for a minimum in the radiation intensity.
2. Now, hold the receiver very steady and have another group member slowly pull the reflector away from the receiver. Because the reflector sits on the table it is easier to measure the distance it has traveled when the next minimum is found.
3. Repeat the measurement several times to get an accurate average distance reading.

Question 6:

What is the distance between intensity minima?

Question 7:

What is the wavelength of the radiation? What frequency does this correspond to?

<u>Further Questions (for experiment, thought, future exam questions…)</u>

- Is there any radiation intensity of any polarization off the ends of the antenna?
- An antenna similar to this was used by Marconi for his first transatlantic broadcast. What issues would you face to receive such a broadcast?
- What happens if the reflector is tilted relative to the transmitter? How tilted can it be for you to still get a reasonable result?
- If you had a second reflector on the other side of the transmitter, could you create a resonant cavity where the reflected waves from each reflector add constructively? Would it matter where the transmitter was in the cavity?

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2006**

Experiment 9: Interference and Diffraction

OBJECTIVES

1. To explore the diffraction of light through a variety of apertures
2. To learn how interference can be used to measure small distances very accurately. By example we will measure the wavelength of the laser, the spacing between tracks on a CD and the thickness of human hair

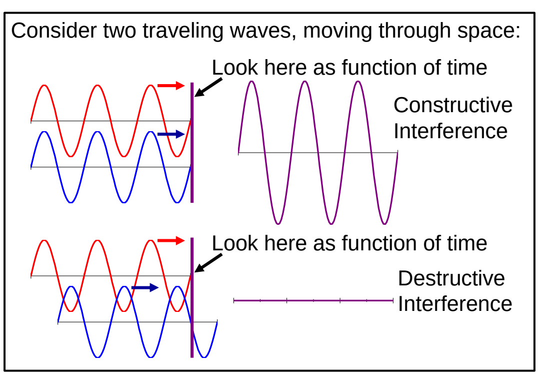
WARNING! The beam of laser pointers is so concentrated that it can cause *real* damage to your retina if you look into the beam either directly or by reflection from a shiny object. Do NOT shine them at others or yourself.

PRE-LAB READING

INTRODUCTION

Electromagnetic radiation propagates as a wave, and as such can exhibit interference and diffraction. This is most strikingly seen with laser light, where light shining on a piece of paper looks speckled (with light and dark spots) rather than evenly illuminated, and where light shining through a small hole makes a pattern of bright and dark spots rather than the single spot you might expect from your everyday experiences with light. In this lab we will use laser light to investigate the phenomena of interference and diffraction and will see how we can use these phenomena to make accurate measurements of very small objects like the spacing between tracks on a CD and the thickness of human hair.

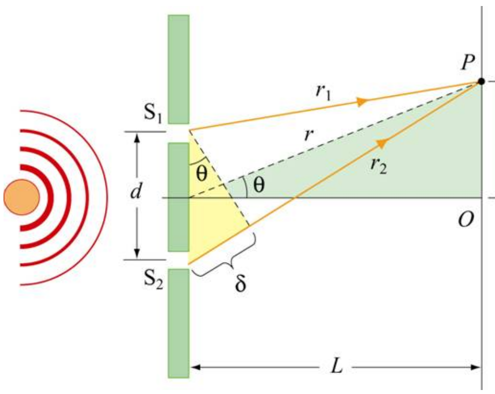
The Details: Interference



The picture at left forms the basis of all the phenomena you will observe in the lab. Two different waves arrive at a single position in space (at the screen). If they are in phase then they add constructively and you see a bright spot. If they are out of phase then they add destructively and you see nothing (dark spot).

The key to creating interference is creating phase shift between two waves that are then brought together at a single position. A common way to do that is to add extra path length to one of the waves relative to the other. In this lab the distance traveled from source to screen, and hence the relative phase of incoming waves, changes as a function of lateral position on the screen, creating a visual interference pattern.

Two Slit Interference



The first phenomenon we consider is two slit interference. Light from the laser hits two very narrow slits, which then act like in-phase point sources of light. In traveling from the slits to the screen, however, the light from the two slits travel different distances. In the picture at left light hitting point P from the bottom slit travels further than the light from the top slit. This extra path length introduces a phase shift between the two waves and leads to a position dependent interference pattern on the screen.

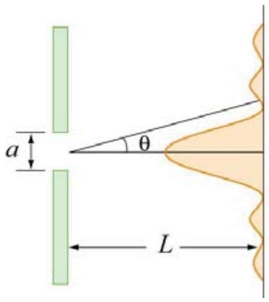
Here the extra path length is $\delta = d\ \sin\left(\theta\right)$, leading to a phase shift ϕ given by $\dfrac{\delta}{\lambda} = \dfrac{\phi}{2\pi}$.

Realizing that phase shifts that are multiples of 2π give us constructive interference while odd multiples of π lead to destructive interference leads to the following conditions: Maxima: $d\ \sin\left(\theta\right) = m\lambda$; Minima: $d\ \sin\left(\theta\right) = \left(m + \tfrac{1}{2}\right)\lambda$

Multiple Slit Interference

If instead of two identical slits separated by a distance *d* there are multiple identical slits, each separated by a distance *d*, the same effect happens. For example, at all angles θ satisfying $d\ \sin\left(\theta\right) = m\lambda$ we find constructive interference, now from all of the holes. The difference in the resulting interference pattern lies in those regions that are neither maxima or minima but rather in between. Here, because more incoming waves are available to interfere, the interference becomes more destructive, making the minima appear broader and the maxima sharper. This explains the appearance of a brilliant array of colors that change as a function of angle when looking at a CD. A CD has a large number of small grooves, each reflecting light and becoming a new source like a small slit. For a given angle, a distinct set of wavelengths will form constructive maxima when the reflected light reaches your eyes.

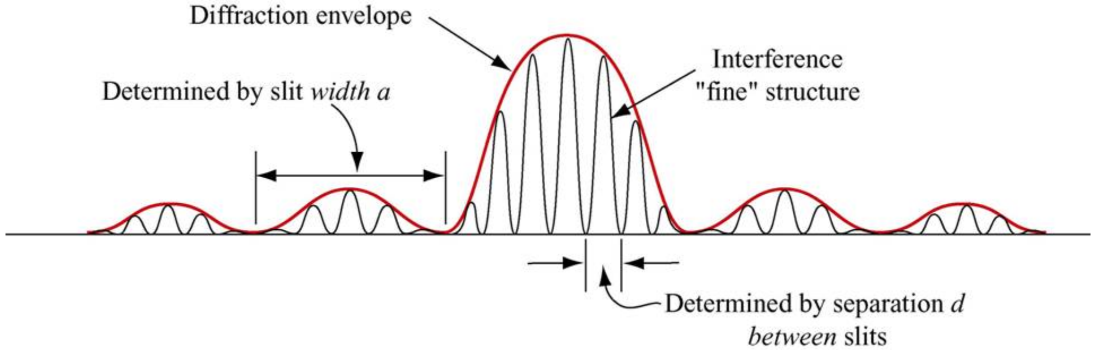
Diffraction



The next kind of interference we consider is light going through a single slit, interfering with itself. This is called diffraction, and arises from the finite width of the slit (*a* in the picture at left). The resultant effect is not nearly as easy to derive as that from two-slit interference (which, as you can see from above, is straight-forward). The result for the anglular locations of the minima is $a\sin\left(\theta\right) = m\lambda$.

Putting it Together

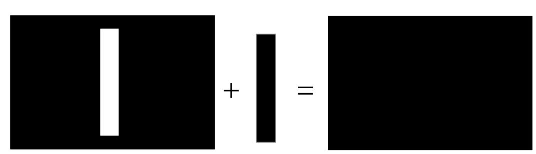
If you have two wide slits, that is, slits that exhibit both diffraction and interference, the pattern observed on a distant screen is as follows:



Here the amplitude modulation (the red envelope) is set by the diffraction (the width of the slits), while the "individual wiggles" are due to the interference between the light coming from the two different slits. You know that this must be the case because d must be larger than a, and hence the minima locations, which go like $1/d$, are closer together for the two slit pattern than for the single slit pattern.

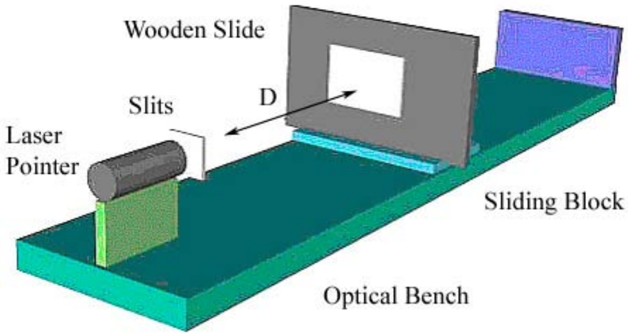
The Opposite of a Slit: Babinet's Principle

So far we have discussed sending light through very narrow slits or reflecting it off of small grooves, in each case creating a series of point-like "new sources" of light that can then go on and interfere. Rather amazingly, light hitting a small solid object, like a piece of hair, creates the same interference pattern as if the object were replaced with a hole of the same dimensions. This idea is Babinet's Principle, and the reason behind it is summed up by the pictoral equation at right. If you add an object to a hole of the same size, you get a filled hole. EM waves hitting those objects must add in the same fashion, that is, the electric fields produced when light hits the hole, when added to the electric fields produced by the small object, must add to the electric fields produced when light hits the filled hole. Since no light can get through the filled hole, $E_{hole} + E_{object} = 0$. Thus we find that the electric fields coming out of the hole are equal and opposite to the electric fields diffracting off of the small object. Since the observed interference pattern depends on intensity, the square of the electric field, the hole and the object will generate identical diffraction patterns. By measuring properties of the diffraction pattern we can thus measure the width of the small object. In this lab the small object will be a piece of your hair.



APPARATUS

1. Optical bench



The optical bench consists of a holder for a laser pointer, a mount for slides (which contain the slits you will shine light through), and a sliding block to which you will attach pieces of paper to mark your observed interference patterns. Note that a small ring can be slid over the button of the laser pointer in order to keep it on while you make your measurements.

2. Slit Slides

You will be given two slides, each containing four sets of slits labeled a through d. One slide contains single slits with widths from 20 μm to 160 μm. The other slide contains double slits with widths of 40 μm or 80 μm, separated by distances of 250 μm or 500 μm.

GENERALIZED PROCEDURE

In this lab you will shine the light through slits, across hairs or off of CDs and make measurements of the resulting interference pattern.

Part 1: Laser Wavelength

In this part you will measure the wavelength of the laser using the two narrow double slits, and making the measurements that you determine are necessary in pre-lab #1.

Part 2: Diffraction from a CD

Finally, you will measure the width of tracks on a CD by reflecting laser light off of it and measuring the resulting diffraction pattern.

Part 3: Thickness of Human Hair

Next you will discover the ability to measure the size of small objects using diffraction, by measuring the width of a human hair.

END OF PRE-LAB READING

Experiment 9: Interference and Diffraction

Answer these questions on a separate sheet of paper and turn them in before the lab

1. Measuring the Wavelength of Laser Light

In the first part of this experiment you will shine a red laser through a pair of narrow slits (a = 40 μm) separated by a known distance (you will use both d = 250 μm and 500 μm) and allow the resulting interference pattern to fall on a screen a distance L away (L ~ 40 cm). This set up is as pictured in Fig. 2 (in the "Two Slit Interference" section above).

(a) Will the center of the pattern (directly between the two holes) be an interference minimum or maximum?

(b) You should be able to easily mark and then measure the locations of the interference maxima. For the sizes given above, will these maxima be roughly equally spaced, or will they spread out away from the central peak? If you find that they are equally spaced, note that you can use this to your advantage by measuring the distance between distance maxima and dividing by the number of intermediate maxima to get an average spacing. If they spread out, which spacing should you use in your measurement to get the most accurate results, one close to the center or one farther away?

(c) Approximately how many interference maxima will you see on one side of the pattern before their intensity is significantly reduced by diffraction due to the finite width a of the slit?

(d) Derive an equation for calculating the wavelength λ of the laser light from your measurement of the distance Δy between interference maxima. Make sure that you keep a copy of this equation in your notes! You will need it for the lab.

(e) In order to most accurately measure the distance between maxima Δy, it helps to have them as far apart as possible. (Why?) Assuming that the slit parameters and light wavelength are fixed, what can we do in order to make Δy bigger? What are some reasons that can we not do this ad infinitum?

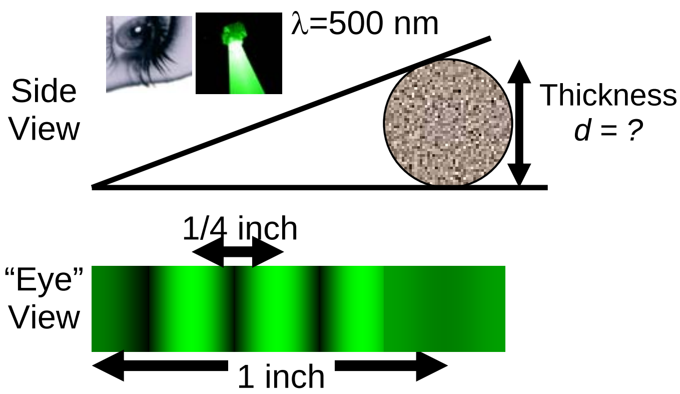
2. Single Slit Interference

Now that you have measured the wavelength λ of the light you are using, you will want to measure the width of some slits from their diffraction pattern. When measuring diffraction patterns (as opposed to the interference patterns of problem 1) it is typically easiest to measure between diffraction minima.

(a) Derive an equation for calculating the width a of a slit from your measurement of the distance Δy between diffraction minima. Make sure that you keep a copy of this equation in your notes! You will need it for the lab. Note that this same equation will be used for measuring the thickness of your hair.

(b) What is the width of the central maximum (the distance on the screen between the m=-1 and m=1 minima)? How does this compare to the distance Δy between other adjacent minima?

3. Another Way to Measure Hair



In addition to using hair as a thin object for diffraction, you can also measure its thickness using an interferometer. In fact, you can use this to measure even smaller objects. Its use on a small fiber is pictured at left. The fiber is placed between two glass slides, lifting one at an angle relative to the other. The slides are illuminated with green light from above, and when the set-up is viewed from above, an interference pattern, pictured in the "Eye View", appears.

What is the thickness *d* of the fiber?

4. CD

In the last part of this lab you will reflect light off of a CD and measure the resulting interference pattern on a screen a distance $L \sim 5$ cm away.

(a) A CD has a number of tracks, each of width *d* (this is what you are going to measure). Each track contains a number of bits, of length $l \sim d/3$. Approximately how many bits are there on a CD? In case you didn't know, CDs sample two channels (left and right) at a rate of 44100 samples/second, with a resolution of 16 bits/sample. In addition to the actual data bits, there are error correction and packing bits that roughly double the number of bits on the CD.

(b) What, approximately, must the track width be in order to accommodate this number of bits on a CD? In case you don't have a ruler, a CD has an inner diameter of 40 mm and an outer diameter of 120 mm.

(c) Derive an equation for calculating the width *d* of the tracks from your measurement of the distance Δy between interference maxima. Make sure that you keep a copy of this equation in your notes! You will need it for the lab.

(d) Using the previous results, what approximately will the distance between interference maxima Δy be on the screen?

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop. Start LabView by double clicking on this file.

MEASUREMENTS

Part 1: Laser Wavelength

In this part you will measure the wavelength λ of the laser light you are using

1. Set up the optical bench as pictured in the apparatus diagram.
    a. Clip paper onto the wooden slide, and place roughly 40 cm away from the slide holder
    b. Place the double slit slide in the slide holder and align so that light from the laser goes through slit pattern *a*.
    c. Turn the laser on (lock it with the clip that slides around the on button)
    d. Adjust the location of the wooden slide so that the pattern is visible but as large as possible
2. Mark the locations of the intensity maxima. If they are too close to measure individually, mark of a set of them and determine the average spacing.

Question 1:

What distance between the slide and the screen did you use? What was the average distance Δy between maxima?

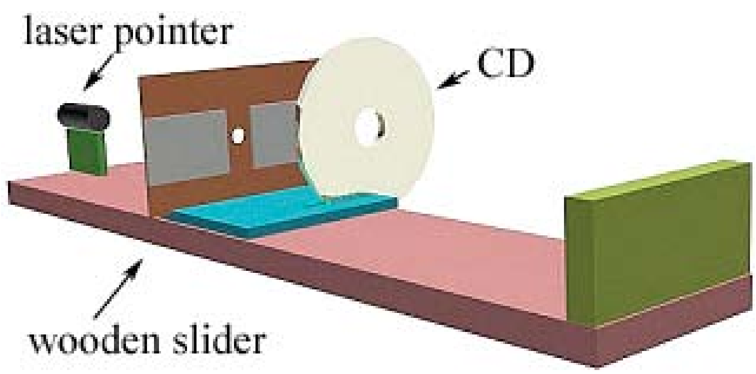
Question 2:

Using $\lambda = \frac{d\Delta y}{L}$, what do you calculate to be the wavelength of the laser light? Does this make sense?

Part 2: Diffraction from a CD

In this part you will determine the track width on a CD by measuring the distance between interference maxima generated by light reflected from it.

1. Remove the slide from in front of the laser pointer
2. Clip a card with a hole in it to the back of the wooden slide.
3. Place a CD in the groove in the back of the wooden slider. Light will pass through the hole in the slider and card, reflect off the CD, and land on the card.
4. Turn on the laser and measure the distance between interference maxima.



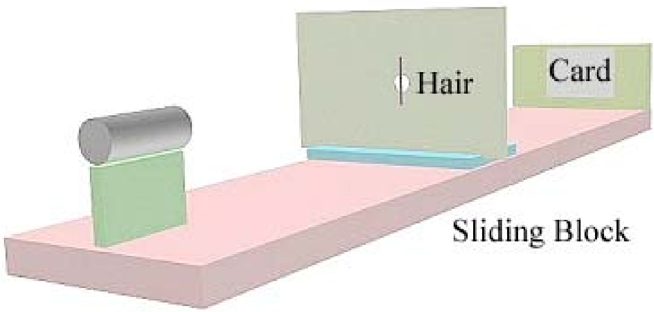
Question 3:

Using $d = \frac{\lambda L}{\Delta y}$, what is the width of the tracks? Does this make sense? Why are they that size?

Part 3: Thickness of Human Hair

Now you will measure the thickness of a human hair using diffraction.

1. Remove the CD and card from the wooden slide, and tape some hair across the hole (the hair should run vertically as pictured below).
2. Clip a card to the block at the end of the apparatus.
3. Shine the laser on the hair, and adjust the distance between the hair and the card so that you obtain a useable diffraction pattern.



Question 4:

What is the thickness of the hair that you measure? Does this seem reasonable?

Further Questions (for experiment, thought, future exam questions…)

- Instead of measuring the wavelength of light from the two slit patterns, you could have instead used single slits. Would that have been more or less accurate? Why?
- Why did you use two slit patterns a to measure the light wavelength rather than pattern d.
- Where does most of the measurement error come from? How would you improve this in future labs?
- If we redid these experiments with a blue laser instead of red, what changes would you have needed to make? Would it have affected the accuracy of the measurements?
- Does the track width change as a function of location on the CD? If so, is it larger or smaller near the outside?
- What is the ratio of the track size to the wavelength of the light that you used (which is very similar to the wavelength of light used in commercial players)?
- What would happen to the diffraction pattern if the track width was smaller?
- Why are people anxious to move to blue lasers in commercial CD and DVD players?