

Topics: Driven LRC Circuits

Related Reading:

Course Notes (Liao et al.): Chapter 12

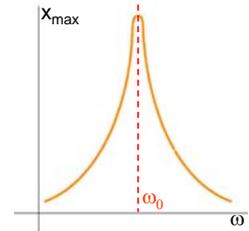
Experiments: (7) Undriven & Driven LRC Circuits Part Two: Driven RLC Circuits

Topic Introduction

In today's class we will investigate the behavior of LRC circuits that we introduced in Monday's class in an extended experiment. First you will study undriven circuits, which behave like a mass on a spring which is pulled and then released. Then you will look at the behavior of driven circuits that exhibit resonance.

Driven LRC Circuits

We can also add a force to our circuits – the AC power supply. In this case the current responds at the drive frequency. However, depending on the frequency of the drive, the current may be out of phase (either *leading* or *lagging* the drive) and its amplitude will also vary. This is easily seen in mechanical systems. For a fantastic example, play with the pendula at the Kendall T station. Depending on how fast you drive them they will respond either in or out of phase with your drive, and they will either move a little or a lot. When you drive at the natural frequency, the amplitude increases greatly, and the system is “in resonance.”



One Element at a Time

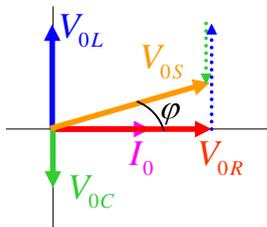
In order driven LRC circuits its easiest to start thinking about individual circuit elements. A resistor obeys Ohm's law: $V=IR$. Neither the amplitude nor phase of the voltage depends on the frequency (the response voltage is always in phase with the current).

A capacitor is different. At low frequencies the capacitor “fills up,” but at high frequencies it begins discharging before “filling up.” The voltage is frequency dependent and the current *leads* the voltage (current flows before the charge/potential on the capacitor appears).

An inductor is the opposite – it hates the change of high frequencies and thus has large voltages there – and the current *lags* – the inductor fights before current flows.

When put together in LRC circuits, the capacitor “dominates” at low frequencies, the inductor at high ones. At resonance ($\omega = \sqrt{1/LC}$) the frequency is such that these two effects balance and the current will be largest in the circuit. Also at this frequency the current is in phase with the driving voltage (the AC power supply).

Seeing it Mathematically – Phasors



To do this mathematically we will use phasor diagrams. A phasor is a vector whose magnitude is the amplitude of either voltage or current and whose angle corresponds to its phase. Phasors rotate CCW about the origin with time as their phase evolves, and their current amplitude is their component along the y-axis, which oscillates as it should.

Phasors allow us to add voltages that are not in phase with each other.

For example, the phasor diagram above illustrates the relationship of voltages in a series LRC circuit. The current I is assigned to be at “0 phase” (along the x-axis). The phase of the voltage across the resistor is the same. The voltage across the

inductor L leads (is ahead of I) and the voltage across the capacitor C lags (is behind I). If you add up (using vector arithmetic) the voltages across R , L & C (the red and dashed blue & green lines respectively) you arrive at the voltage across the power supply. This then gives you a rapid way of understanding the phase between the drive (the power supply) and the response (the current) – here labeled ϕ .

Natural Frequency

Last class we derived the natural frequency $\omega = \sqrt{1/LC}$ at which an undriven, undamped circuit will ring. This is also the frequency at which a driven circuit will be in resonance, that is, where it will exhibit the maximum current for a given drive voltage and where the current and drive voltage will be in phase.

Reactance & Impedance

The reactance X is the proportionality constant relating voltage across a circuit element and current through it ($V_0 = I_0 X$). The difference between resistance and reactance is that the current through and voltage across a resistance are always in phase, while for a reactance they are always 90° out of phase. The generic term, impedance (Z), is used when the current and voltage have an arbitrary phase relationship.

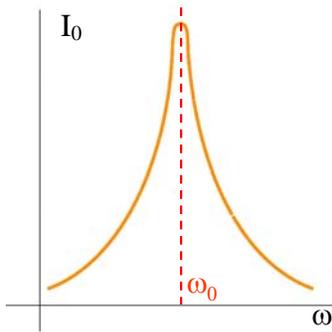
The reactance of a capacitor is $X_c = 1/\omega C$. If you drive current at a low frequency the capacitor will fill up and have a large voltage across it, whereas if you drive current a high frequency the capacitor will begin discharging before it has a chance to completely charge, and hence it won't build up as large a voltage. The current *leads* the voltage by 90° , as is most clearly seen in an uncharged capacitor where the current must flow before the charge/potential on the capacitor can build up.

The reactance of an inductor is $X_L = \omega L$. It has a larger voltage when the current frequency is high, because it doesn't like change and high frequency means lots of change. Now the current *lags* the voltage by 90° – if you try to drive a current through an inductor with no current in it, the inductor will immediately put up a fight (create an EMF) and then later allow current to flow.

Using phasors, we added the voltages across a resistor, inductor and capacitor in series and found the impedance and phase of the circuit is given by:

$$Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (\omega L - 1/\omega C)^2} \text{ and } \tan \varphi = X/R = (\omega L - 1/\omega C)/R$$

As you can see from the phase relationship (or by simply thinking about the circuit), at low frequencies the capacitor will “dominate” (it fills up, limiting the current) whereas at high frequencies the inductor will dominate (it fights the rapid changes). At resonance ($\omega = \sqrt{1/LC}$) these two effects balance, the impedance is a minimum and the phase is zero (current is in phase with the driving voltage of the AC power supply).



The amplitude of the current for a given drive thus varies as a function of frequency, resulting in a frequency response curve similar to that pictured at left. Mathematically,

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

Important Equations

Impedance of R, L, C: $R = R$ (in phase), $X_C = \frac{1}{\omega C}$ (I leads), $X_L = \omega L$ (I lags)

Impedance of series RLC: $Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

Phase in series RLC: $\tan \varphi = X/R = (\omega L - 1/\omega C)/R$

NOTE: Although I place these equations here they are not as fundamental as the impedance of R, L & C individually. Those resistance/reactances always hold, while this impedance and phase are only valid in series RLC circuits. Furthermore, these equations are readily derivable, and you should definitely know how to do so.

Experiment 7: Undriven and Driven LRC Circuits Part Two: driven RLC Circuits

Preparation: Read pre-lab and answer pre-lab questions.

This lab consists of two main parts. You did the first part last class involving an undriven LRC circuit and determined its natural frequency. In the second part you will drive the same circuit around that natural frequency to confirm that it is the resonance frequency (or at least close to it) and to determine the properties of the circuit both on and off resonance.