

Topics: Faraday's Law

Related Reading:

Course Notes (Liao et al.): Chapter 10

Topic Introduction

So far in this class magnetic fields and electric fields have been fairly well isolated. Electric fields are generated by static charges, magnetic fields by moving charges (currents). In each of these cases the fields have been static – we have had constant charges or currents making constant electric or magnetic fields. Today we make two major changes to what we have seen before: we consider the interaction of these two types of fields, and we consider what happens when they are not static. We will discuss the final Maxwell's equation, Faraday's law, which explains that electric fields can be generated not only by charges but also by magnetic fields that vary in time.

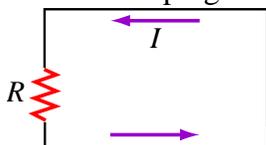
Faraday's Law

It is not entirely surprising that electricity and magnetism are connected. We have seen, after all, that if an electric field is used to accelerate charges (make a current) that a magnetic field can result. Faraday's law, however, is something completely new. We can now forget about charges completely. What Faraday discovered is that a changing magnetic flux generates an EMF (electromotive force). Mathematically:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}, \text{ where } \Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} \text{ is the magnetic flux, and } \mathcal{E} = \oint \vec{\mathbf{E}}' \cdot d\vec{\mathbf{s}} \text{ is the EMF}$$

In the formula above, $\vec{\mathbf{E}}'$ is the electric field measured in the rest frame of the circuit, if the circuit is moving. The above formula is deceptively simple, so I will discuss several important points to consider when thinking about Faraday's law.

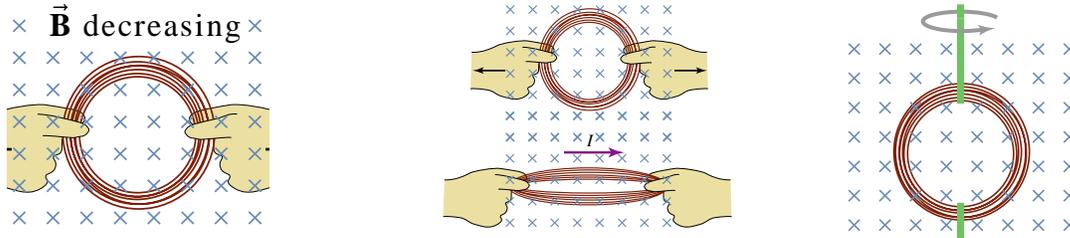
WARNING: First, a warning. Many students confuse Faraday's Law with Ampere's Law. Both involve integrating around a loop and comparing that to an integral across the area bounded by that loop. Aside from this mathematical similarity, however, the two laws are completely different. In Ampere's law the field that is "curling around the loop" is the magnetic field, created by a "current flux" $(I = \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}})$ that is penetrating the looping B field. In Faraday's law the electric field is curling, created by a *changing* magnetic flux. In fact, there need not be any currents at all in the problem, although as we will see below typically the EMF is measured by its ability to drive a current around a physical loop – a circuit. Keeping these differences in mind, let's continue to some details of Faraday's law.



EMF: How does the EMF become apparent? Typically, when doing Faraday's law problems there will be a physical loop, a closed circuit, such as the one pictured at left. The EMF is then observed as an electromotive force that drives a current in the circuit: $\mathcal{E} = IR$. In

this case, the path walked around in calculating the EMF is the circuit, and hence the associated area across which the magnetic flux is calculated is the rectangular area bordered by the circuit. Although this is the most typical initial use of Faraday's law, it is not the only one – we will see that it can be applied in "empty space" space as well, to determine the creation of electric fields.

Changing Magnetic Flux: How do we get the magnetic flux Φ_B to change? Looking at the integral $\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = BA \cos(\theta)$, hints at three distinct methods: changing the strength of the field, the area of the loop, or the angle of the loop. These methods are shown below.



In each of the cases pictured above, the magnetic flux into the page is decreasing with time (because the (1) B field, (2) loop area or (3) projected area are decreasing with time). This decreasing flux creates an EMF. In which direction? We can use Lenz's Law to find out.

Lenz's Law

Lenz's Law is a non-mathematical statement of Faraday's Law. It says that systems will always act to *oppose* changes in magnetic flux. For example, in each of the above cases the flux into the page is decreasing with time. The loop doesn't want a decreased flux, so it will generate a clockwise EMF, which will drive a clockwise current, creating a B field into the page (inside the loop) to make up for the lost flux. This, by the way, is the meaning of the minus sign in Faraday's law. I recommend that you use Lenz's Law to determine the direction of the EMF and then use Faraday's Law to calculate the amplitude. By the way, just as with Faraday's Law, you don't need a physical circuit to use Lenz's Law. Just pretend that there is a wire in which current could flow and ask what direction it would need to flow in order to *oppose* the changing flux. In general, *opposing* a change in flux means *opposing* what is happening to change the flux (e.g. forces or torques *oppose* the change).

Applications

A number of technologies rely on induction to work – generators, microphones, metal detectors, and electric guitars to name a few. Another common application is eddy current braking. A magnetic field penetrating a metal spinning disk (like a wheel) will induce eddy currents in the disk, currents which circle inside the disk and exert a torque on the disk, trying to stop it from rotating. This kind of braking system is commonly used in trains. Its major benefit (aside from eliminating costly service to maintain brake pads) is that the braking torque is proportional to angular velocity of the wheel, meaning that the ride smoothly comes to a halt.

Important Equations

Faraday's Law:
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Magnetic Flux:
$$\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

EMF:
$$\mathcal{E} = \oint \vec{\mathbf{E}}' \cdot d\vec{\mathbf{s}}$$
 where $\vec{\mathbf{E}}'$ is the electric field measured in the rest frame of the circuit, if the circuit is moving.