Class 28: Outline

Hour 1:

Displacement Current

Maxwell's Equations

Hour 2:

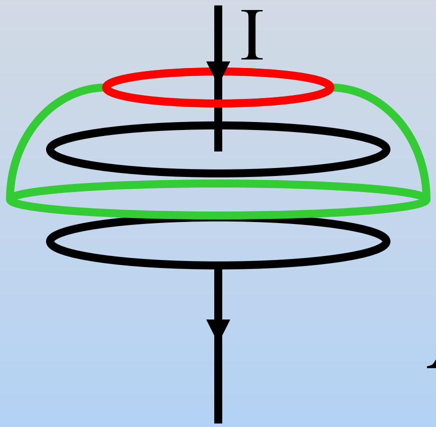
Electromagnetic waves

Finally:
Bringing it All Together

Displacement Current

Ampere's Law: Capacitor

Consider a charging capacitor:



Use Ampere's Law to calculate the magnetic field just above the top plate

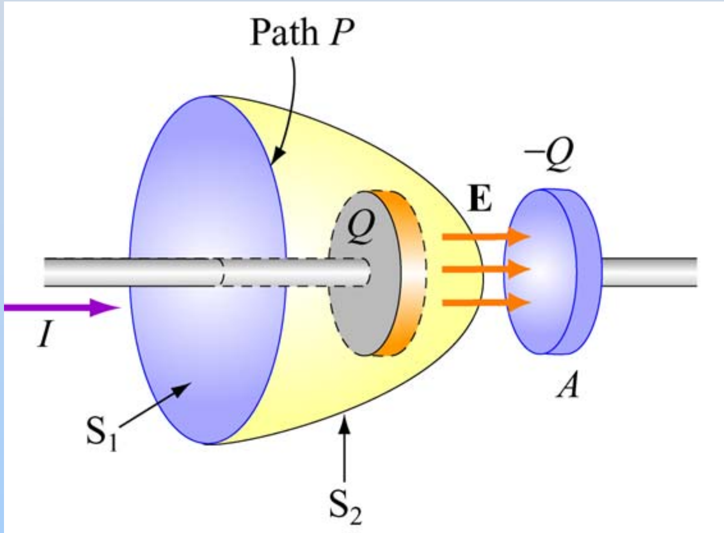
$$\text{Ampere's law: } \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$$

1) Red Amperian Area, $I_{enc} = I$
2) Green Amperian Area, $I = 0$

What's Going On?

Displacement Current

We don’t have current between the capacitor plates but we do have a changing E field. Can we “make” a current out of that?



$$E = \frac{Q}{\varepsilon_0 A} \Rightarrow Q = \varepsilon_0 EA = \varepsilon_0 \Phi_E$$

$$\boxed{\frac{dQ}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} \equiv I_d}$$

This is called (for historic reasons) the **Displacement Current**

Maxwell-Ampere's Law

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 (I_{encl} + I_d)$$

$$= \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

PRS Questions:
Capacitor

Maxwell's Equations

Electromagnetism Review

- E fields are created by:
  (1) electric charges — Gauss's Law
  (2) time changing B fields — Faraday's Law
- B fields are created by
  (1) moving electric charges (*NOT* magnetic charges) — Ampere's Law
  (2) time changing E fields — Maxwell's Addition
- E (B) fields exert forces on (moving) electric charges

Lorentz Force

Maxwell's Equations

$$\oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$ (Gauss's Law)

$$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$ (Faraday's Law)

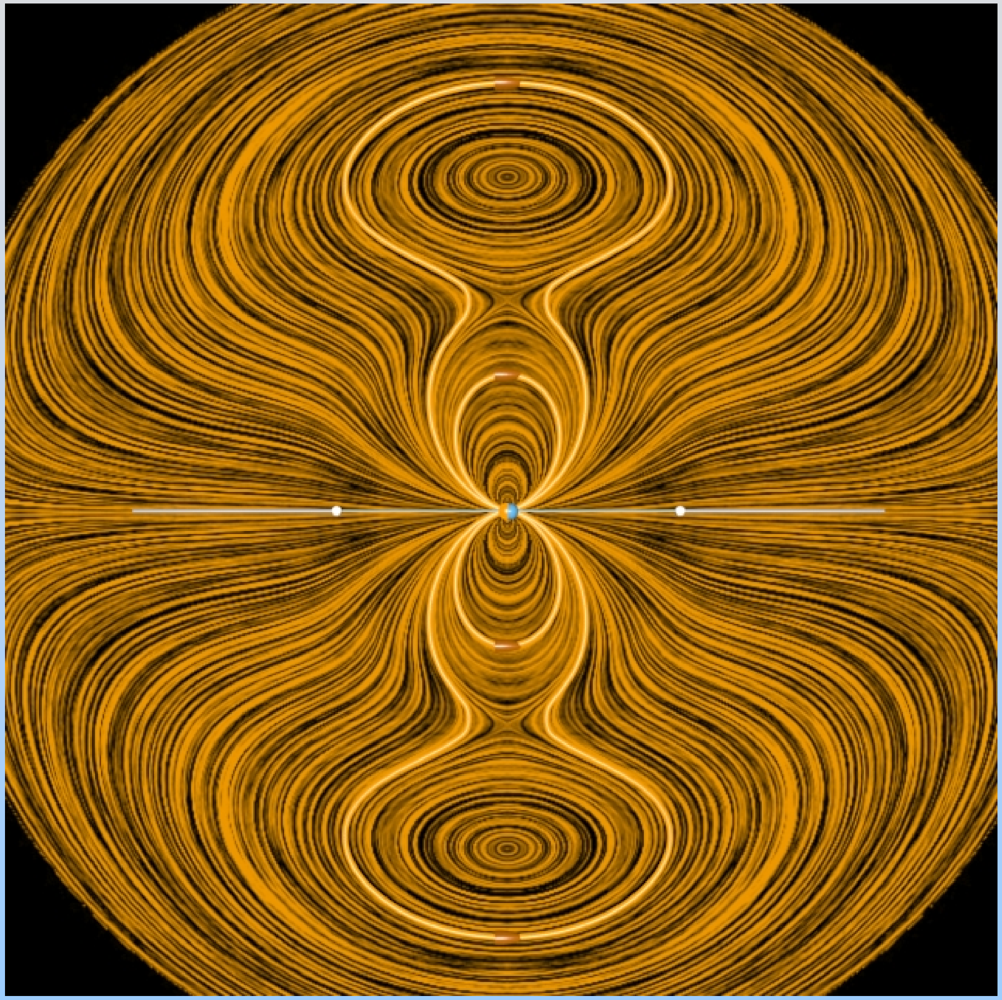
$$\oint\!\!\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$ (Magnetic Gauss's Law)

$$\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$ (Ampere-Maxwell Law)

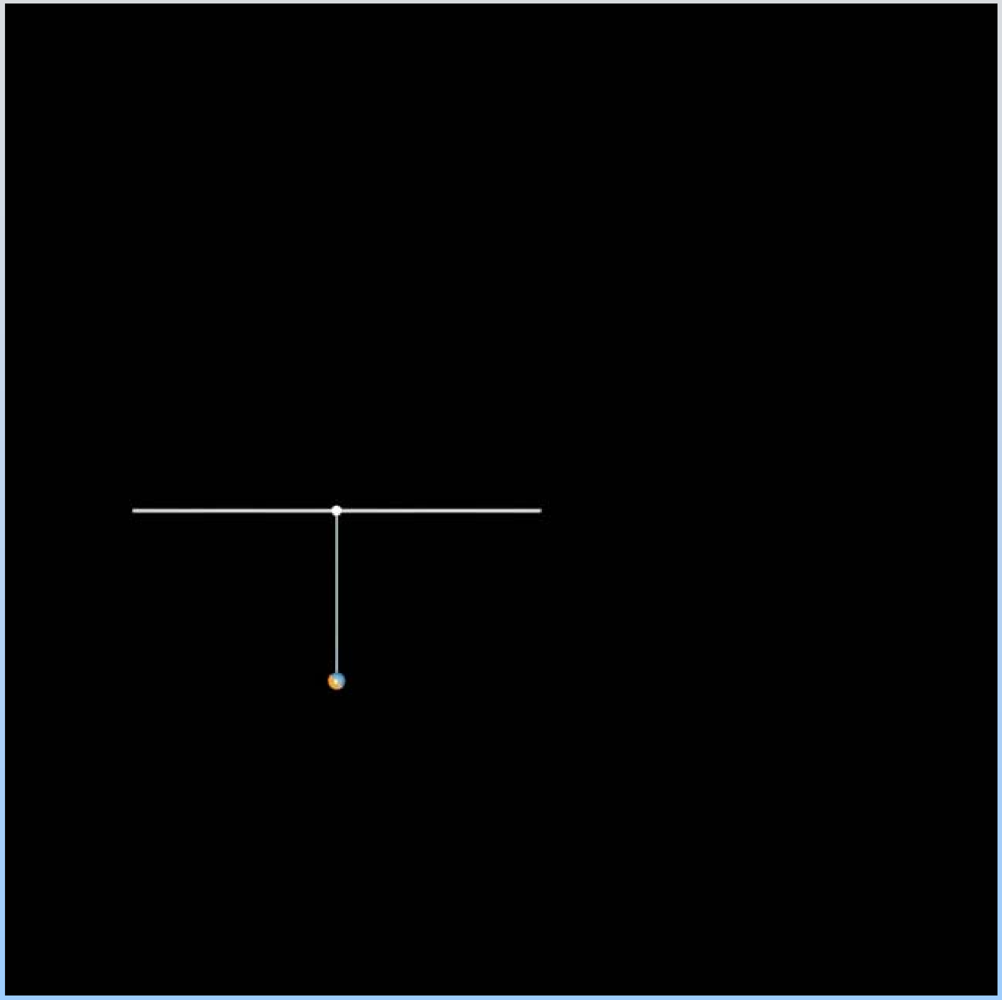
$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$ (Lorentz force Law)

Electromagnetic Radiation

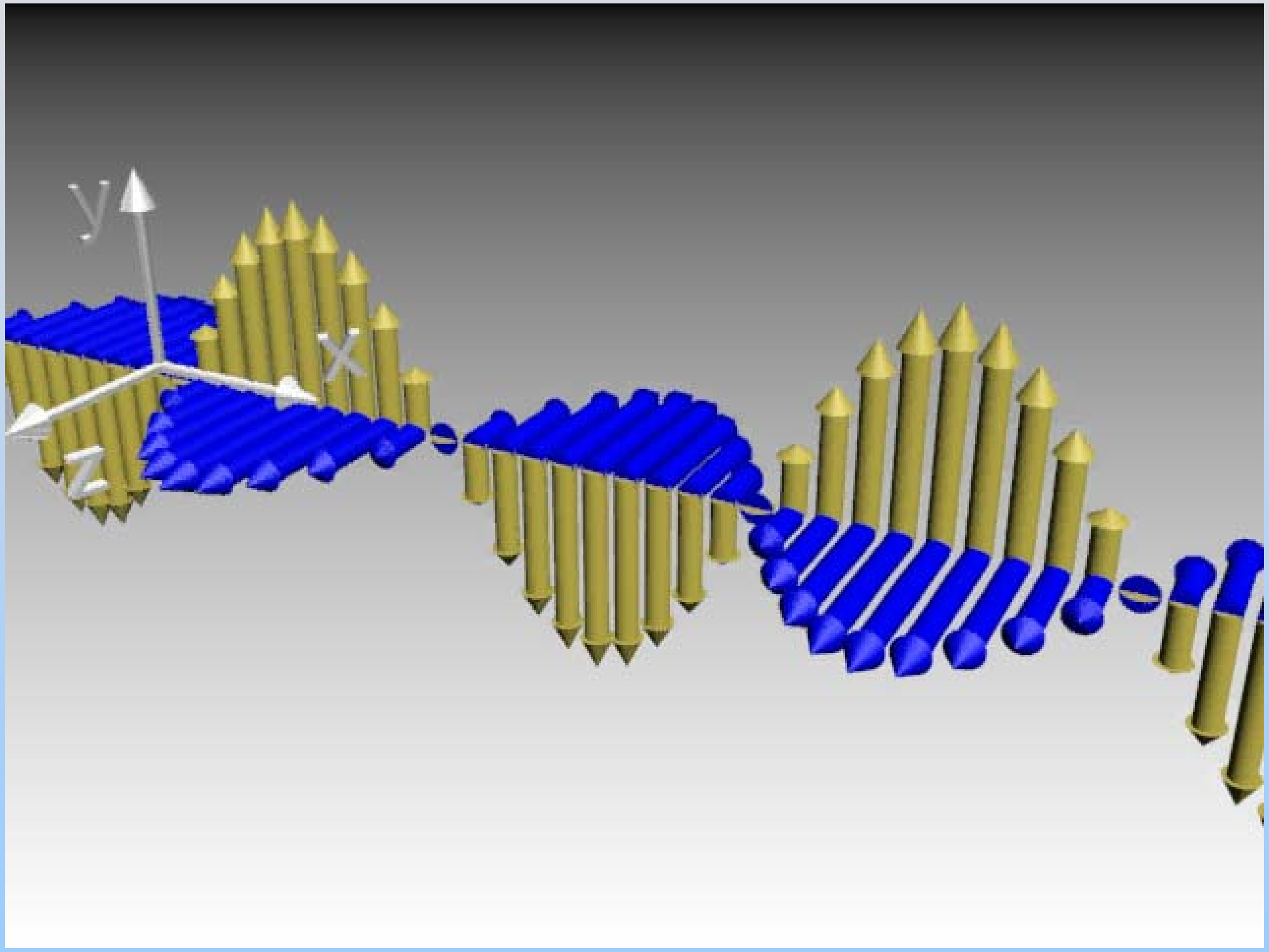
A Question of Time…



http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/light/05-CreatingRadiation/05-pith_f220_320.html



Electromagnetic Radiation: Plane Waves



http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/light/07-EBlight/07-EB_Light_320.html

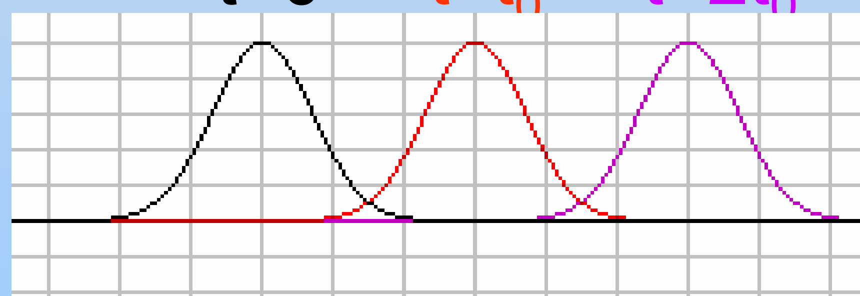
Traveling Waves

Consider f(x) = 

x=0

What is g(x,t) = f(x-vt)?

t=0 $t=t_0$ $t=2t_0$



x=0 $x=vt_0$ $x=2vt_0$

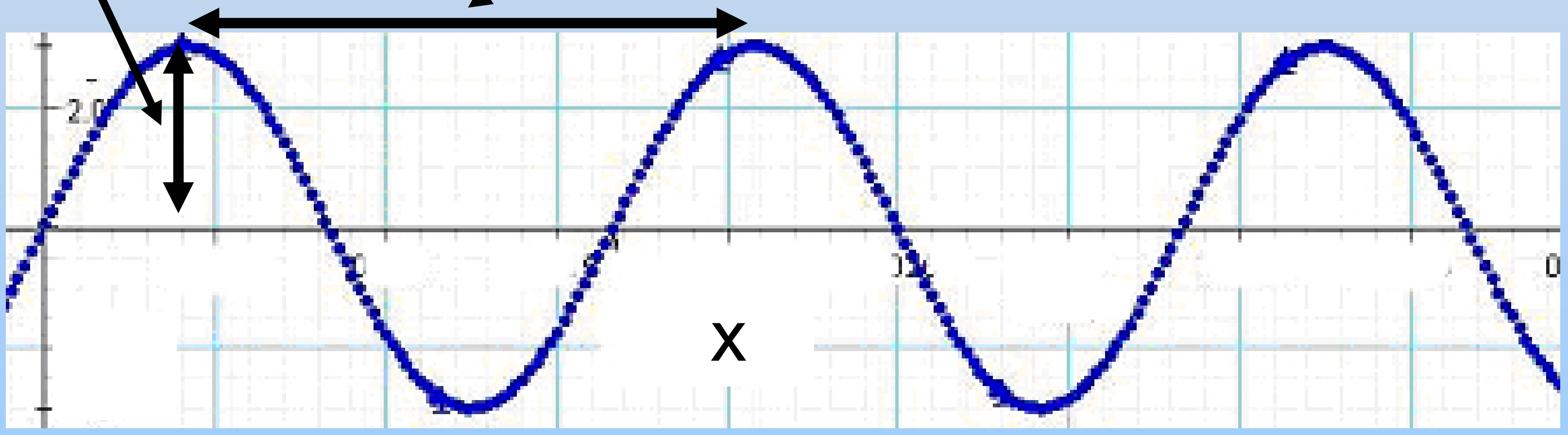
f(x-vt) is traveling wave moving to the right!

Traveling Sine Wave

Now consider $f(x) = y = y_0\sin(kx)$:

$$\text{Wavelength } (\lambda) = \frac{2\pi}{\text{wavenumber } (k)}$$

Amplitude (y_0)



What is $g(x,t) = f(x+vt)$? Travels to left at velocity v

$$y = y_0\sin(k(x+vt)) = y_0\sin(kx+kvt)$$

Traveling Sine Wave

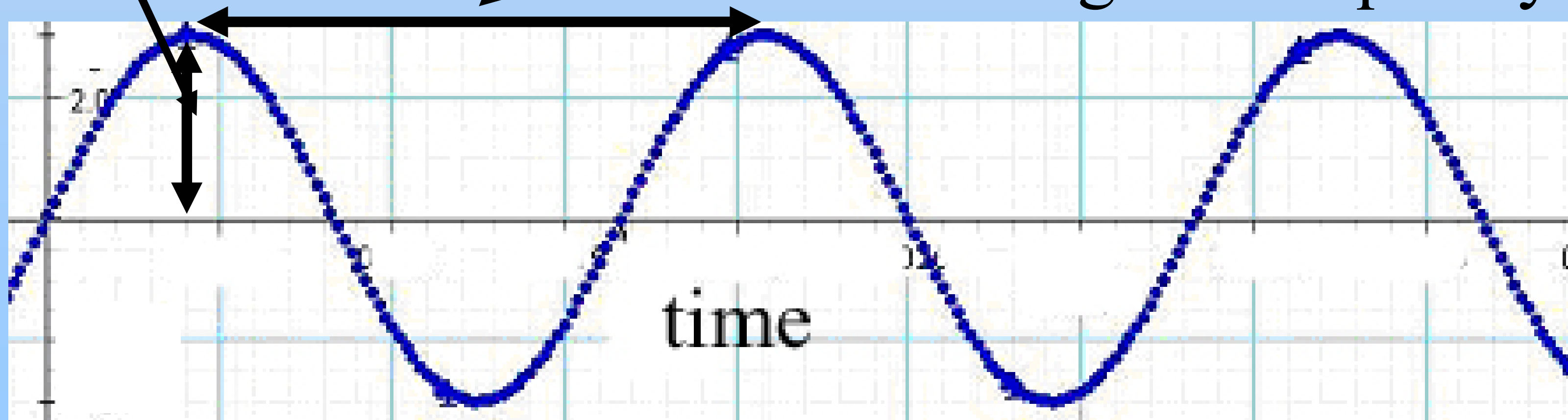
$$y = y_0 \sin\left(kx + kvt\right)$$

At x=0, just a function of time: $y = y_0 \sin(kvt) \equiv y_0 \sin(\omega t)$

$$\text{Period } (T) = \frac{1}{\text{frequency } (f)}$$

$$= \frac{2\pi}{\text{angular frequency } (\omega)}$$

Amplitude (y_0)

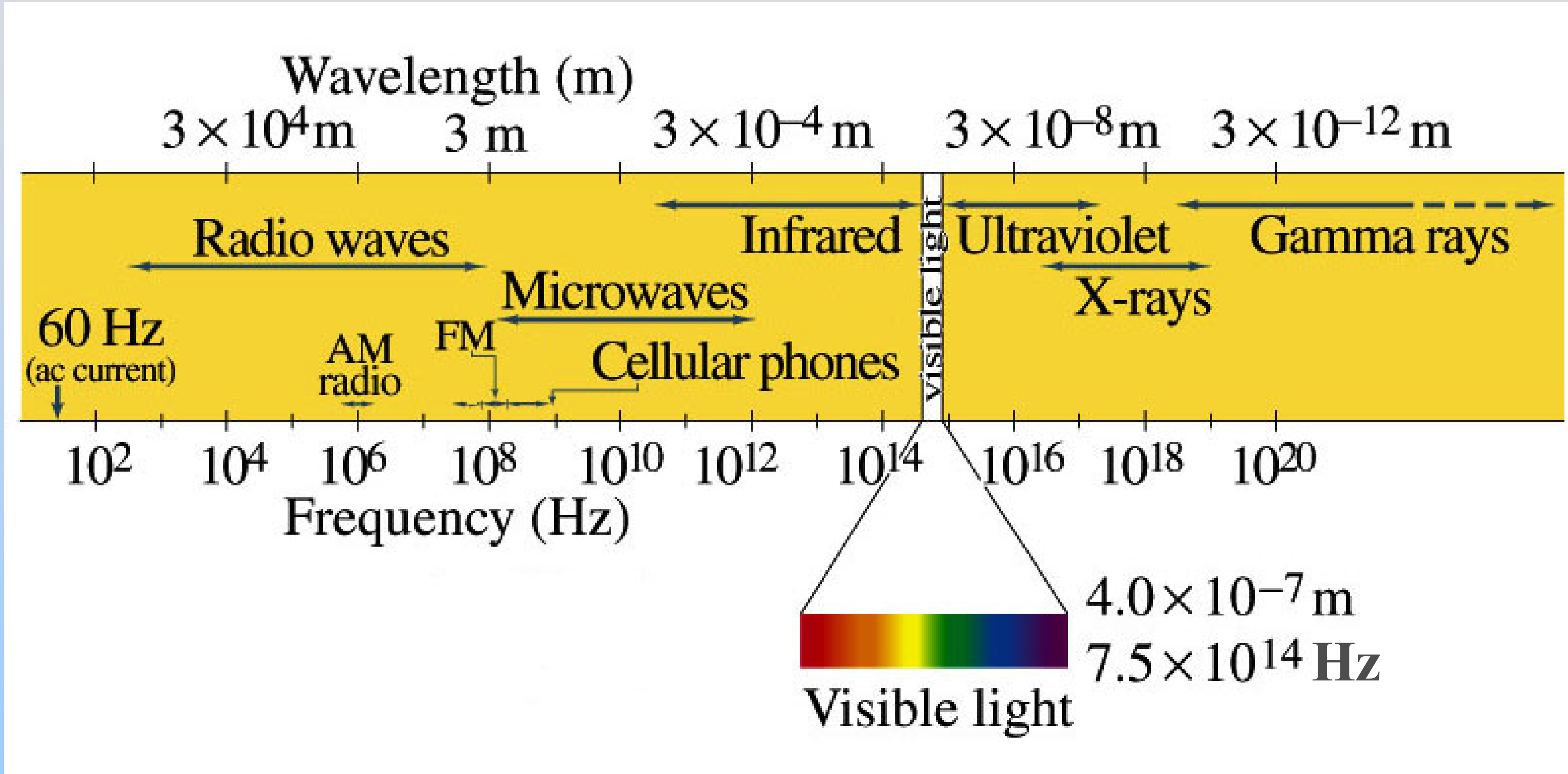


Traveling Sine Wave

$$y = y_0 \sin(kx - \omega t)$$

- Wavelength: λ
- Frequency : f
- Wave Number: $k = \dfrac{2\pi}{\lambda}$
- Angular Frequency: $\omega = 2\pi f$
- Period: $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$
- Speed of Propagation: $v = \dfrac{\omega}{k} = \lambda f$
- Direction of Propagation: $+x$

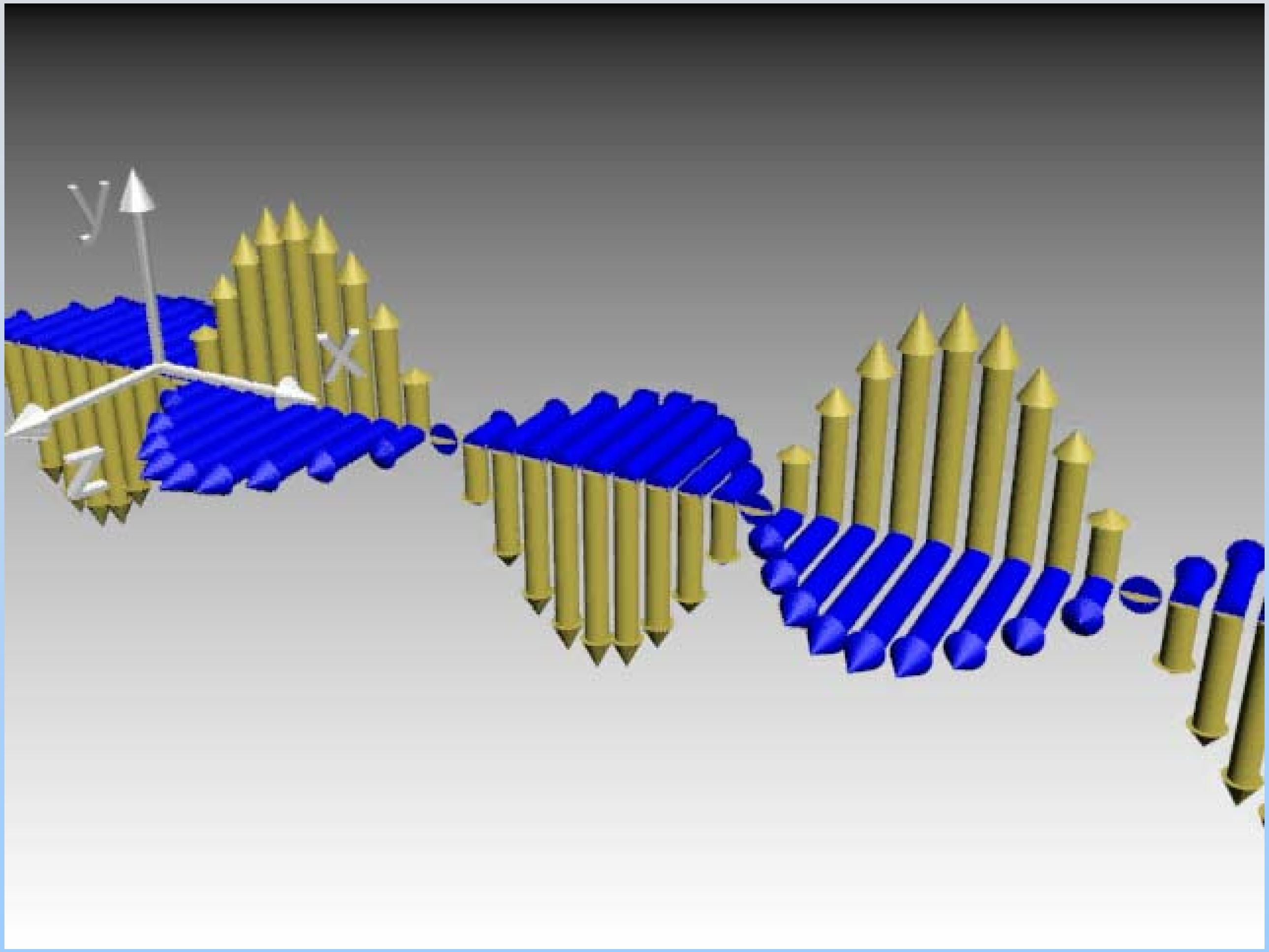
Electromagnetic Waves



Remember: $\lambda f = c$

Electromagnetic Radiation: Plane Waves

http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/light/07-EBlight/07-EB_Light_320.html



Watch 2 Ways:

1) Sine wave traveling to right (+x)
2) Collection of out of phase oscillators (watch one position)

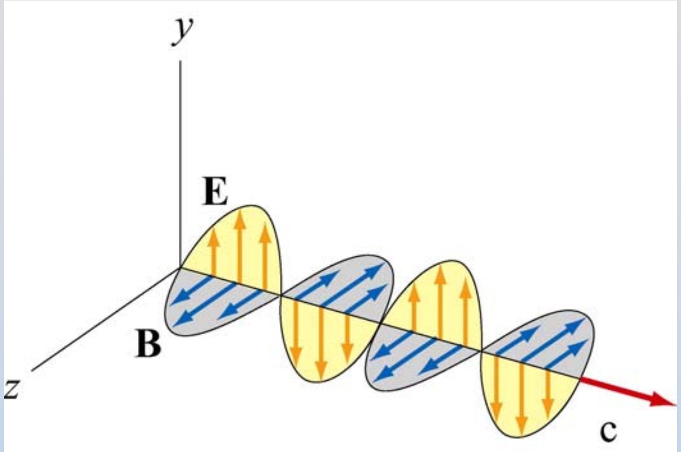
Don’t confuse vectors with heights – they are magnitudes of E (gold) and B (blue)

PRS Question: Wave

Group Work:
Do Problem 1

Properties of EM Waves

Travel (through vacuum) with speed of light



$$v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times 10^8 \frac{m}{s}$$

At every point in the wave and any instant of time, E and B are in phase with one another, with

$$\frac{E}{B} = \frac{E_0}{B_0} = c$$

E and B fields perpendicular to one another, and to the direction of propagation (they are **transverse**):

$$\text{Direction of propagation} = \text{Direction of } \vec{\mathbf{E}}\times\vec{\mathbf{B}}$$

Direction of Propagation

$$\vec{\mathbf{E}} = \hat{\mathbf{E}} E_0 \sin(k(\hat{\mathbf{p}} \cdot \vec{\mathbf{r}}) - \omega t); \quad \vec{\mathbf{B}} = \hat{\mathbf{B}} B_0 \sin(k(\hat{\mathbf{p}} \cdot \vec{\mathbf{r}}) - \omega t)$$

$$\boxed{\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{p}}}$$

$\hat{\mathbf{E}}$	$\hat{\mathbf{B}}$	$\hat{\mathbf{p}}$	$(\hat{\mathbf{p}} \cdot \vec{\mathbf{r}})$
$\hat{\mathbf{i}}$	$\hat{\mathbf{j}}$	$\hat{\mathbf{k}}$	z
$\hat{\mathbf{j}}$	$\hat{\mathbf{k}}$	$\hat{\mathbf{i}}$	x
$\hat{\mathbf{k}}$	$\hat{\mathbf{i}}$	$\hat{\mathbf{j}}$	y
$\hat{\mathbf{j}}$	$\hat{\mathbf{i}}$	$-\hat{\mathbf{k}}$	$-z$
$\hat{\mathbf{k}}$	$\hat{\mathbf{j}}$	$-\hat{\mathbf{i}}$	$-x$
$\hat{\mathbf{i}}$	$\hat{\mathbf{k}}$	$-\hat{\mathbf{j}}$	$-y$

PRS Question: Direction of Propagation

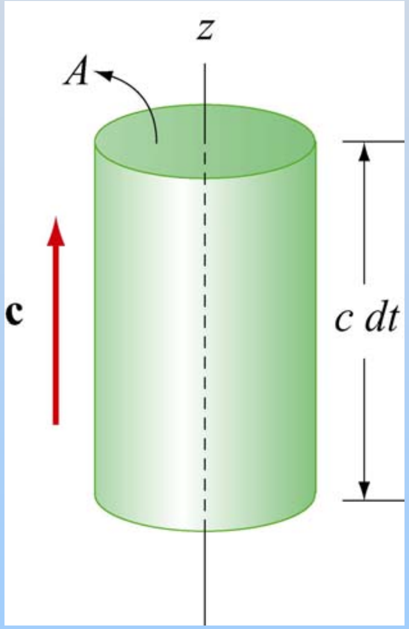
In Class Problem: Plane EM Waves

Energy & the Poynting Vector

Energy in EM Waves

Energy densities: $u_E = \frac{1}{2}\varepsilon_0 E^2,\ u_B = \frac{1}{2\mu_0}B^2$

Consider cylinder:



$$dU = (u_E + u_B)A\,dz = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right)A c\,dt$$

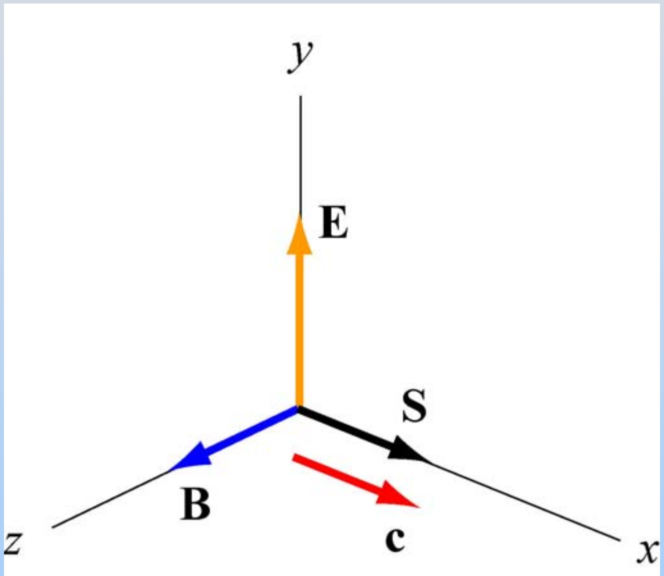
What is rate of energy flow per unit area?

$$S = \frac{1}{A}\frac{dU}{dt} = \frac{c}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right) = \frac{c}{2}\left(\varepsilon_0 cEB + \frac{EB}{c\mu_0}\right)$$

$$= \frac{EB}{2\mu_0}\left(\varepsilon_0\mu_0 c^2 + 1\right) = \frac{EB}{\mu_0}$$

Poynting Vector and Intensity

Direction of energy flow = direction of wave propagation



$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$: Poynting vector

units: Joules per square meter per sec

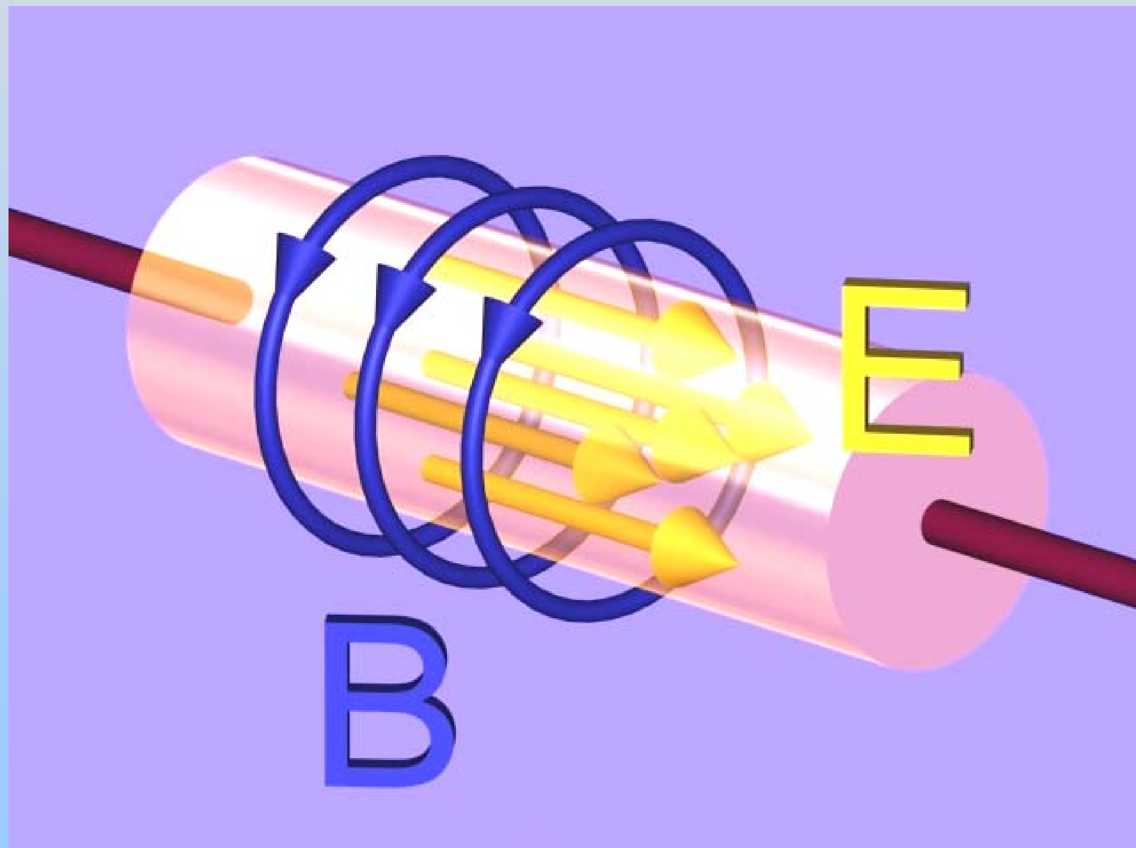
Intensity *I*:

$$I \equiv \langle S \rangle = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2\mu_0 c} = \frac{c B_0^2}{2\mu_0}$$

Energy Flow: Resistor

$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$

On surface of resistor is INWARD

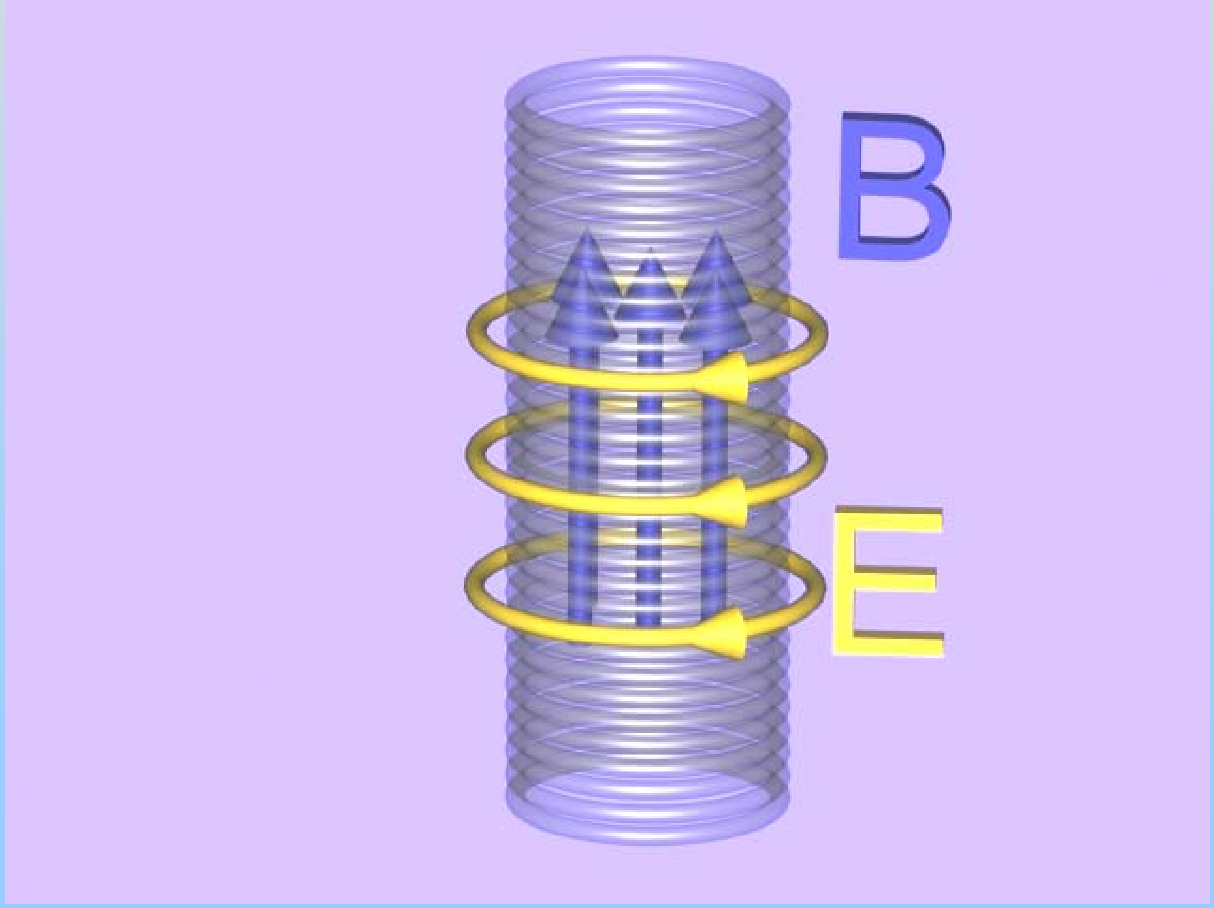


PRS Questions:
Poynting Vector

Energy Flow: Inductor

$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$

On surface of inductor with increasing current is INWARD



Energy Flow: Inductor

$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$

On surface of inductor with decreasing current is OUTWARD