Class 24: Outline

Hour 1:

Inductance & LR Circuits

Hour 2:

Energy in Inductors

Last Time:
Faraday's Law
Mutual Inductance

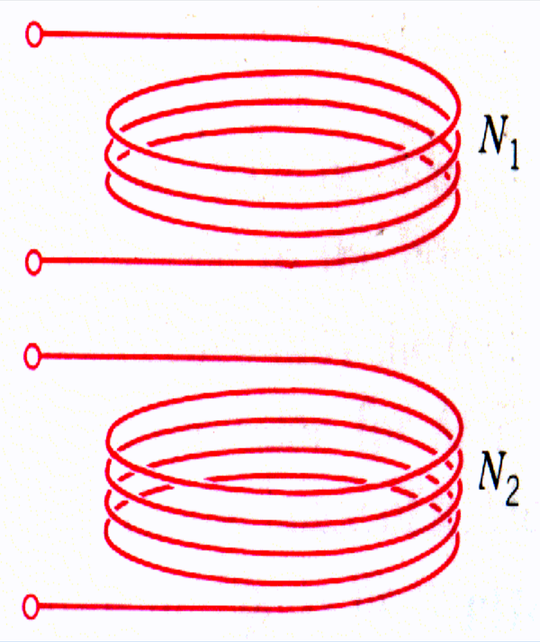
Faraday's Law of Induction

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

Changing magnetic flux *induces* an EMF

Lenz: Induction ***opposes*** change

Mutual Inductance



A current I_2 in coil 2, induces some magnetic flux Φ_{12} in coil 1. We define the flux in terms of a "mutual inductance" M_{12}:

$$N_1 \Phi_{12} \equiv M_{12} I_2$$

$$\rightarrow M_{12} = \frac{N_1 \Phi_{12}}{I_2}$$

$$M_{12} = M_{21} = M$$

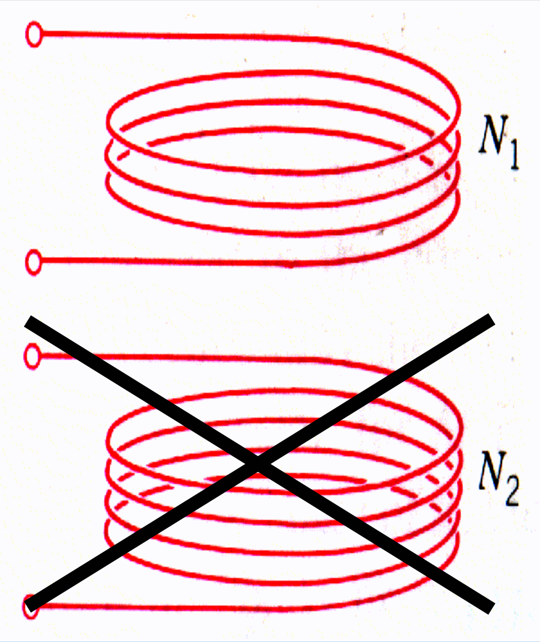
$$\boxed{\mathcal{E}_{12} \equiv -M \frac{dI_2}{dt}}$$

You need AC currents!

Demonstration: Remote Speaker

This Time:
Self Inductance

Self Inductance



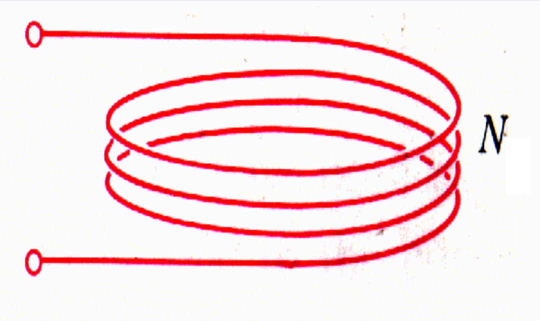
What if we forget about coil 2 and ask about putting current into coil 1? There is "self flux":

$$N_1\Phi_{11} \equiv M_{11}I_1 \equiv LI$$

$$\rightarrow L = \frac{N\Phi}{I}$$

$$\boxed{\mathcal{E} \equiv -L\frac{dI}{dt}}$$

Calculating Self Inductance



$$L = \frac{N\Phi}{I}$$

Unit: Henry

$$1\,\mathrm{H} = 1\,\frac{\mathrm{V}\cdot\mathrm{s}}{\mathrm{A}}$$

1. Assume a current I is flowing in your device
2. Calculate the B field due to that I
3. Calculate the flux due to that B field
4. Calculate the self inductance (divide out I)

Group Problem: Solenoid

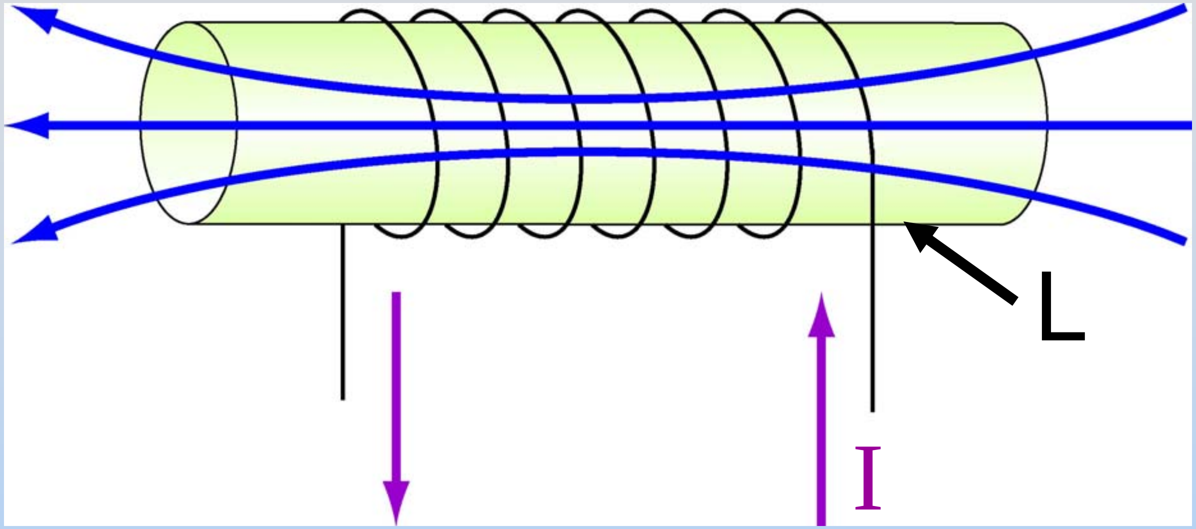
Calculate the self-inductance L of a solenoid (n turns per meter, length ℓ, radius R)

REMEMBER

1. Assume a current I is flowing in your device
2. Calculate the B field due to that I
3. Calculate the flux due to that B field
4. Calculate the self inductance (divide out I)

$$L = N\Phi / I$$

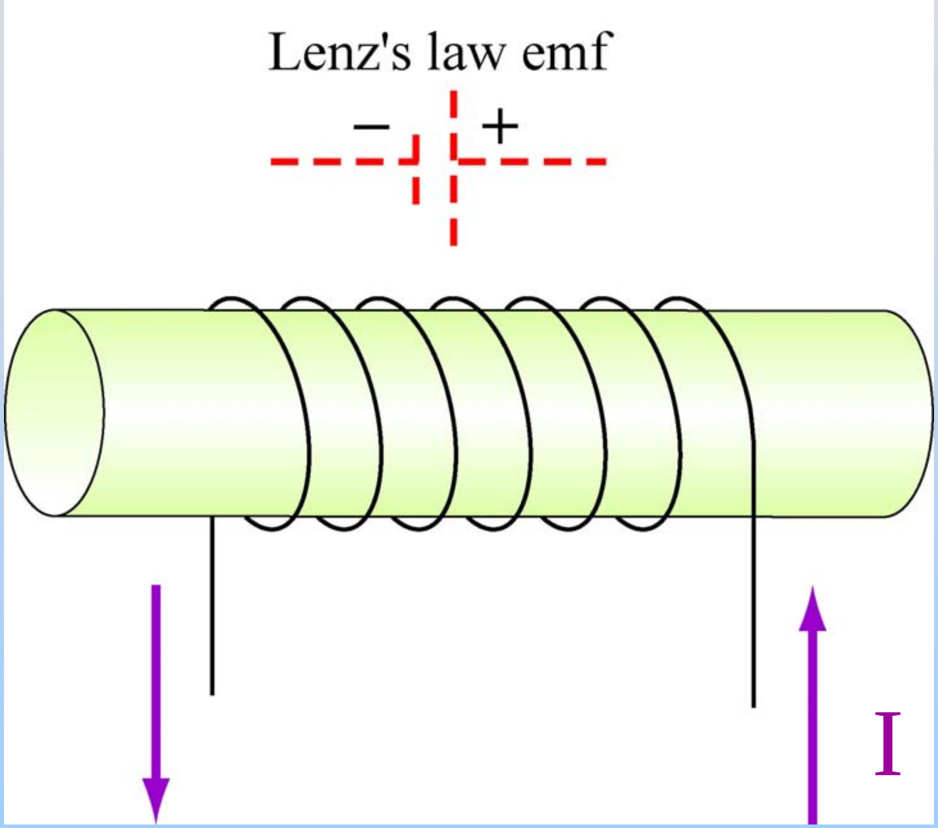
Inductor Behavior

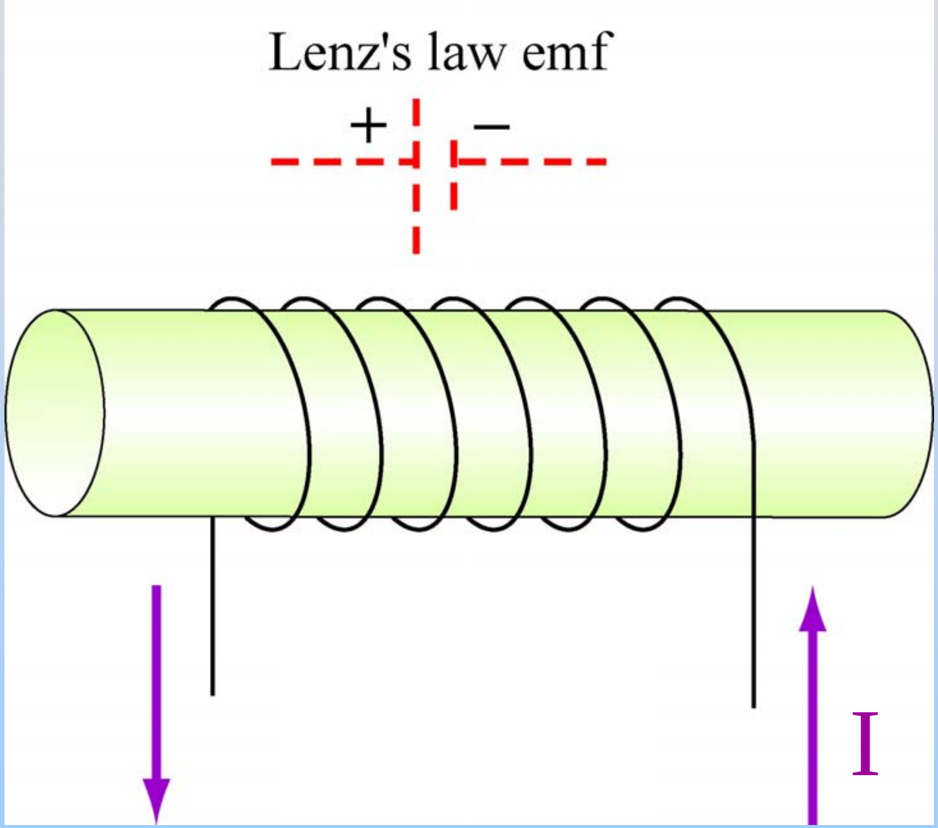


$$\mathcal{E} = -L\frac{dI}{dt}$$

Inductor with constant current does nothing

Back EMF $\varepsilon = -L\frac{dI}{dt}$



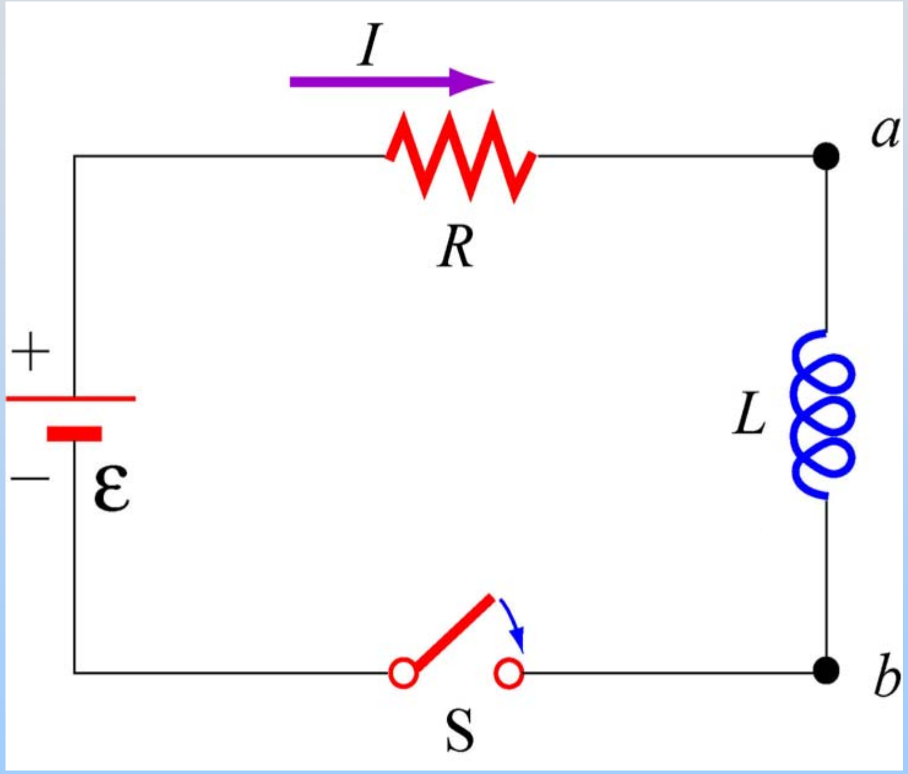


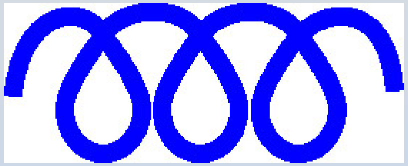
$$\frac{dI}{dt} > 0,\ \ \varepsilon_L < 0$$

$$\frac{dI}{dt} < 0,\ \ \varepsilon_L > 0$$

Inductors in Circuits

Inductor: Circuit element which exhibits self-inductance



Symbol: 

When traveling in direction of current:

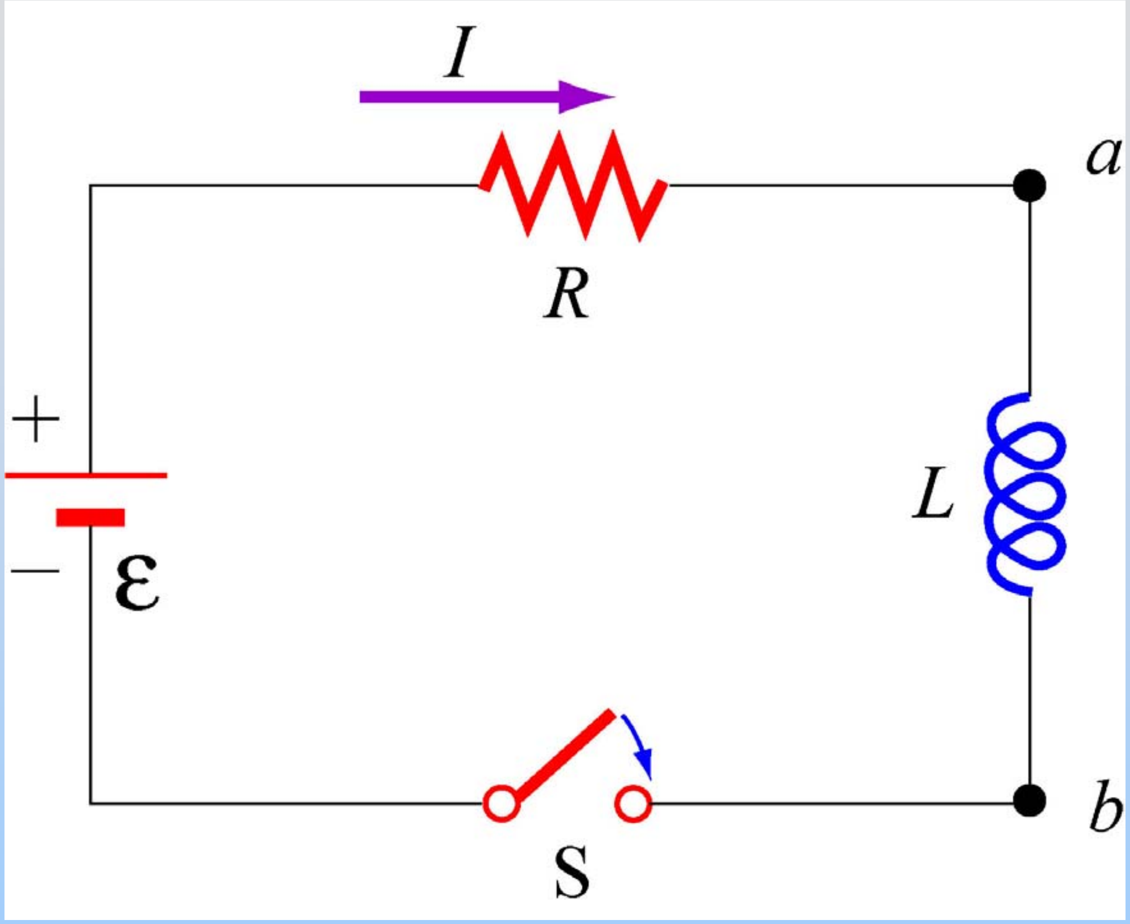
$$\varepsilon = -L\frac{dI}{dt}$$

Inductors hate change, like steady state

They are the opposite of capacitors!

PRS Question: Closing a Switch

LR Circuit

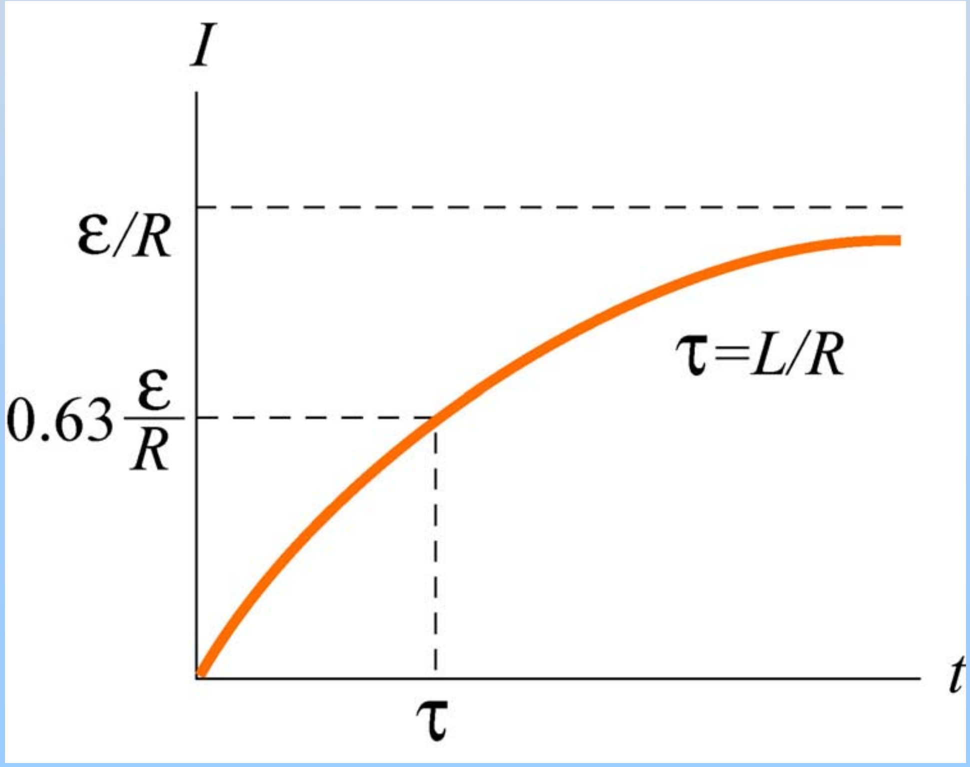


$$\sum_i V_i = \varepsilon - IR - L\frac{dI}{dt} = 0$$

LR Circuit

$$\varepsilon - IR - L\frac{dI}{dt} = 0 \quad \Rightarrow \quad \frac{L}{R}\frac{dI}{dt} = -\left(I - \frac{\varepsilon}{R}\right)$$

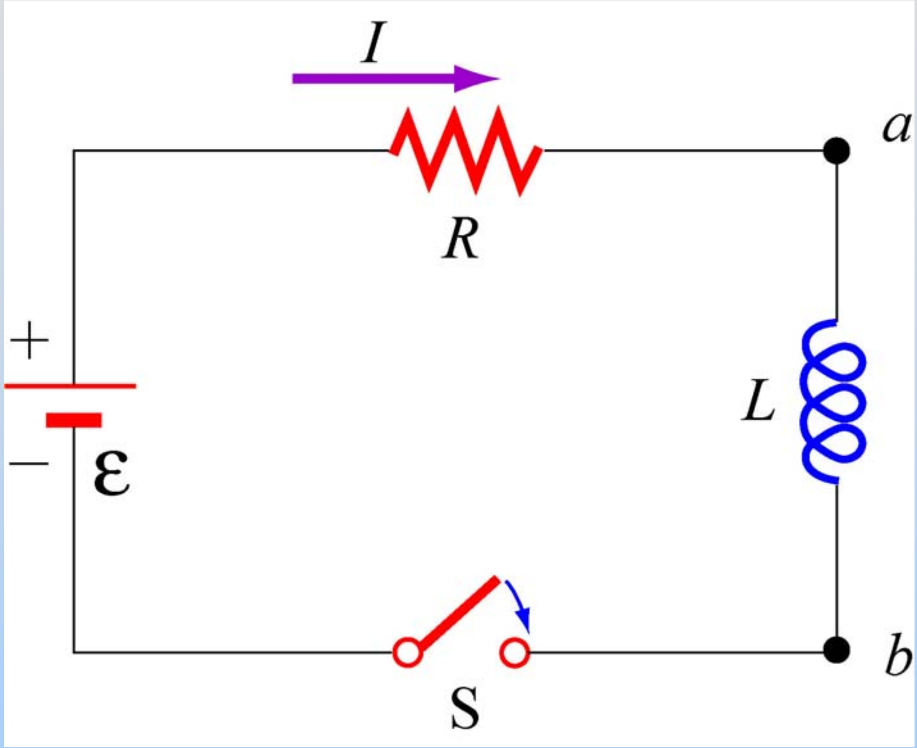
Solution to this equation when switch is closed at t = 0:

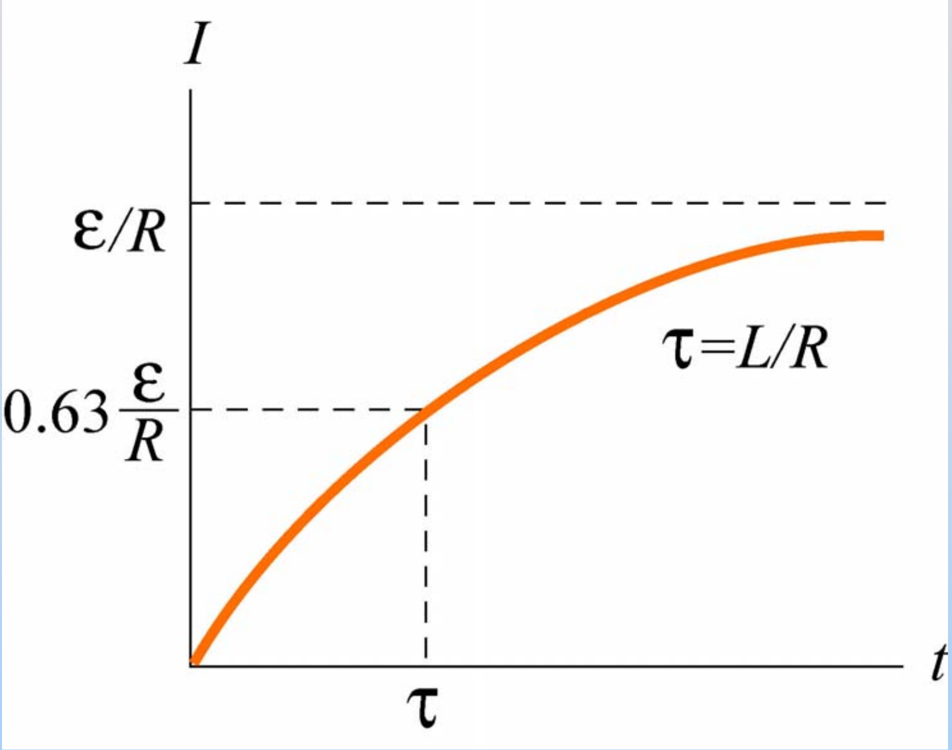


$$I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R}: \text{LR time constant}$$

LR Circuit

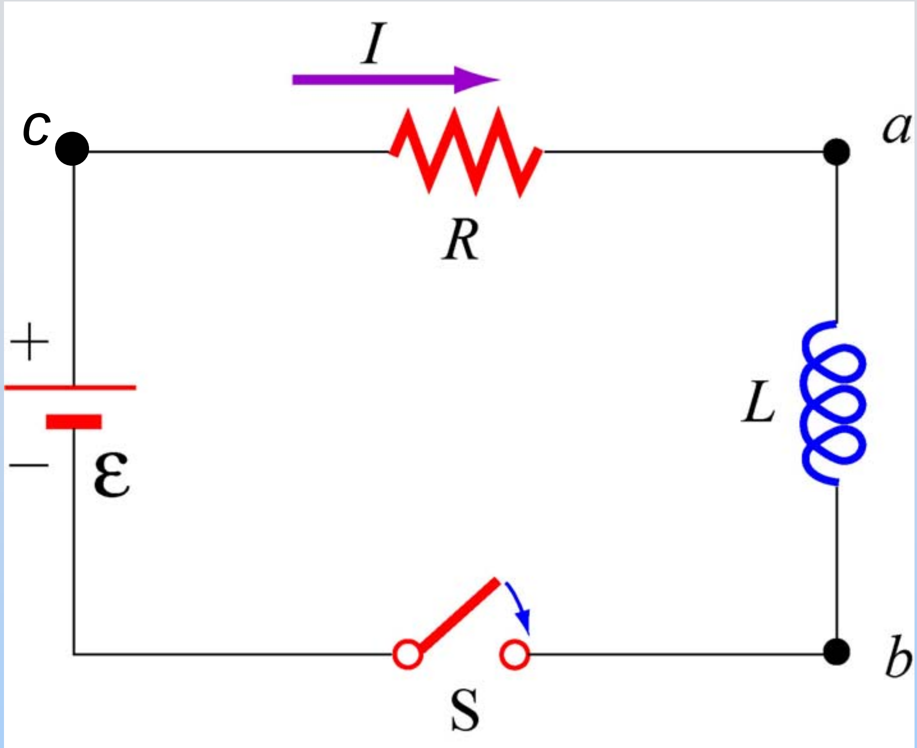


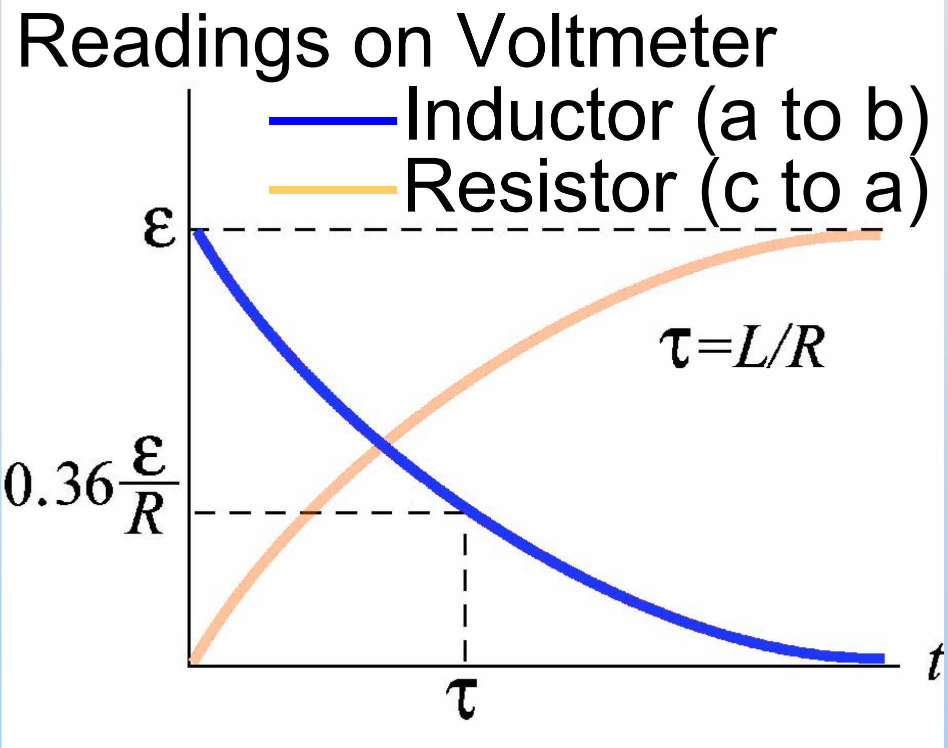


t=0⁺: Current is trying to change. Inductor works as hard as it needs to to stop it

t=∞: Current is steady. Inductor does nothing.

LR Circuit



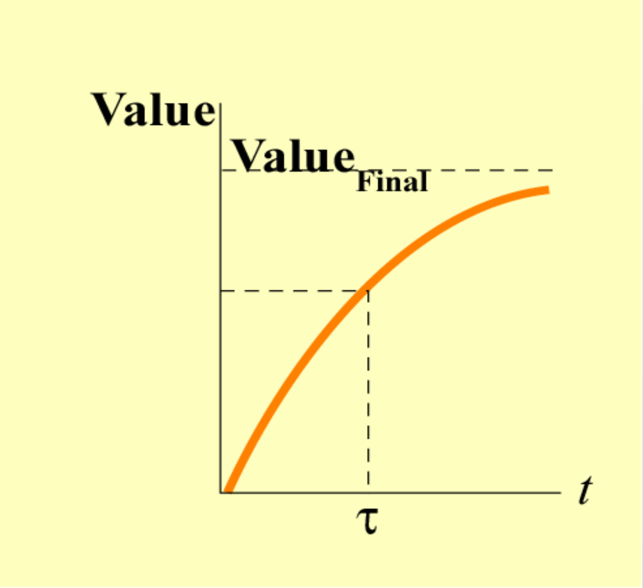


$t=0^+$: Current is trying to change. Inductor works as hard as it needs to to stop it

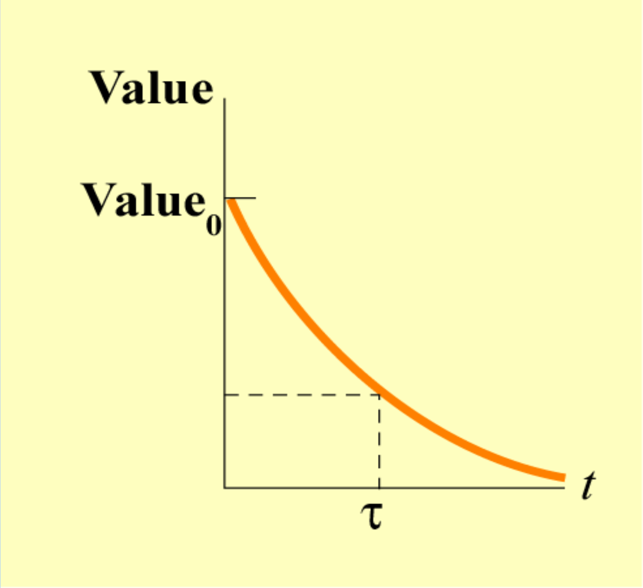
$t=\infty$: Current is steady. Inductor does nothing.

General Comment: LR/RC

All Quantities Either:



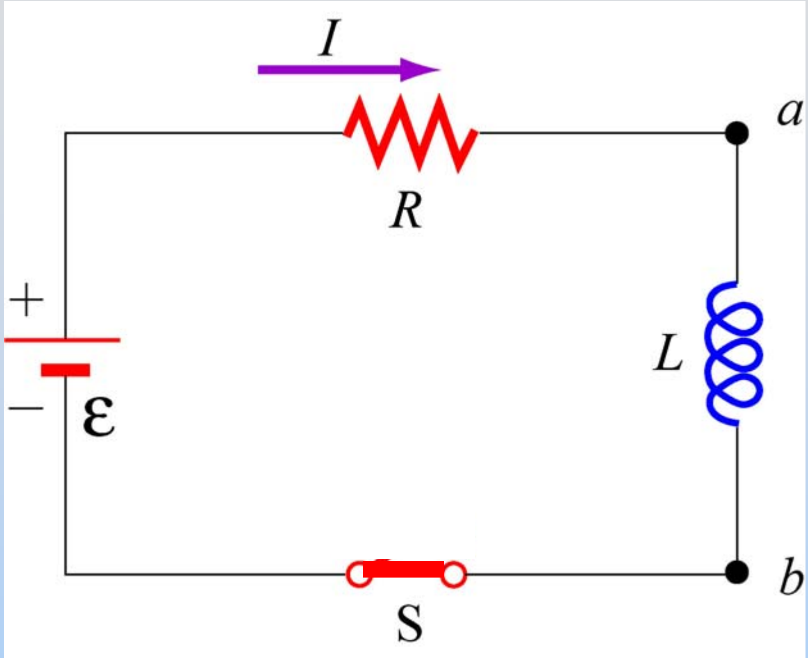
$$\text{Value}(t) = \text{Value}_{\text{Final}}\left(1 - e^{-t/\tau}\right)$$

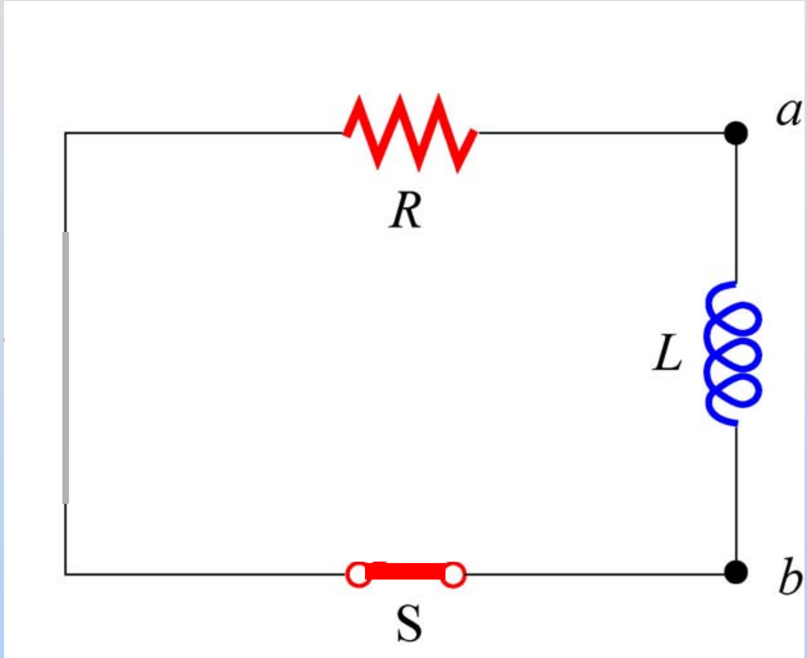


$$\text{Value}(t) = \text{Value}_0 e^{-t/\tau}$$

τ can be obtained from differential equation (prefactor on d/dt) e.g. τ = L/R or τ = RC

Group Problem: LR Circuit

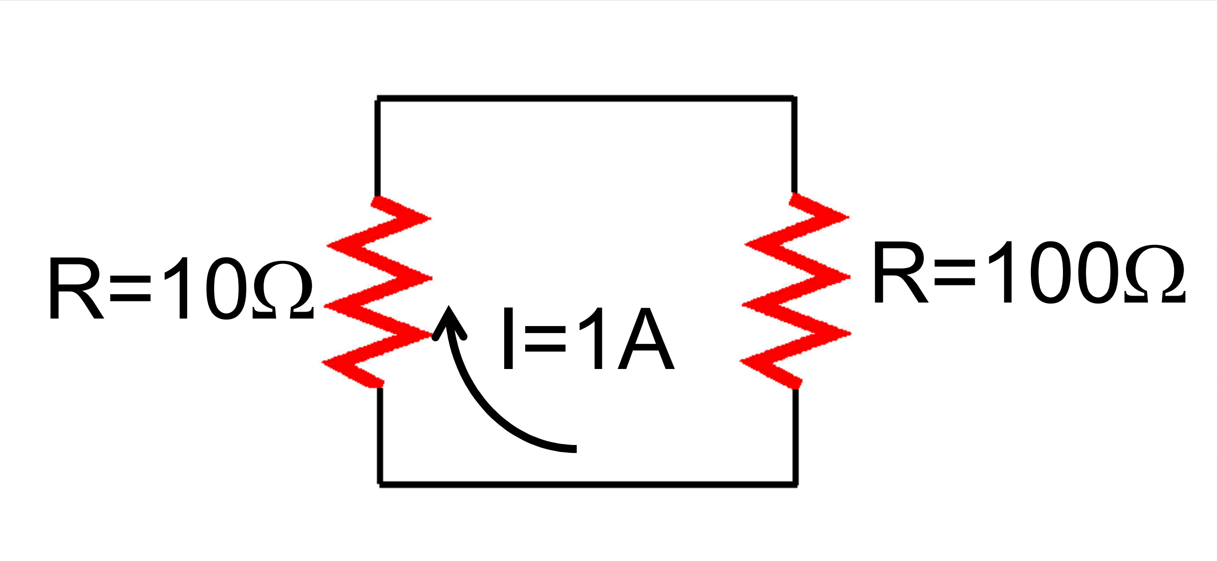




1. What direction does the current flow just after turning off the battery (at t=0+)? At t=∞?
2. Write a differential equation for the circuit
3. Solve and plot I vs. t and voltmeters vs. t

PRS Questions:
LR Circuit & Problem…

Non-Conservative Fields



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

E is no longer a conservative field – Potential now meaningless

This concept
(& next 3 slides)
are complicated.
Bare with me and try not to get confused

Kirchhoff's Modified 2nd Rule

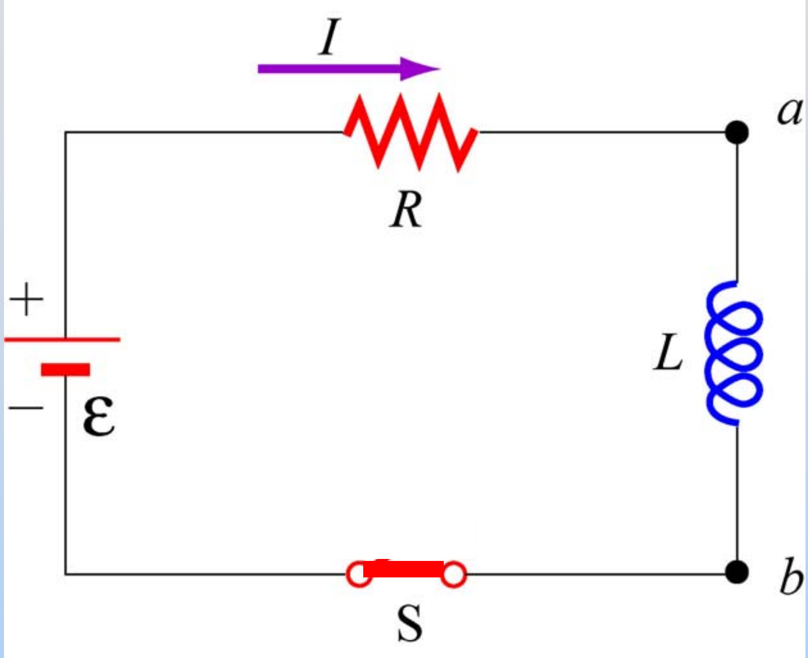
$$\sum_i \Delta V_i = -\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = +N \frac{d\Phi_B}{dt}$$

$$\Rightarrow \sum_i \Delta V_i - N \frac{d\Phi_B}{dt} = 0$$

If all inductance is 'localized' in inductors then our problems go away – we just have:

$$\sum_i \Delta V_i - L \frac{dI}{dt} = 0$$

Ideal Inductor



BUT, EMF generated in an inductor is not a voltage drop across the inductor!

$$\varepsilon = -L\frac{dI}{dt}$$

$$\Delta V_{\text{inductor}} \equiv -\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

Because resistance is 0, E must be 0!

Conclusion:
Be mindful of physics
Don't think too hard doing it

Demos:
Breaking circuits with inductors

Internal Combustion Engine

See figure 1:

http://auto.howstuffworks.com/engine3.htm

Ignition System

The Distributor:

http://auto.howstuffworks.com/ignition-system4.htm

(A) High Voltage Lead
(B) Cap/Rotor Contact
(C) Distributor Cap
(D) To Spark Plug

(A) Coil connection
(B) Breaker Points
(D) Cam Follower
(E) Distributor Cam

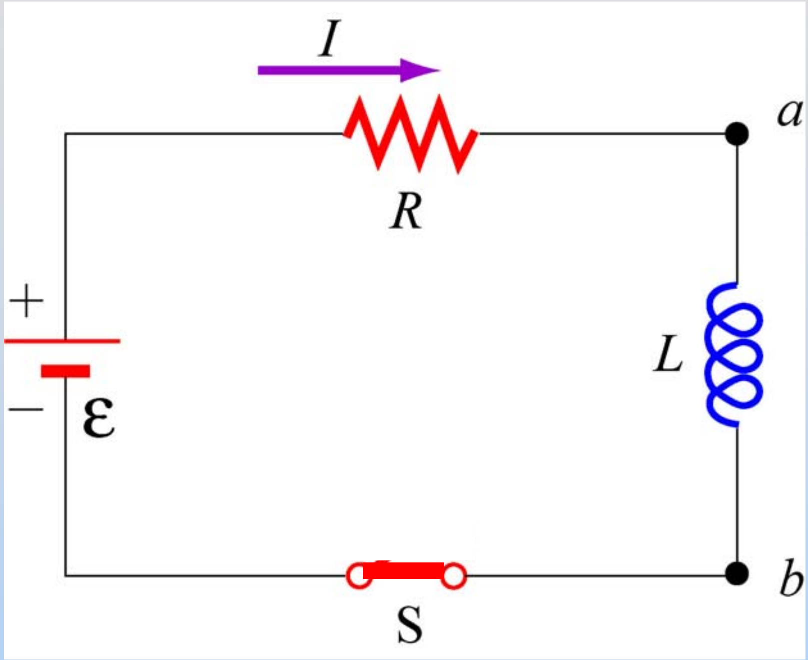
Modern Ignition

See figure:

http://auto.howstuffworks.com/ignition-system.htm

Energy in Inductor

Energy Stored in Inductor



$$\mathcal{E} = +IR + L\frac{dI}{dt}$$

$$I\mathcal{E} = I^2R + L\,I\frac{dI}{dt}$$

$$I\mathcal{E} = I^2R + \frac{d}{dt}\left(\tfrac{1}{2}L\,I^2\right)$$

Battery Supplies

Resistor Dissipates

Inductor Stores

Energy Stored in Inductor

$$U_L = \frac{1}{2} L I^2$$

But where is energy stored?

Example: Solenoid

Ideal solenoid, length *l*, radius *R*, *n* turns/length, current *I*:

$$B = \mu_0 n I \qquad L = \mu_o n^2 \pi R^2 l$$

$$U_B = \tfrac{1}{2} L I^2 = \tfrac{1}{2}\left(\mu_o n^2 \pi R^2 l\right) I^2$$

$$U_B = \left(\frac{B^2}{2\mu_o}\right) \pi R^2 l$$

Energy Density

Volume

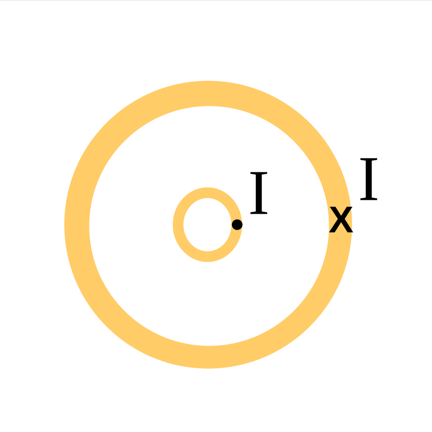
Energy Density

Energy is stored in the magnetic field!

$$u_B = \frac{B^2}{2\mu_o}$$: Magnetic Energy Density

$$u_E = \frac{\varepsilon_o E^2}{2}$$: Electric Energy Density

Group Problem: Coaxial Cable



Inner wire: r=a

Outer wire: r=b

1. How much energy is stored per unit length?
2. What is inductance per unit length?

HINTS: This does require an integral

The EASIEST way to do (2) is to use (1)

Back to Back EMF

PRS Question: Stopping a Motor