Lecture 23: Outline

Hour 1:

Concept Review / Overview

PRS Questions – possible exam questions

Hour 2:

Sample Exam

Yell if you have any questions

7:30-9 pm Tuesday

Exam 2 Topics

- DC Circuits
  - Current & Ohm's Law (Macro- and Microscopic)
  - Power
  - Kirchhoff's Loop Rules
  - Charging/Discharging Capacitor (RC Circuits)
- Magnetic Fields
  - Force due to Magnetic Field (Lorentz Force)
  - Magnetic Dipoles
  - Generating Magnetic Fields
    - Biot-Savart Law & Ampere's Law

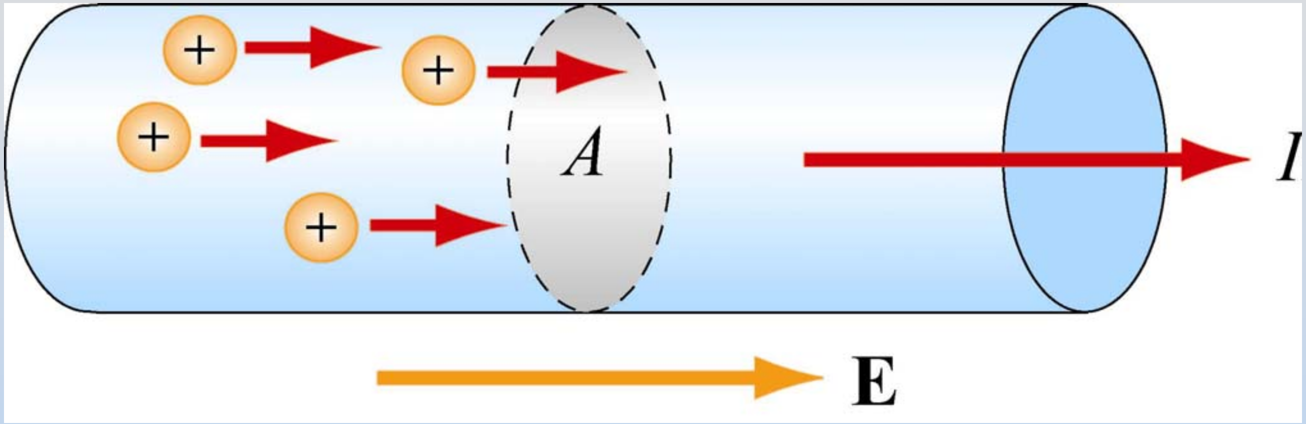
General Exam Suggestions

- You should be able to complete every problem
  - If you are confused, ask
  - If it seems too hard, you aren't thinking enough
  - Look for hints in other problems
  - If you are doing math, you're doing too much
- Read directions completely (before & after)
- Write down what you know before starting
- Draw pictures, define (label) variables
  - Make sure that unknowns drop out of solution
- Don't forget units!

What You Should Study

- Review Friday Problem Solving (& Solutions)
- Review In Class Problems (& Solutions)
- Review PRS Questions (& Solutions)
- Review Problem Sets (& Solutions)
- Review PowerPoint Presentations
- Review Relevant Parts of Study Guide
  (& Included Examples)

Current & Ohm's Law



$$I = \frac{dQ}{dt}$$

$$\vec{\mathbf{J}} \equiv \frac{I}{A}\hat{\mathbf{I}}$$

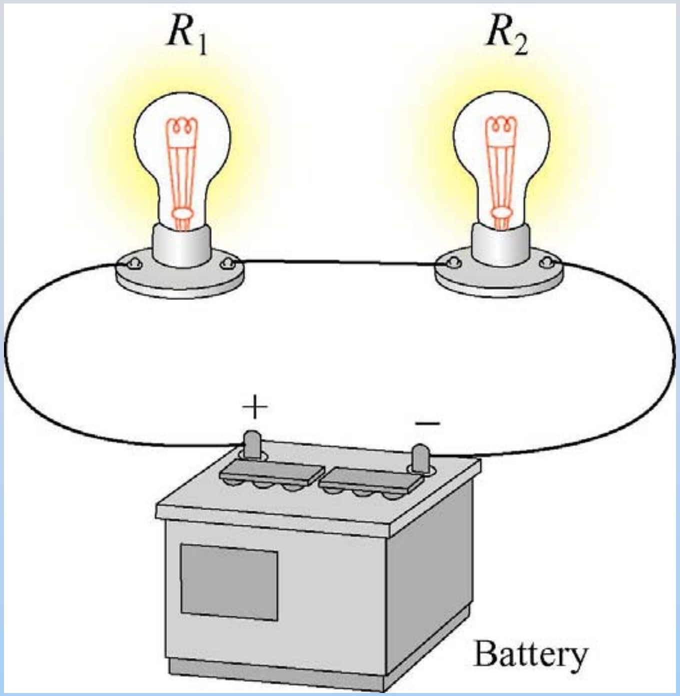
Ohm's Laws

$$\vec{\mathbf{E}} = \rho\vec{\mathbf{J}} = \left(1/\sigma\right)\vec{\mathbf{J}}$$

$$\boxed{\Delta V = IR}$$

$$R = \frac{\rho\ell}{A}$$

Series vs. Parallel

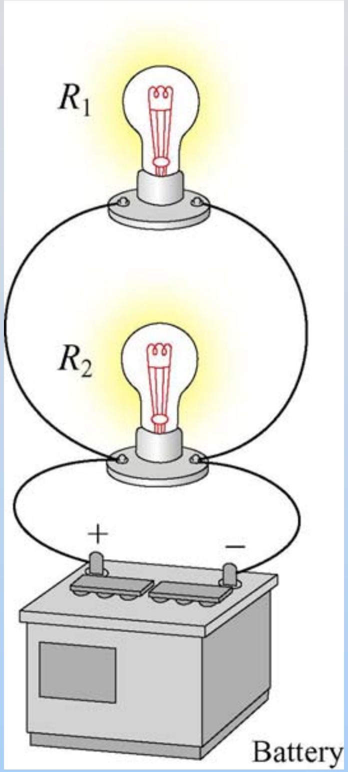


$$R_s = R_1 + R_2$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$C_P = C_1 + C_2$$



Series

- Current same
- Voltages add

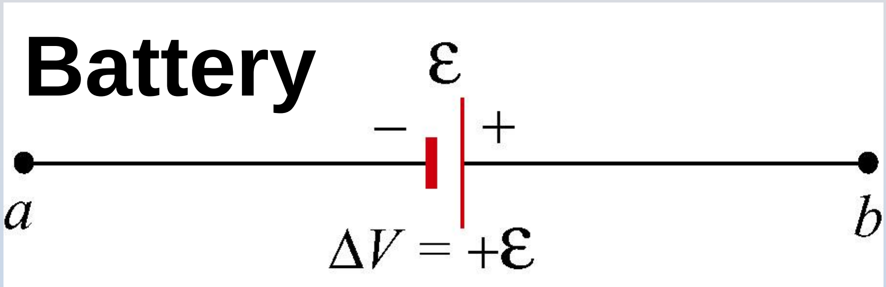
Parallel

- Currents add
- Voltages same

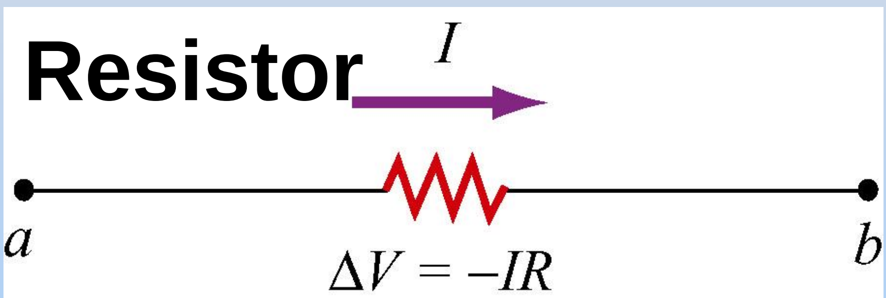
PRS Questions: Light Bulbs

Class 10

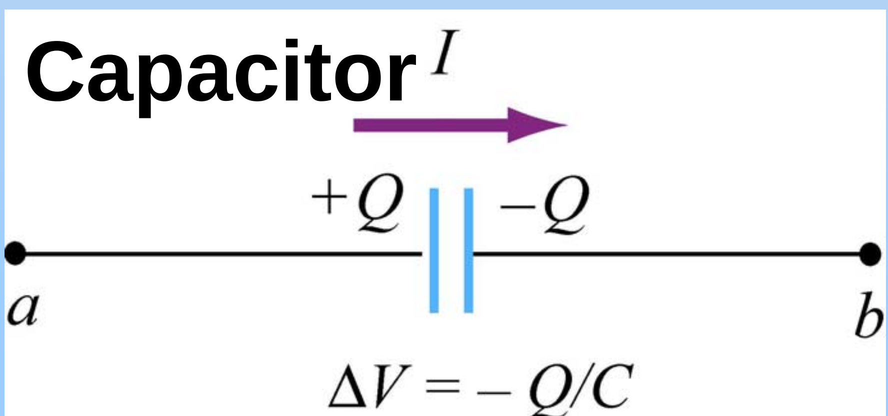
Current, Voltage & Power



$$P_{\text{supplied}} = I\,\Delta V = I\,\varepsilon$$



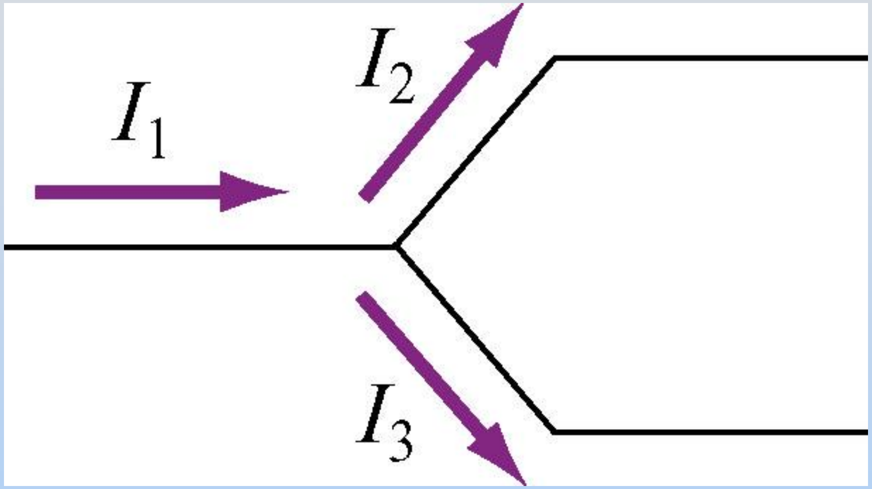
$$P_{\text{dissipated}} = I\,\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

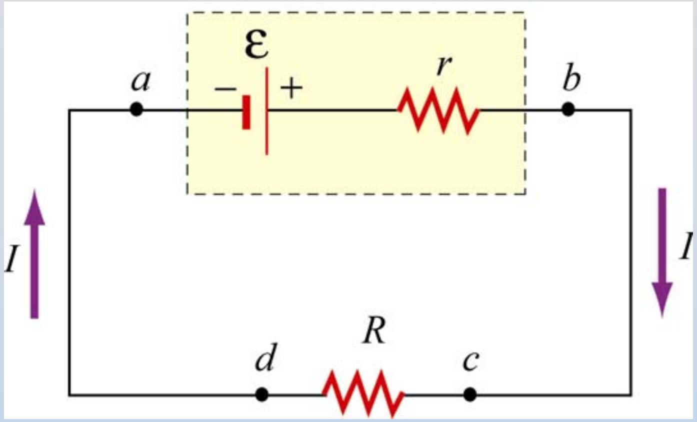


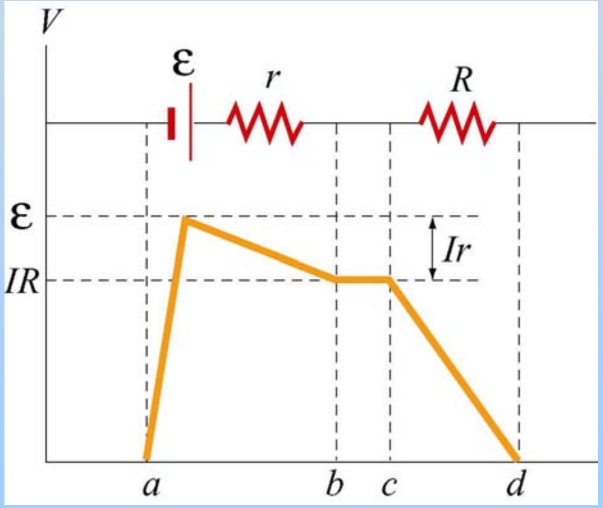
$$P_{\text{absorbed}} = I\,\Delta V = \frac{dQ}{dt}\frac{Q}{C}$$

$$= \frac{d}{dt}\frac{Q^2}{2C} = \frac{dU}{dt}$$

Kirchhoff's Rules





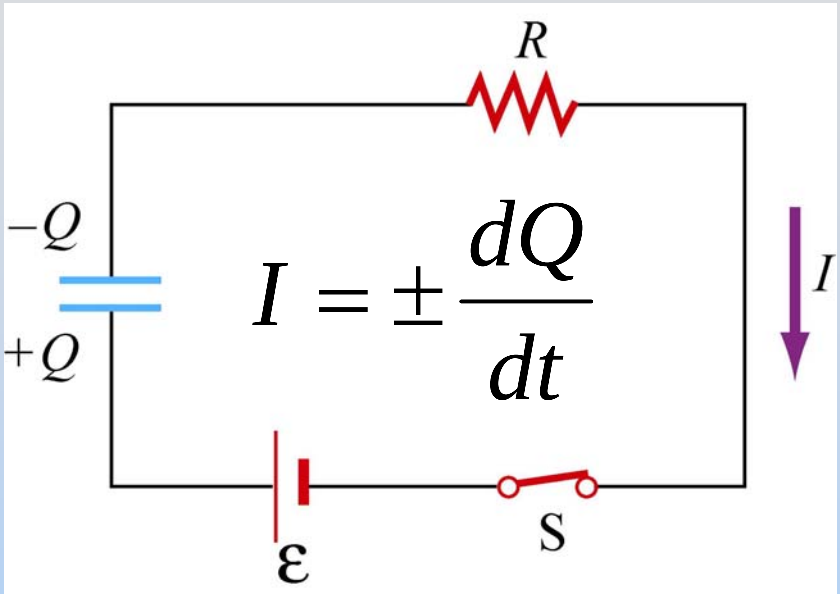


$$I_1 = I_2 + I_3$$

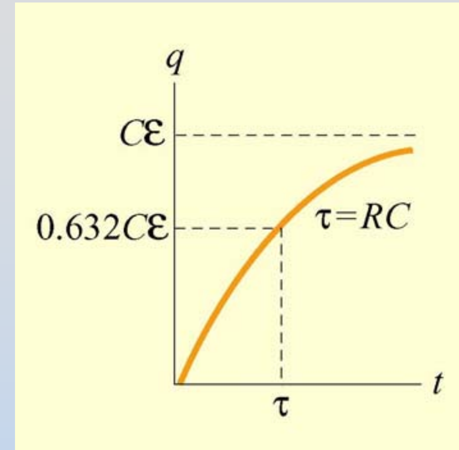
$$\Delta V = -\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

Closed Path

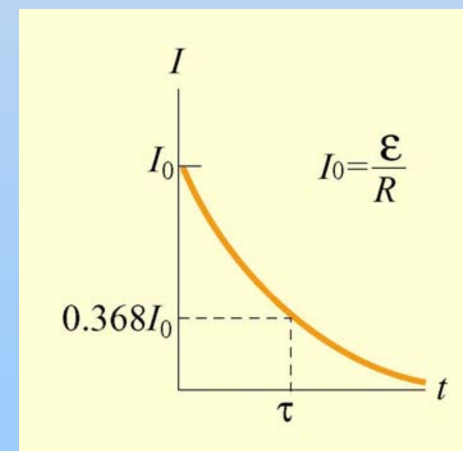
(Dis)Charging A Capacitor



$$I = \pm \frac{dQ}{dt}$$



$$Q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$



$$I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$$

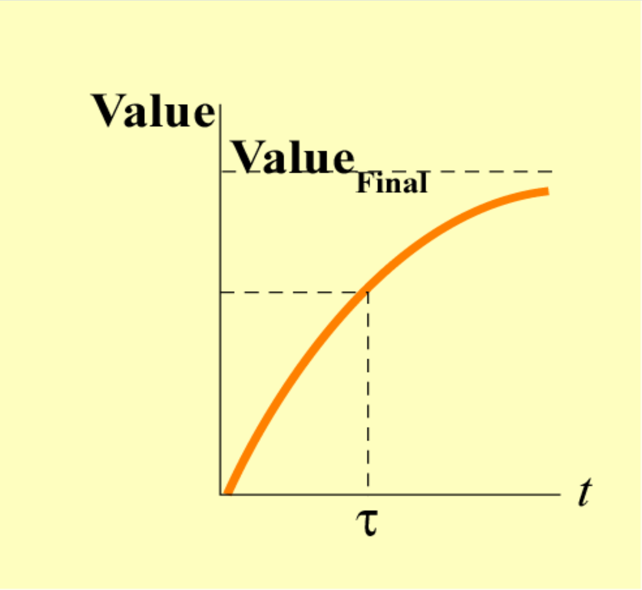
$$\sum_i \Delta V_i = \varepsilon - \frac{Q}{C} - IR = 0$$

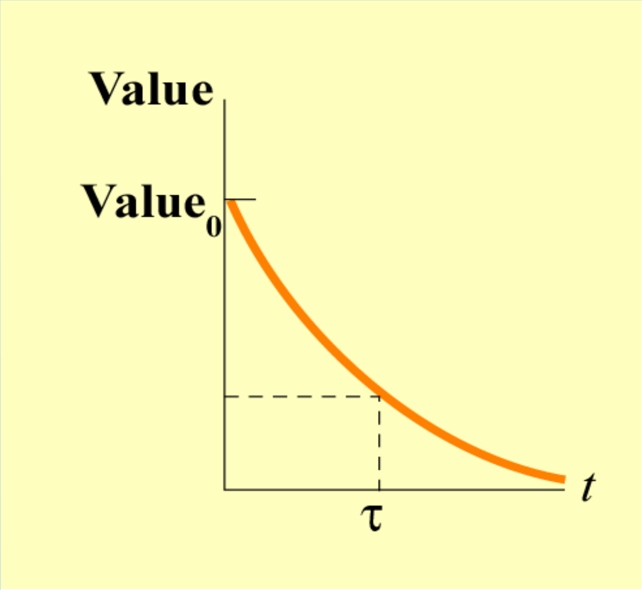
Q_{final} τ

$$C\mathcal{E} - Q - RC\frac{dQ}{dt} = 0$$

General Comment: RC

All Quantities Either:





$$\text{Value}(t) = \text{Value}_{\text{Final}}\left(1 - e^{-t/\tau}\right)$$

$$\text{Value}(t) = \text{Value}_0 e^{-t/\tau}$$

τ can be obtained from differential equation (prefactor on d/dt) e.g. τ = RC

PRS Questions: DC Circuits with Capacitors

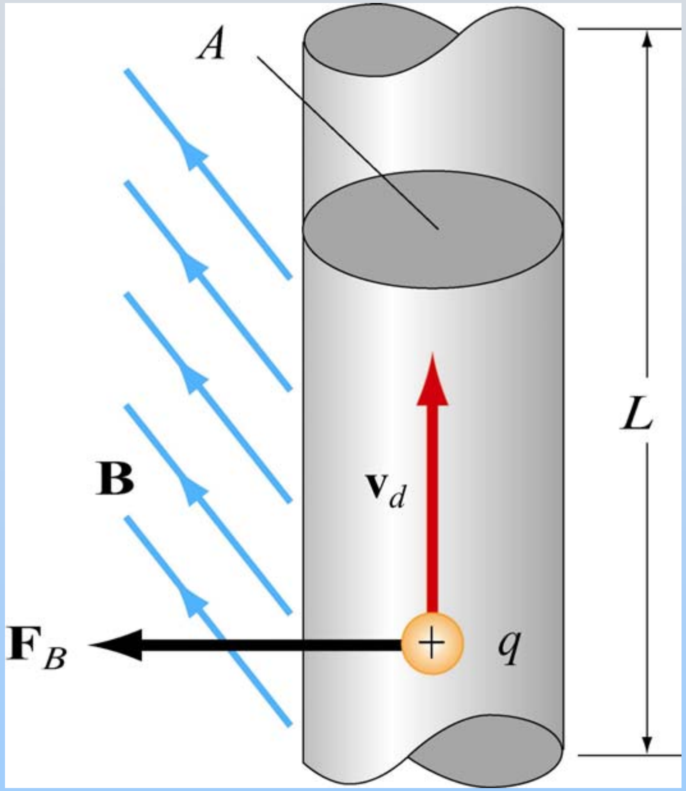
Class 12

Right Hand Rules

1. Torque: Thumb = torque,
   Fingers show rotation
2. Feel: Thumb = I,
   Fingers = B,
   Palm = F
3. Create: Thumb = I
   Fingers (curl) = B
4. Moment: Fingers (curl) = I
   Thumb = Moment (=B inside loop)

Magnetic Force

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$



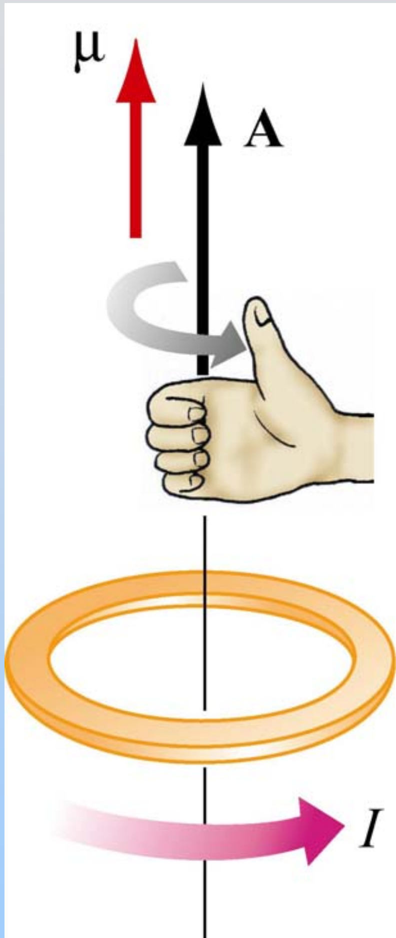
$$d\vec{\mathbf{F}}_B = I d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

$$\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right)$$

PRS Questions: Right Hand Rule

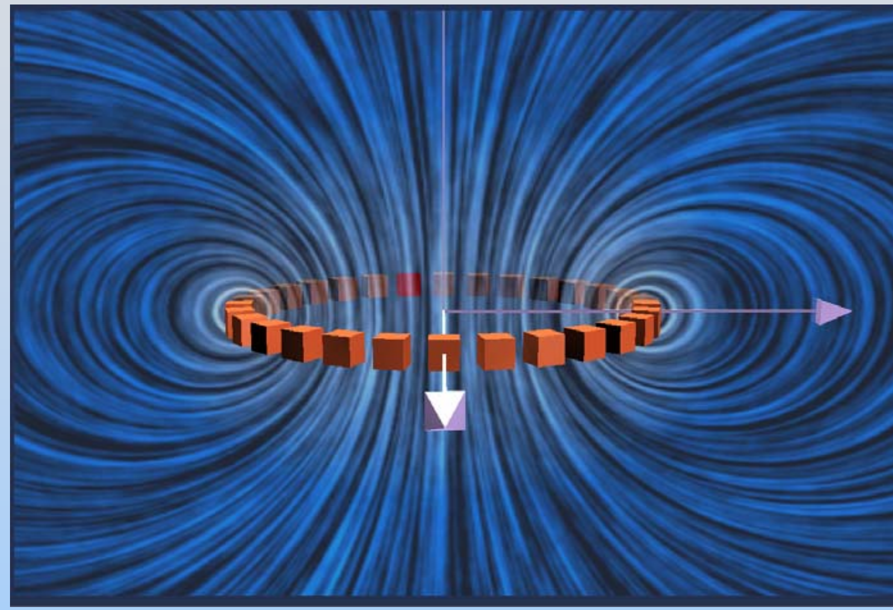
Class 14

Magnetic Dipole Moments



$$\vec{\boldsymbol{\mu}} \equiv IA\hat{\mathbf{n}} \equiv I\vec{\mathbf{A}}$$

Generate:



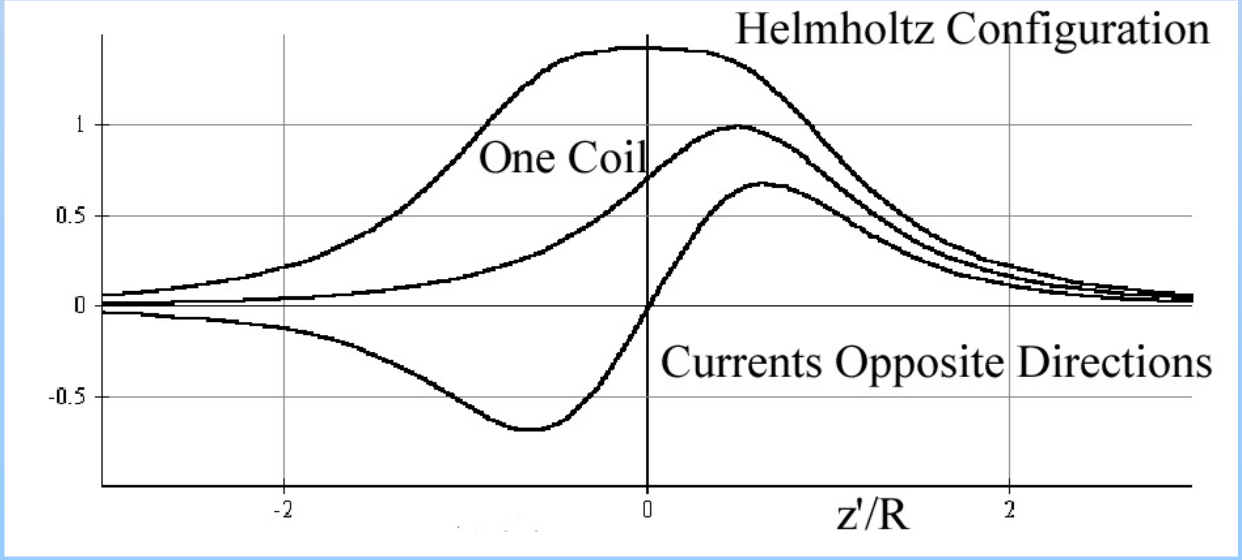
Feel:

1) Torque to align with external field
2) Forces as for bar magnets

Helmholtz Coil

Common Concept Question

- Parallel (Helmholtz) makes uniform field (torque, no force)
- Anti-parallel makes zero, non-uniform field (force, no torque)

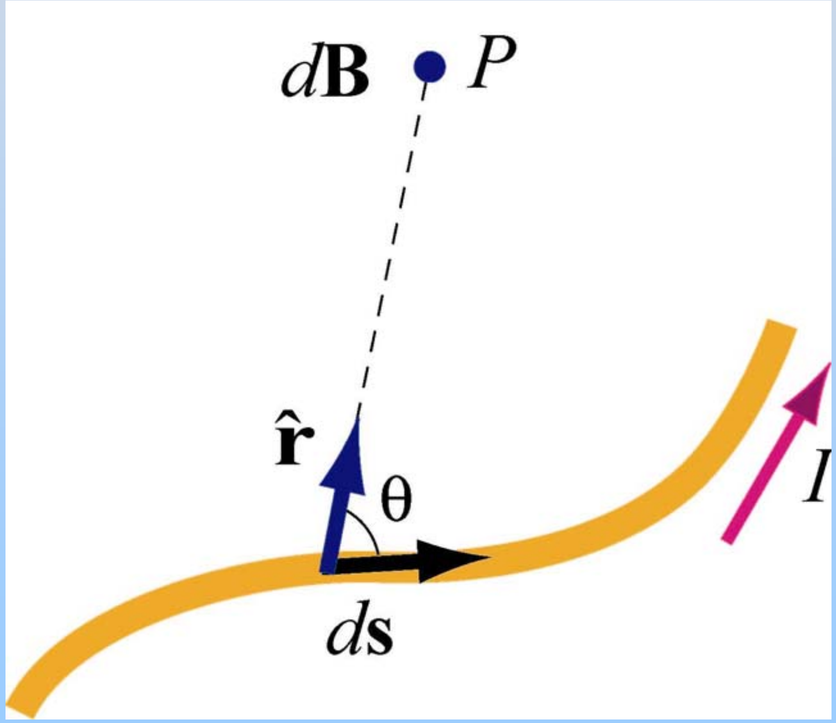


PRS Questions: Magnetic Dipole Moments

Class 17

The Biot-Savart Law

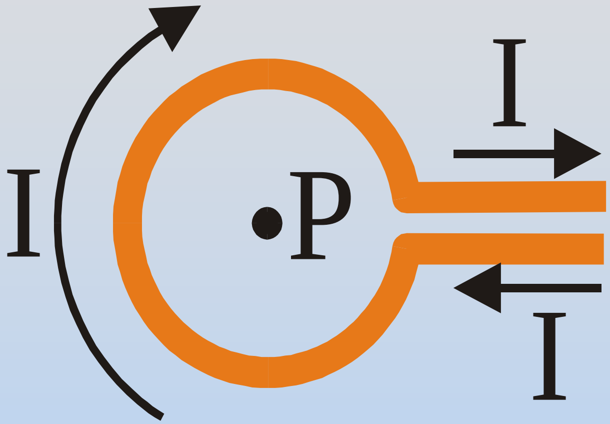
Current element of length ds carrying current I (or equivalently charge q with velocity v) produces a magnetic field:

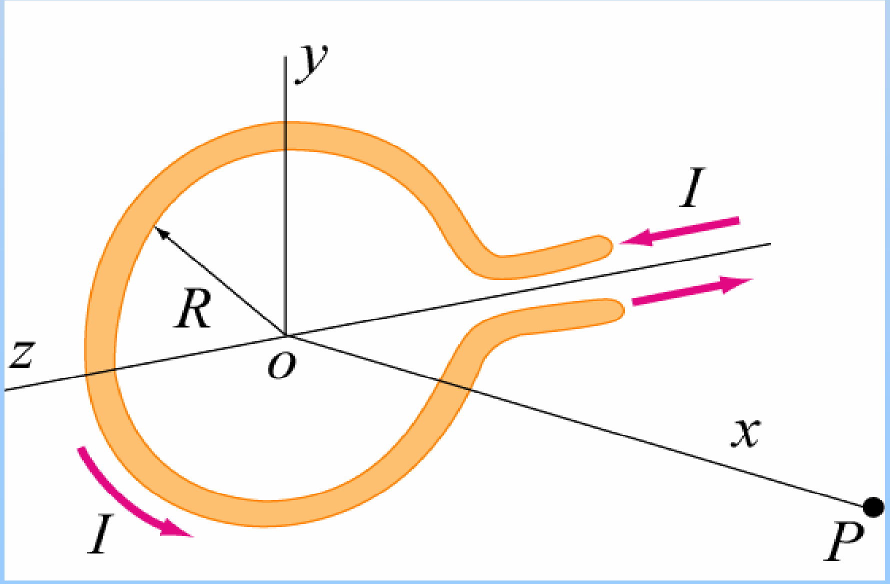


$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi} \frac{q\,\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

Biot-Savart: 2 Problem Types

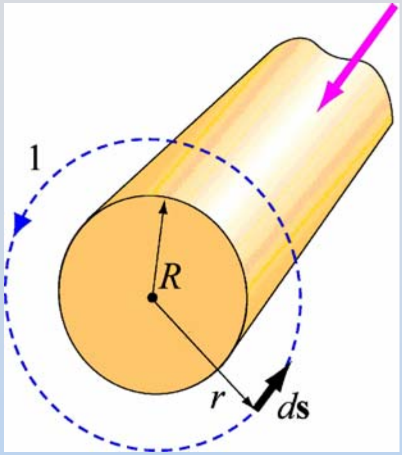


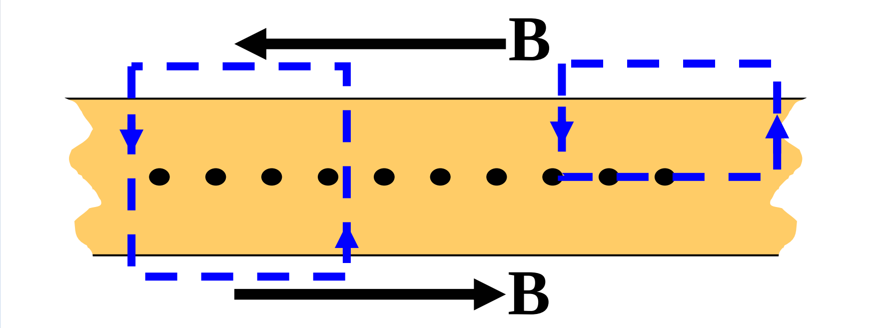


Notice that r is the same for every point on the loop. You don't really need to integrate (except to find path length)

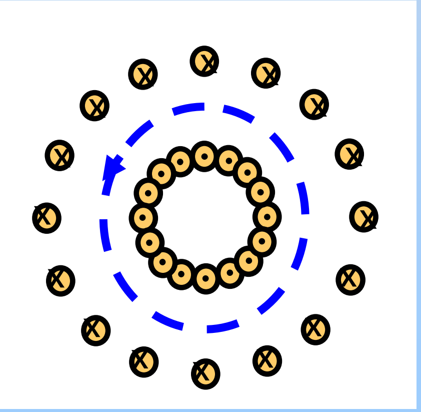
Ampere's Law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$

Long Circular Symmetry



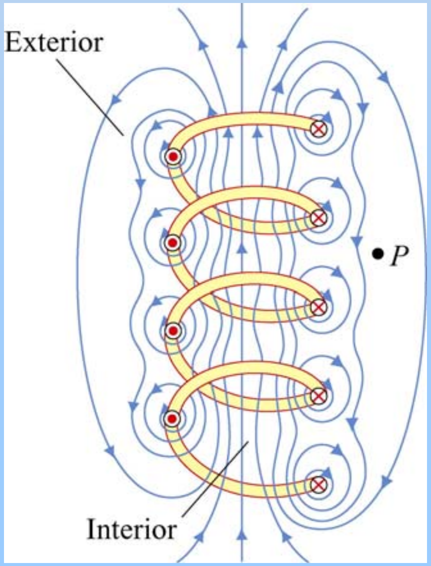


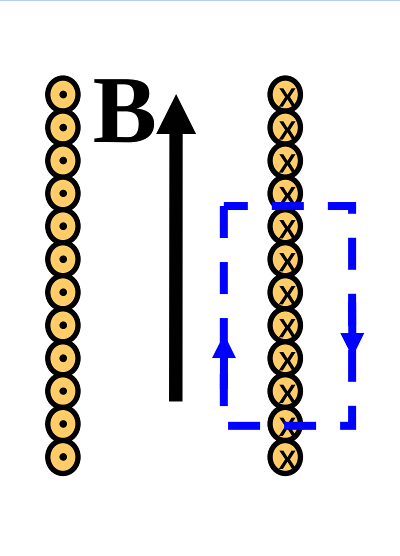
(Infinite) Current Sheet



Torus/Coax

Solenoid = 2 Current Sheets



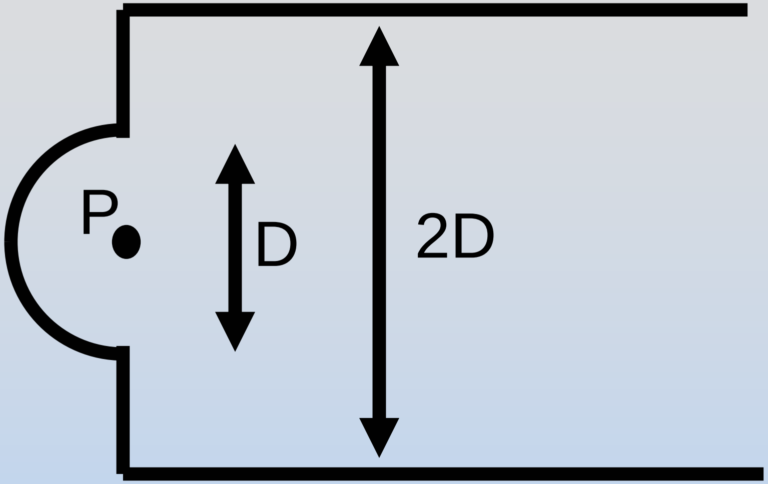


PRS Questions: Making B Fields

Classes 14-19

SAMPLE EXAM:

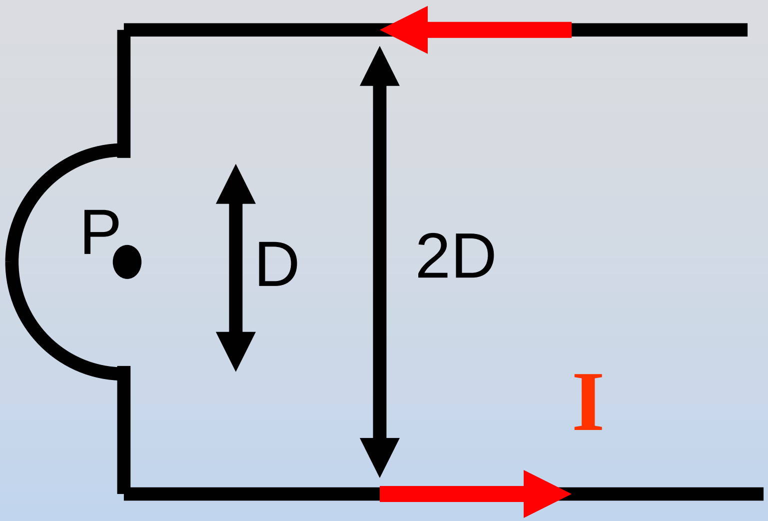
Problem 1: Wire Loop



A current flowing in the circuit pictured produces a magnetic field at point P pointing out of the page with magnitude B.

a) What direction is the current flowing in the circuit?
b) What is the magnitude of the current flow?

Solution 1: Wire Loop



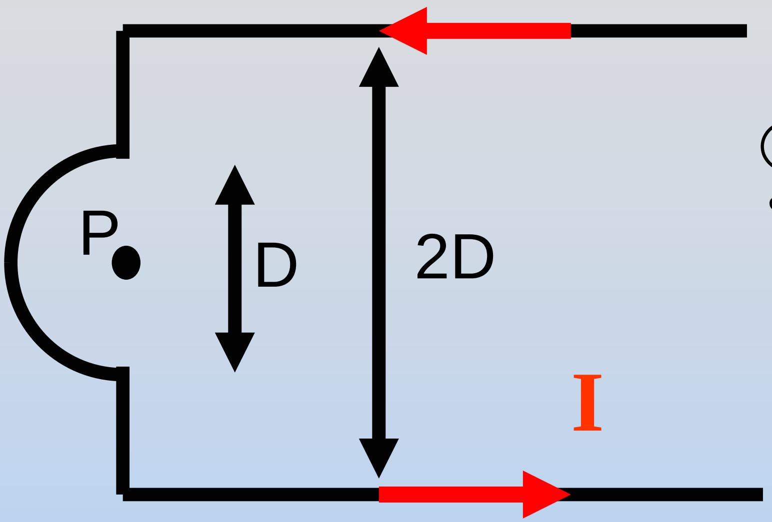
a) The current is flowing counter-clockwise, as shown above

b) There are three segments of the wire: the semi-circle, the two horizontal leads, and the two vertical leads.

The two vertical leads do not contribute to the B field (ds || r)

The two horizontal leads make an infinite wire a distance D from the field point.

Solution 1: Wire Loop



For infinite wire use Ampere's Law:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} \Rightarrow B \cdot 2\pi D = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi D}$$

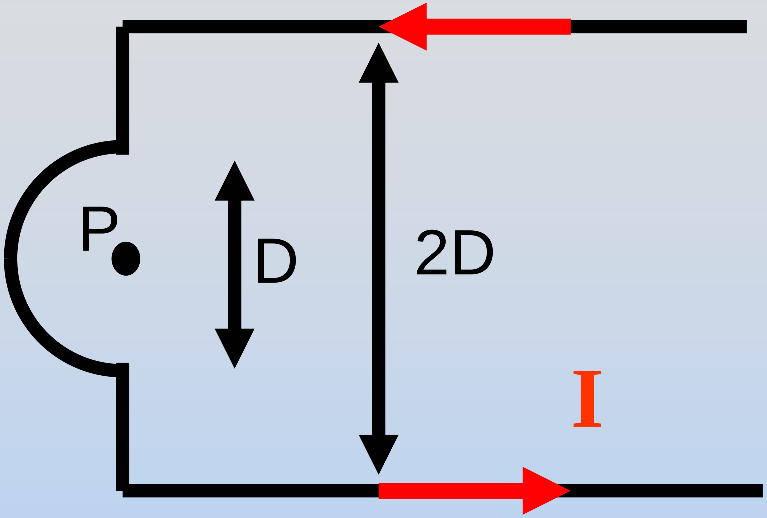
For the semi-circle use Biot-Savart:

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2} \qquad r = \frac{D}{2} \text{ and } d\vec{\mathbf{s}} \perp \hat{\mathbf{r}}$$

$$B = \int dB = \int \frac{\mu_0}{4\pi} \frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I}{r^2} (\pi r) = \frac{\mu_0 I}{4r} = \frac{\mu_0 I}{2D}$$

Solution 1: Wire Loop



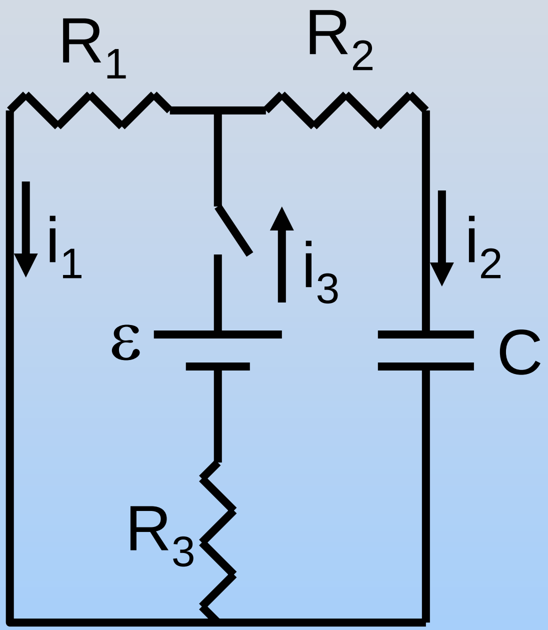
Adding together the two parts:

$$B = \frac{\mu_0 I}{2\pi D} + \frac{\mu_0 I}{2D} = \frac{\mu_0 I}{2D}\left(1 + \frac{1}{\pi}\right)$$

They gave us B and want I to make that B:

$$\boxed{I = \frac{2DB}{\mu_0\left(1 + \frac{1}{\pi}\right)}}$$

Problem 2: RC Circuit



Initially C is uncharged.

1. When the switch is first closed, what is the current i_3?
2. After a very long time, how much charge is stored on the capacitor?
3. Obtain a differential equation for the charge on the capacitor

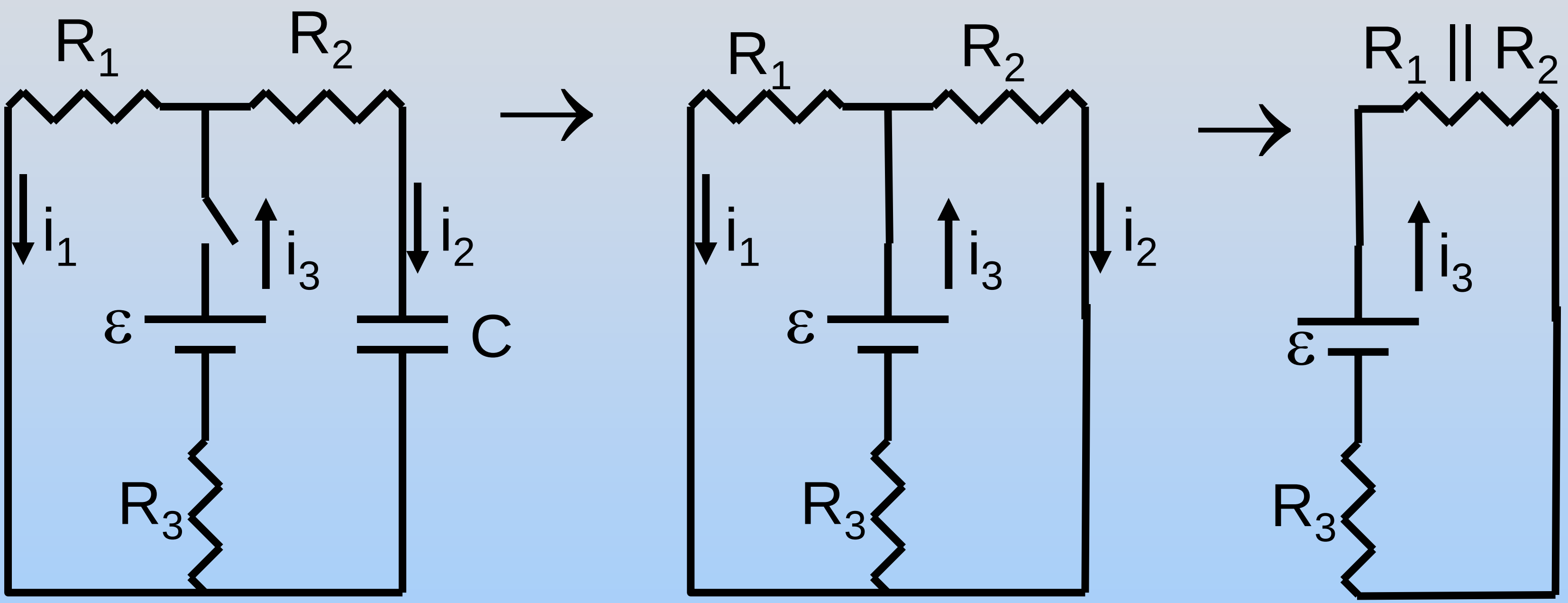
   (Here only, let $R_1=R_2=R_3=R$)

Now the switch is opened

4. Immediately after opening the switch, what is i_1? i_2? i_3?
5. How long before i_2 falls to 1/e of this initial value?

Solution 2: RC Circuit

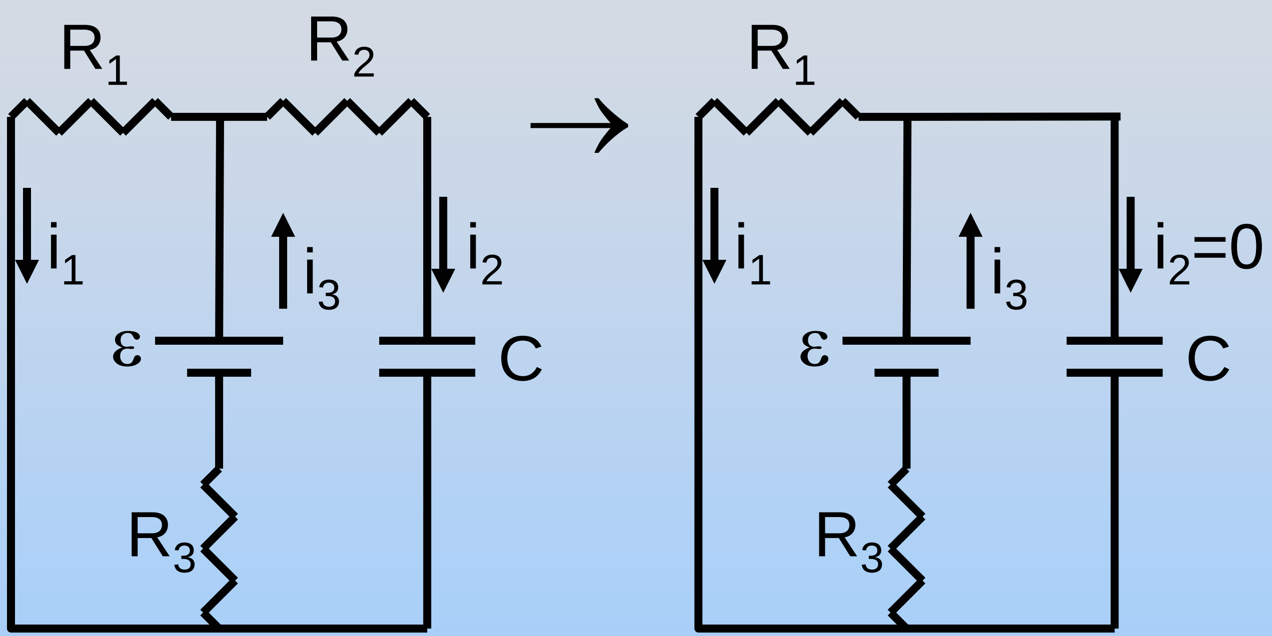
Initially C is uncharged → Looks like short



$$R_{eq} = R_3 + \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \Rightarrow \boxed{i_3 = \frac{\varepsilon}{R_{eq}}}$$

Solution 2: RC Circuit

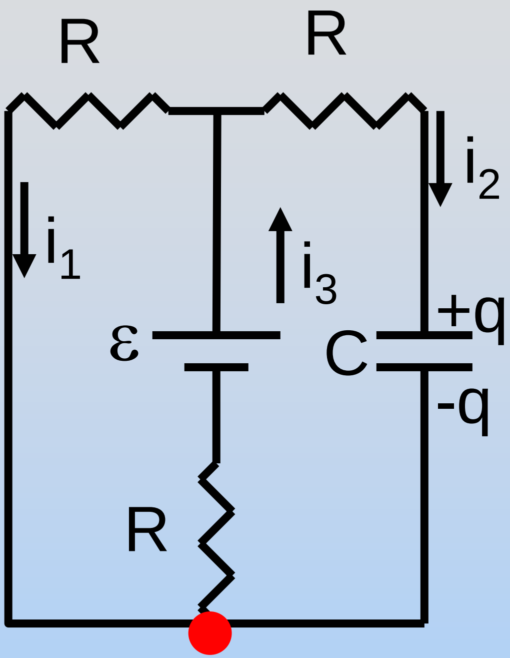
After a long time, C is full $\rightarrow i_2 = 0$



$$i_1 = i_3 = \frac{\varepsilon}{R_1 + R_3}$$

$$Q = CV_C = C\left(i_1 R_1\right) = \boxed{C\varepsilon \frac{R_1}{R_1 + R_3}}$$

Solution 2: RC Circuit



Kirchhoff's Loop Rules

$$\text{Left: } -i_3R+\varepsilon-i_1R=0$$

$$\text{Right: } -i_3R+\varepsilon-i_2R-{q}/{c}=0$$

$$\text{Current: } i_3=i_1+i_2$$

$$\text{Want to have } i_2 \text{ and } q \text{ only } (L-2R):$$

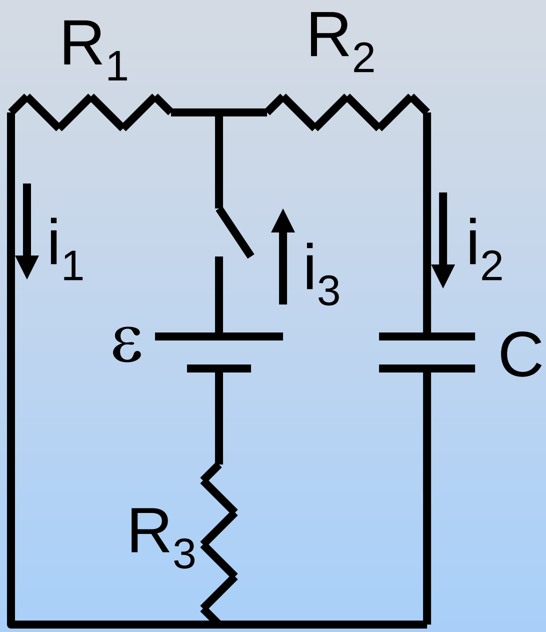
$$0=-\left(i_1+i_2\right)R+\varepsilon-i_1R+2\left(i_1+i_2\right)R-2\varepsilon+2i_2R+{2q}/{c}$$

$$=3i_2R-\varepsilon+{2q}/{c}$$

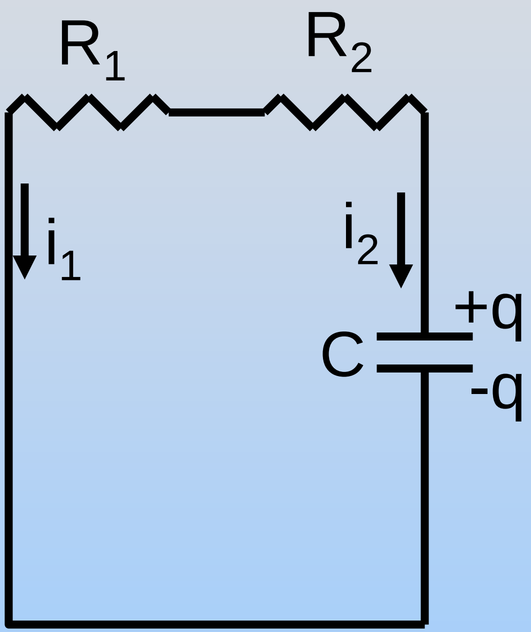
$$i_2=+\frac{dq}{dt} \quad \rightarrow \quad \boxed{\frac{dq}{dt}=\frac{\varepsilon}{3R}-\frac{2q}{3RC}}$$

Solution 2: RC Circuit

Now open the switch.



$$i_3 = 0 \rightarrow$$



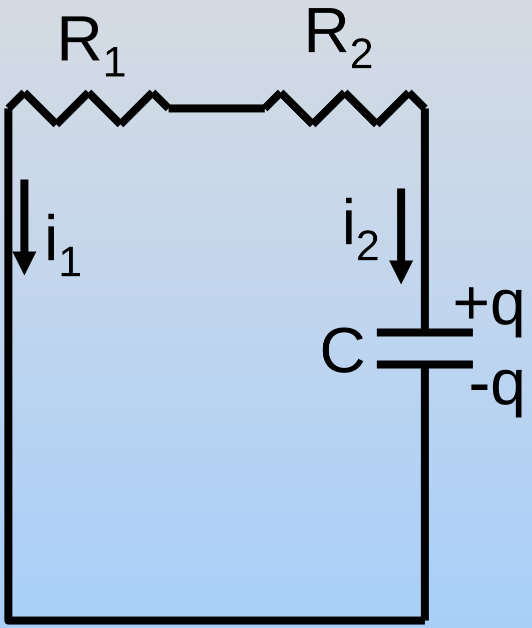
Capacitor now like a battery, with:

$$V_C = \frac{Q}{C} = \varepsilon \frac{R_1}{R_1 + R_3}$$

$$i_1 = -i_2 = \frac{V_C}{R_1 + R_2} = \varepsilon \frac{R_1}{R_1 + R_3} \frac{1}{R_1 + R_2}$$

Solution 2: RC Circuit

How long to fall to 1/e of initial current? The time constant!

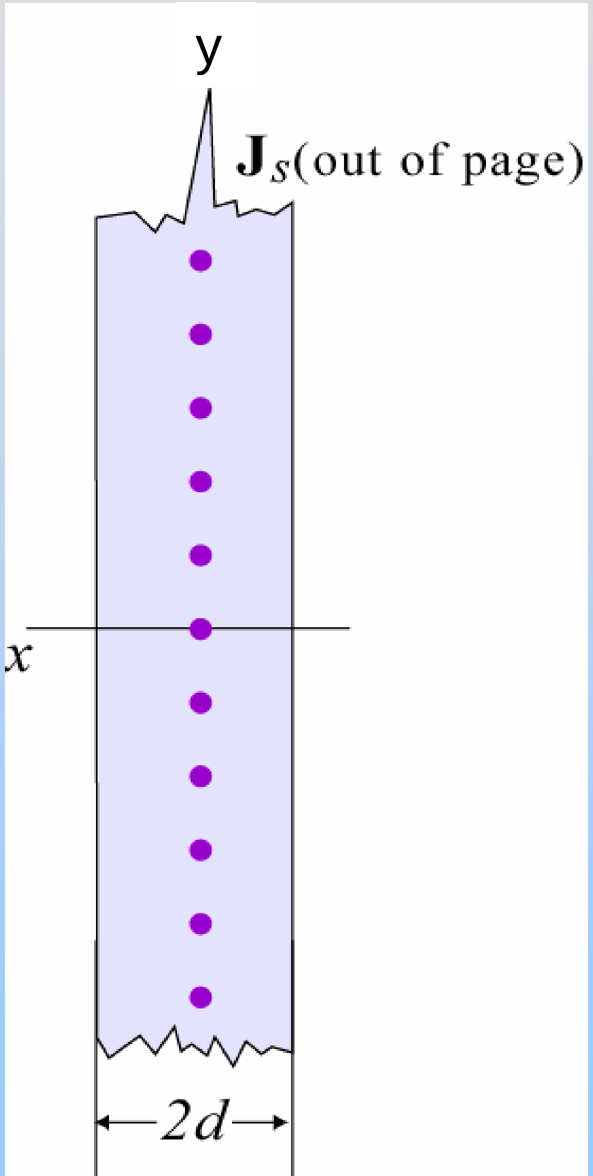


This is an easy circuit since it just looks like a resistor and capacitor in series, so:

$$\tau = (R_1 + R_2)C$$

Notice that this is different than the charging time constant, because there was another resistor in the circuit during the charging

Problem 3: Non-Uniform Slab

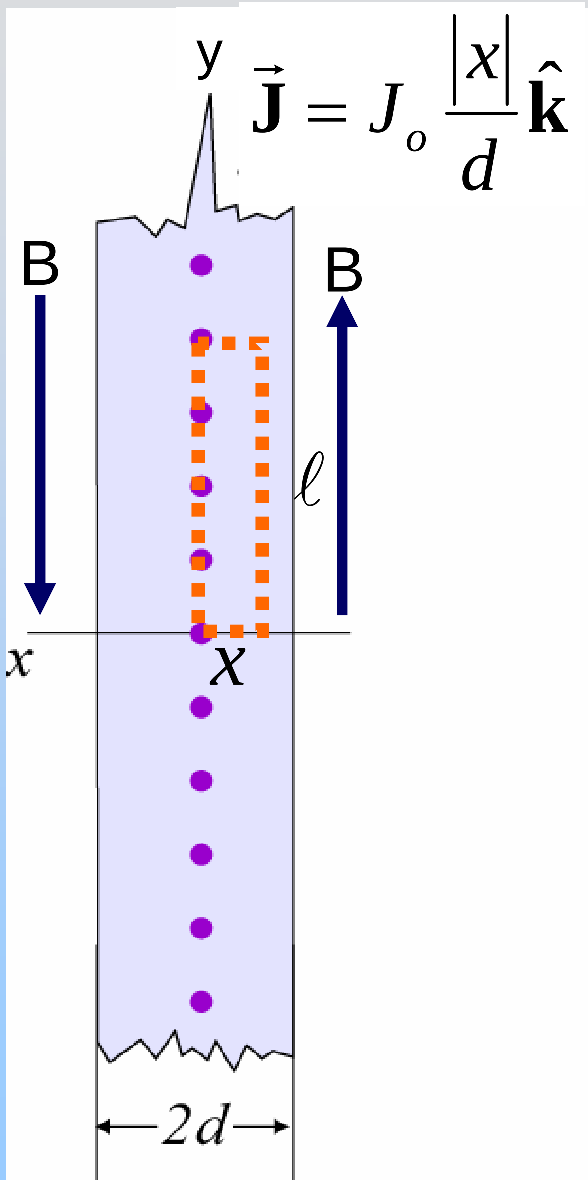


Consider the slab at left with non-uniform current density:

$$\vec{\mathbf{J}} = J_o \frac{|x|}{d} \hat{\mathbf{k}}$$

Find B everywhere

Solution 3: Non-Uniform Slab



Direction: Up on right, down on left

Inside: (at 0<x<d): $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$

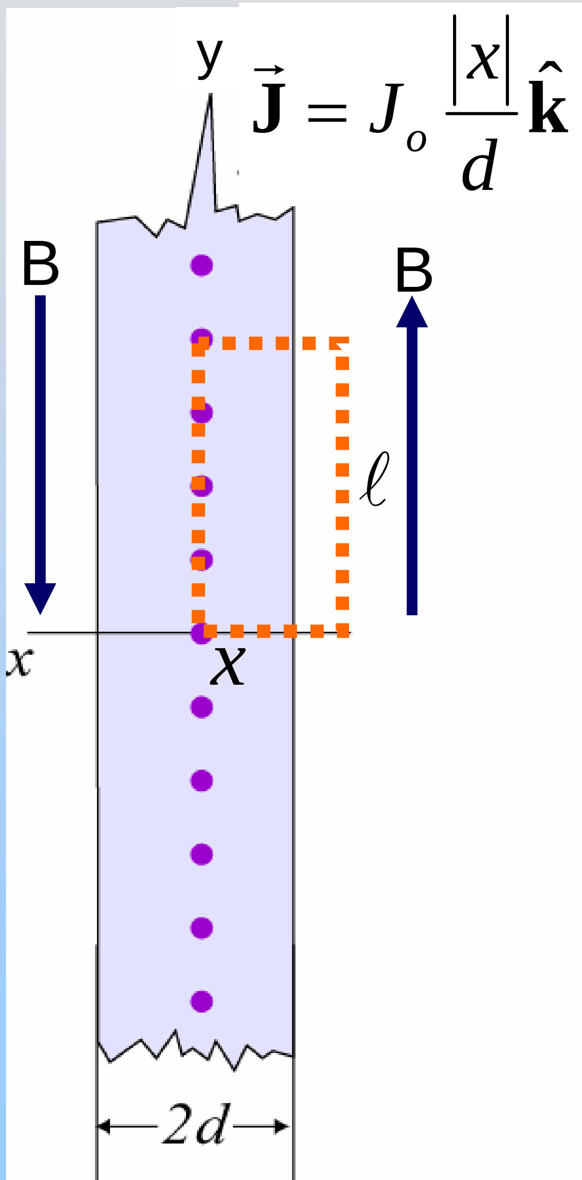
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell + 0 + 0 + 0$$

$$\mu_0 I_{enc} = \mu_0 \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} = \mu_0 \int_0^x \frac{J_0 x}{d} \ \ell dx$$

$$= \mu_0 \frac{J_0 \ell}{d} \frac{x^2}{2}$$

$$\boxed{B = \mu_0 \frac{J_0}{d} \frac{x^2}{2} \quad \text{up}}$$

Solution 3: Non-Uniform Slab



Direction: Up on right, down on left

Outside: (x > d): $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$

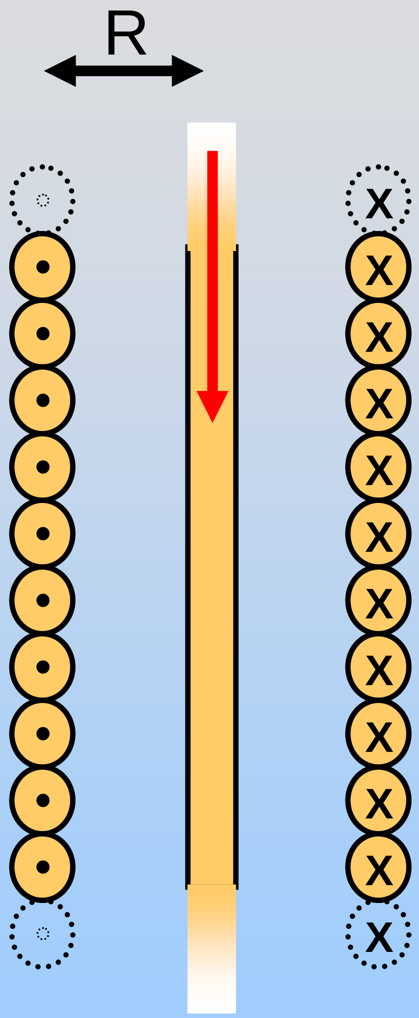
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell + 0 + 0 + 0$$

$$\mu_0 I_{enc} = \mu_0 \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} = \mu_0 \int_0^d \frac{J_0 x}{d} \;\ell dx$$

$$= \mu_0 \frac{J_0 \ell}{d} \frac{d^2}{2}$$

$$\boxed{B = \tfrac{1}{2}\mu_0 J_0 d \;\text{ up}}$$

Problem 4: Solenoid

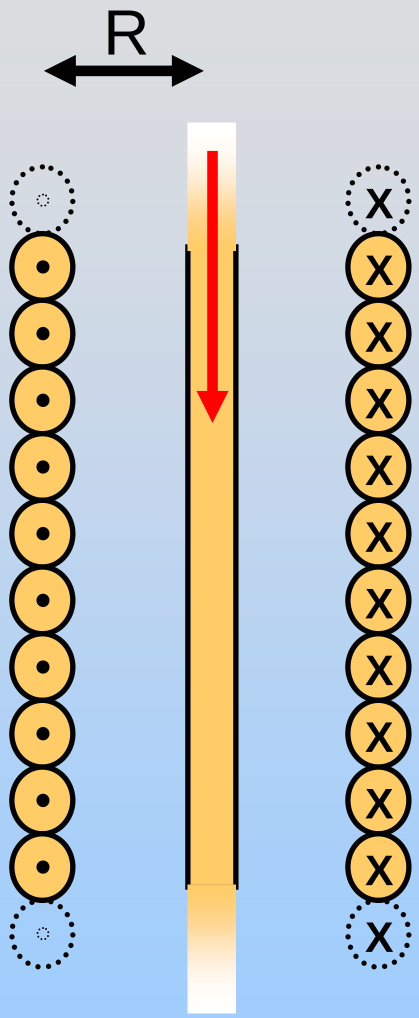


A current I flows up a very long solenoid and then back down a wire lying along its axis, as pictured. The wires are negligibly small (i.e. their radius is 0) and are wrapped at n turns per meter.

a) What is the force per unit length (magnitude and direction) on the straight wire due to the current in the solenoid?

b) A positive particle (mass m, charge q) is launched inside of the solenoid, at a distance $r = a$ to the right of the center. What velocity (direction and non-zero magnitude) must it have so that the field created by the wire along the axis never exerts a force on it?

Solution 4: Solenoid



SUPERPOSITION

You can just add the two fields from each part individually

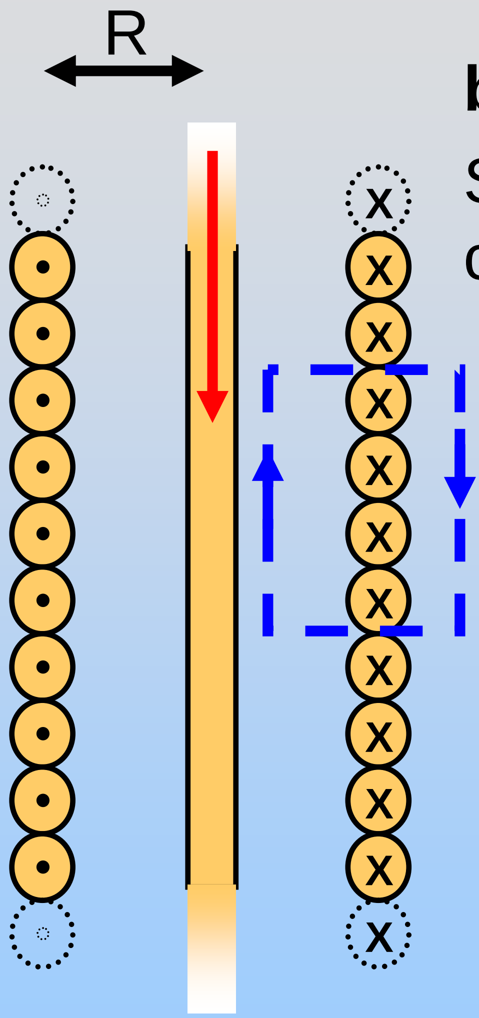
a) Force on wire down axis

Since the current is anti-parallel to the field produced by the solenoid, there is no force (F=0) on this wire

b) Launching Charge q

The central wire produces a field that wraps in circles around it. To not feel a force due to this field, the particle must always move parallel to it – it must move in a circle of radius *a* (since that is the radius it was launched from).

Solution 4: Solenoid



b) Launching Charge q

So first we should use Ampere's law to calculate the field due to the solenoid:

$$\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = Bl = \mu_0 NI$$

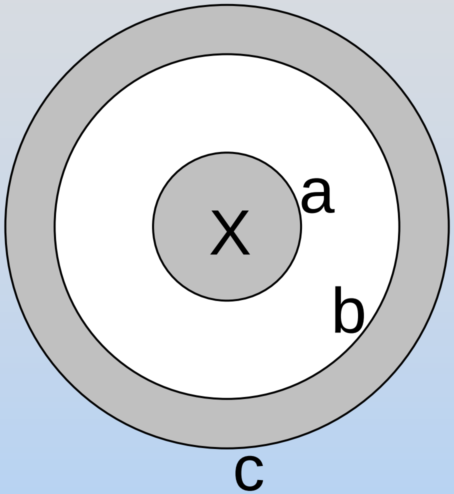
$$B = \frac{\mu_0 NI}{l} = \mu_0 nI \text{ up the solenoid}$$

Now we just need to make a charge q move in a circular orbit with *r* = *a*:

$$\vec{\mathbf{F}}_B = q\,\vec{\mathbf{v}} \times \vec{\mathbf{B}} = qvB = m\,{v^2}/{r} = {mv^2}/{a}$$

$$\boxed{v = \frac{qBa}{m} = \frac{q\mu_0 nIa}{m} \quad \text{out of the page}}$$

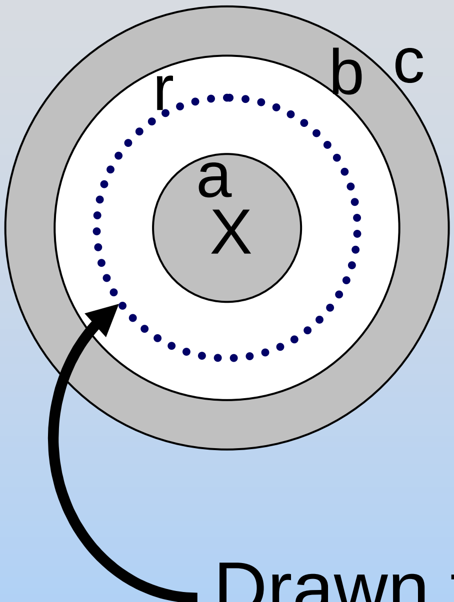
Problem 5: Coaxial Cable



Consider a coaxial cable of with inner conductor of radius *a* and outer conductor of inner radius *b* and outer radius *c*. A current *I* flows into the page on the inner conductor and out of the page on the outer conductor.

What is the magnetic field everywhere (magnitude and direction) as a function of distance *r* from the center of the wire?

Solution 5: Coaxial Cable



Drawn for *a* < *r* < *b*

Everywhere the magnetic field is clockwise. To figure out the magnitude use Ampere's Law:

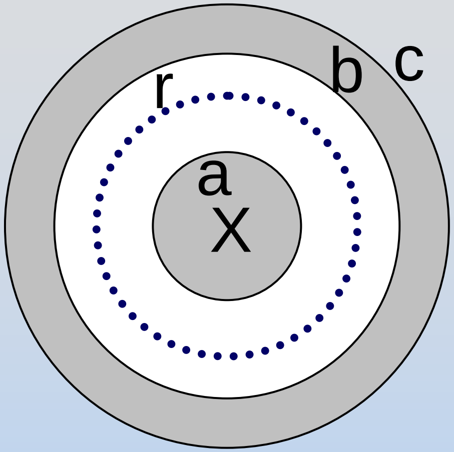
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} \Rightarrow B \cdot 2\pi r = \mu_0 I_{enc}$$

$$\Rightarrow B = \frac{\mu_0 I_{enc}}{2\pi r}$$

The amount of current penetrating our Amperian loop depends on the radius *r*:

$$r \le a: \; I_{enc} = I \frac{r^2}{a^2} \qquad \Rightarrow \boxed{B = \frac{\mu_0 I r}{2\pi a^2} \text{clockwise}}$$

Solution 5: Coaxial Cable



Remember: Everywhere

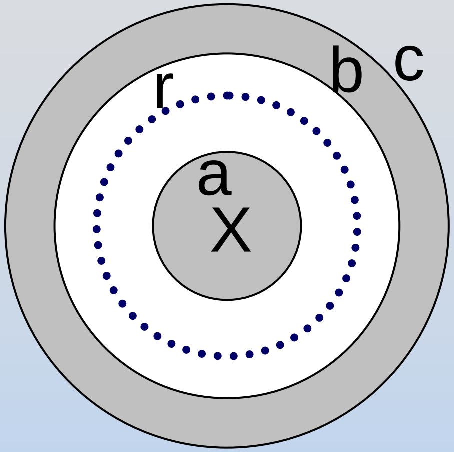
$$B = \frac{\mu_0 I_{enc}}{2\pi r} \text{ clockwise}$$

$$a \le r \le b: \; I_{Encl} = I \quad \Rightarrow \quad \boxed{B = \frac{\mu_0 I}{2\pi r} \text{clockwise}}$$

$$b \le r \le c: \; I_{Encl} = I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)$$

$$\Rightarrow \quad \boxed{B = \frac{\mu_0 I}{2\pi r}\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) \text{clockwise}}$$

Solution 5: Coaxial Cable



Remember: Everywhere

$$B = \frac{\mu_0 I_{enc}}{2\pi r} \text{ clockwise}$$

$$r \geq c: \; I_{Encl} = 0 \quad \Rightarrow \quad \boxed{B = 0}$$