Class 17: Outline

Hour 1:

Dipoles & Magnetic Fields

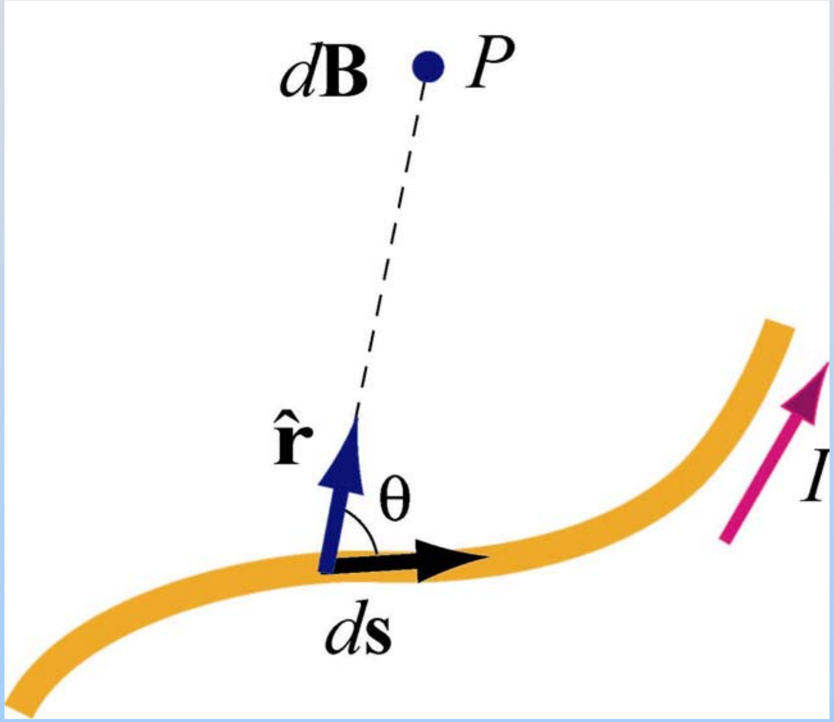
Hour 2:

Expt. 7: Dipoles in B Fields

Last Time:
Biot-Savart

The Biot-Savart Law

Current element of length $d\mathbf{s}$ carrying current I produces a magnetic field:



$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

Moving charges are currents too…

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi} \frac{q\, \vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

(http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/magnetostatics/03-CurrentElement3d/03-cElement320.html)

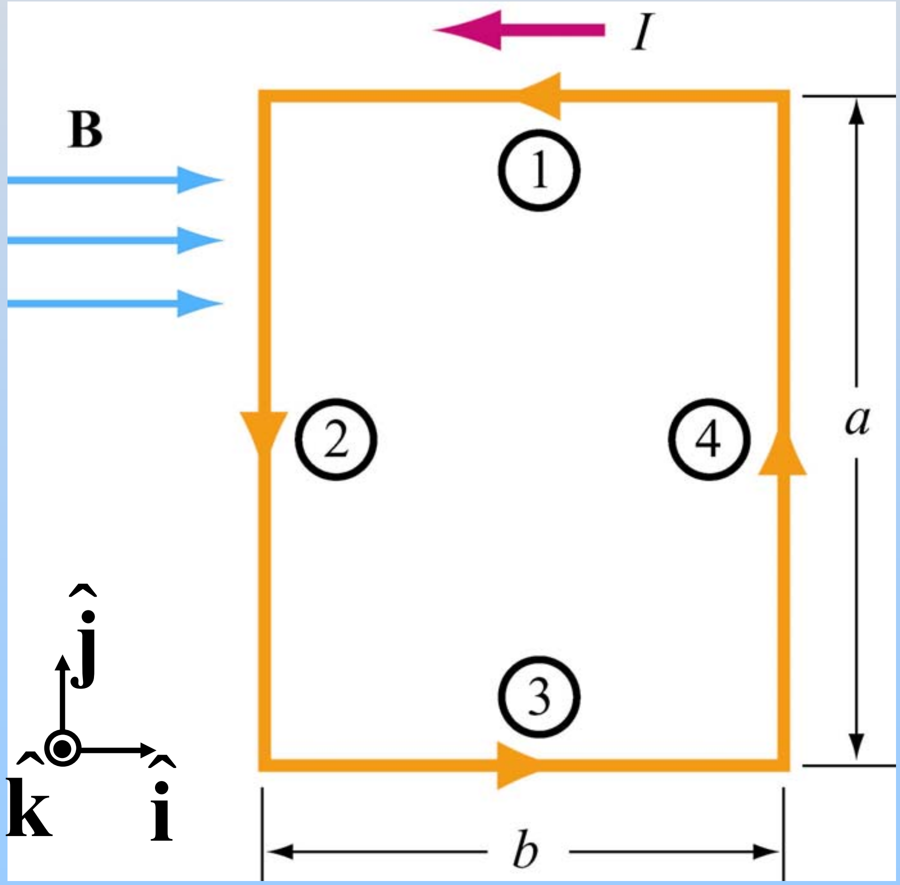
PRS Question:
Force between wires

Magnetic Dipoles: Torque & Force

First: Review From Friday

Rectangular Current Loop

Place rectangular current loop in uniform B field



$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_3 = 0 \quad \left(I\vec{\mathbf{L}} \parallel \vec{\mathbf{B}} \right)$$

$$\vec{\mathbf{F}}_2 = I(-a\hat{\mathbf{j}}) \times (B\hat{\mathbf{i}}) = IaB\hat{\mathbf{k}}$$

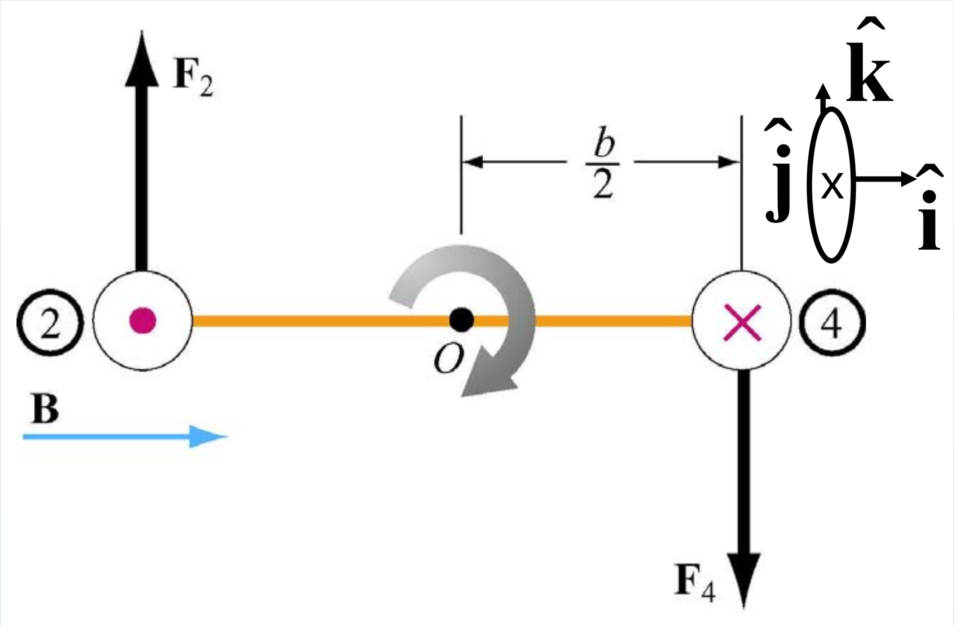
$$\vec{\mathbf{F}}_4 = I(a\hat{\mathbf{j}}) \times (B\hat{\mathbf{i}}) = -IaB\hat{\mathbf{k}}$$

$$\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \vec{\mathbf{F}}_4 = 0$$

No net force on the loop!!

Torque on Rectangular Loop

Recall: $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$



$$\vec{\boldsymbol{\tau}} = \left(-\frac{b}{2}\hat{\mathbf{i}}\right) \times \vec{\mathbf{F}}_2 + \left(\frac{b}{2}\hat{\mathbf{i}}\right) \times \vec{\mathbf{F}}_4$$

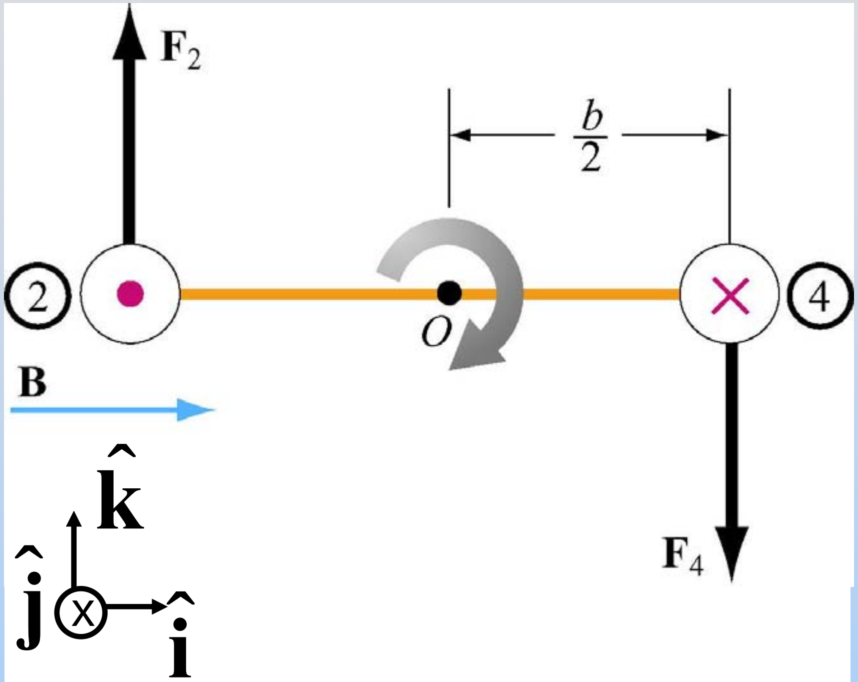
$$= \left(-\frac{b}{2}\hat{\mathbf{i}}\right) \times \left(IaB\hat{\mathbf{k}}\right) + \left(\frac{b}{2}\hat{\mathbf{i}}\right) \times \left(-IaB\hat{\mathbf{k}}\right)$$

$$= \frac{IabB}{2}\hat{\mathbf{j}} + \frac{IabB}{2}\hat{\mathbf{j}} = IabB\hat{\mathbf{j}}$$

Torque Direction:

Thumb in torque direction, fingers rotate with object

Torque on Rectangular Loop



$$\vec{\boldsymbol{\tau}} = IAB\hat{\mathbf{j}}$$

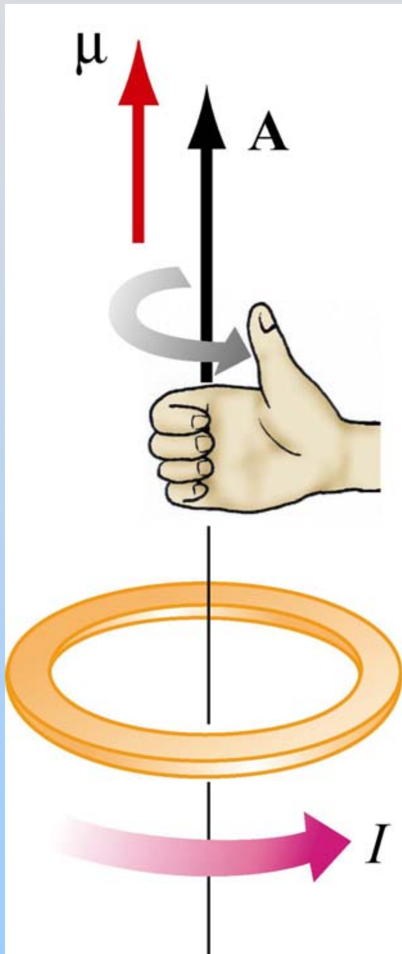
$\vec{\mathbf{A}} = A\hat{\mathbf{n}} = ab\hat{\mathbf{n}}$: area vector

$$\hat{\mathbf{n}} = +\hat{\mathbf{k}}, \quad \vec{\mathbf{B}} = B\hat{\mathbf{i}}$$

$$\boxed{\vec{\boldsymbol{\tau}} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}}$$

Familiar? No net force but there is a torque

Magnetic Dipole Moment



Define Magnetic Dipole Moment:

$$\vec{\mu} \equiv IA\hat{\mathbf{n}} \equiv I\vec{\mathbf{A}}$$

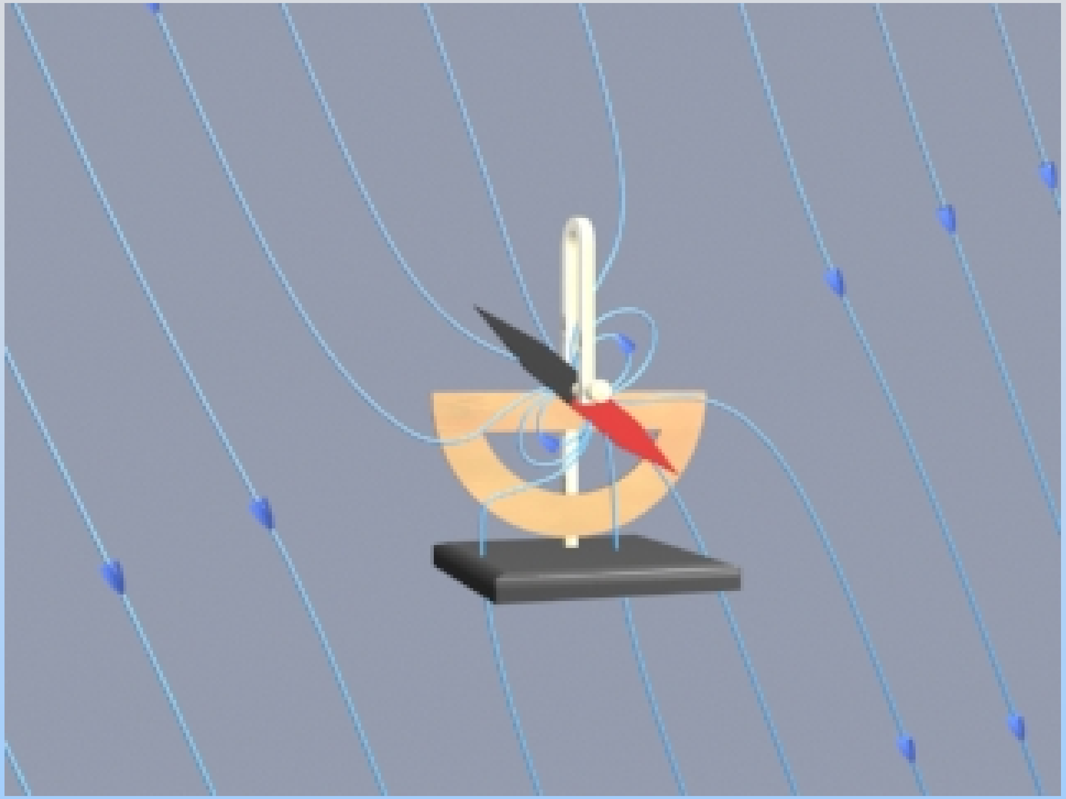
Then:

$$\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$$

Analogous to $\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$

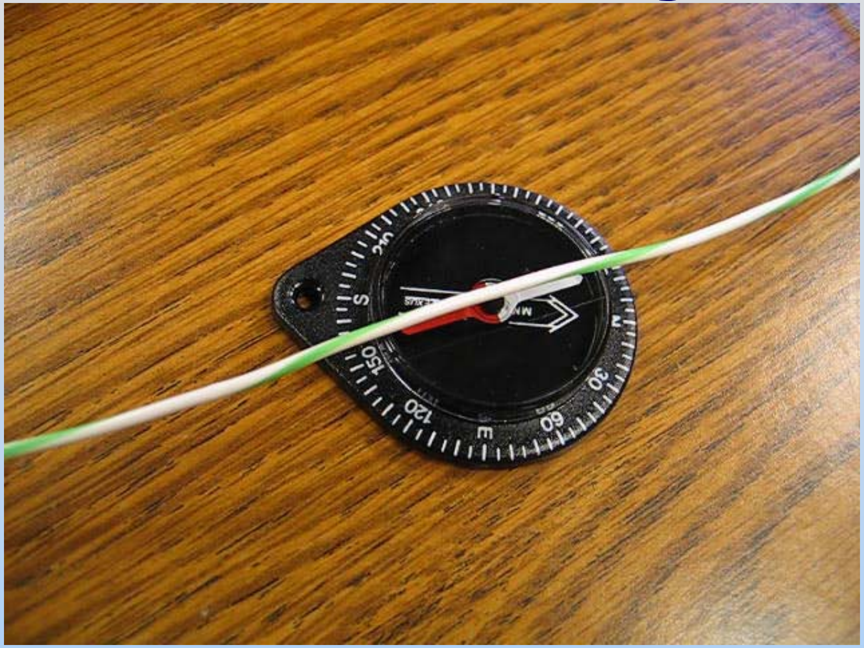
τ tends to align μ with B

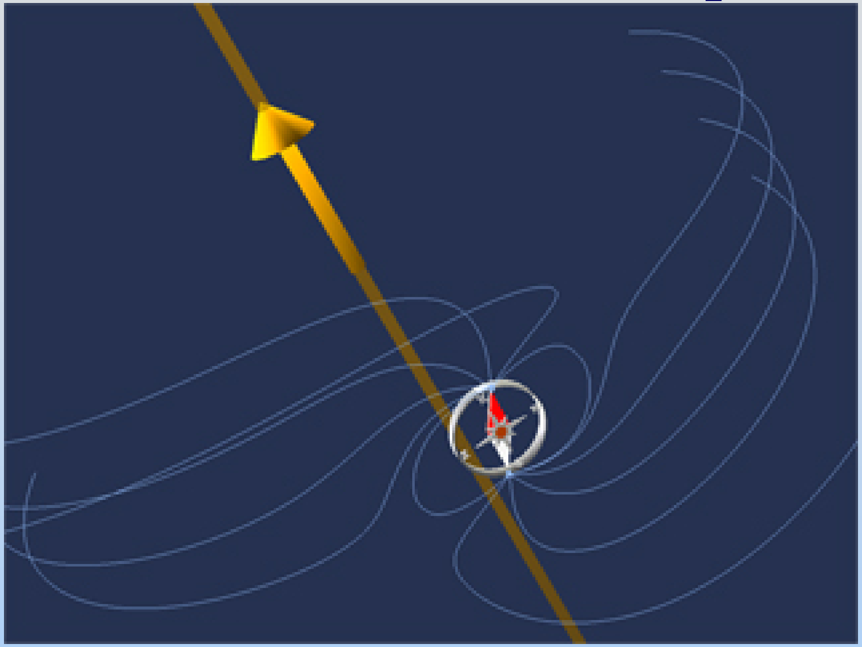
Animation:
Another Way To Look At Torque



External field connects to field of compass needle and “pulls” the dipole into alignment
(http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/magnetostatics/18-dipNeedle/18-Dip_320.html)

Interactive Java Applet: Another Way To Look At Torque



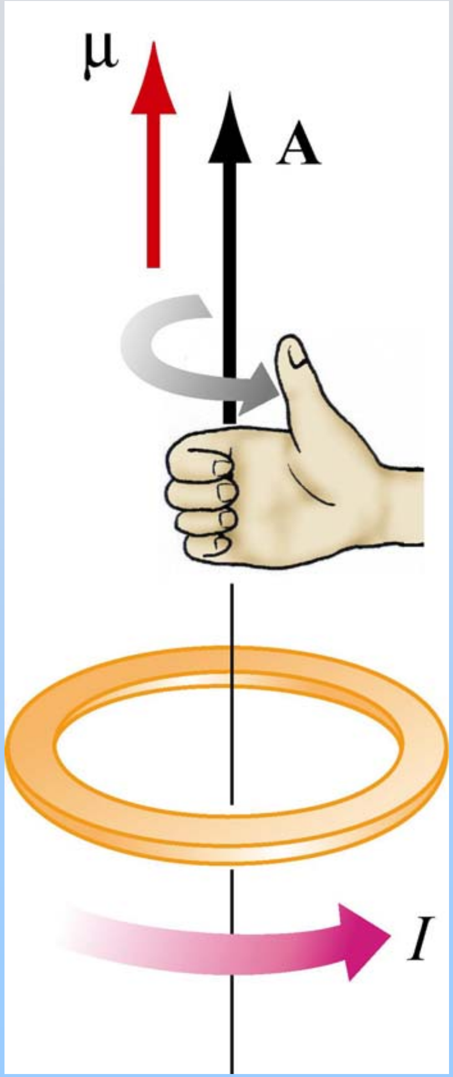


http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/magnetostatics/35-wireandmagnetapp/35-wirecompass320.html

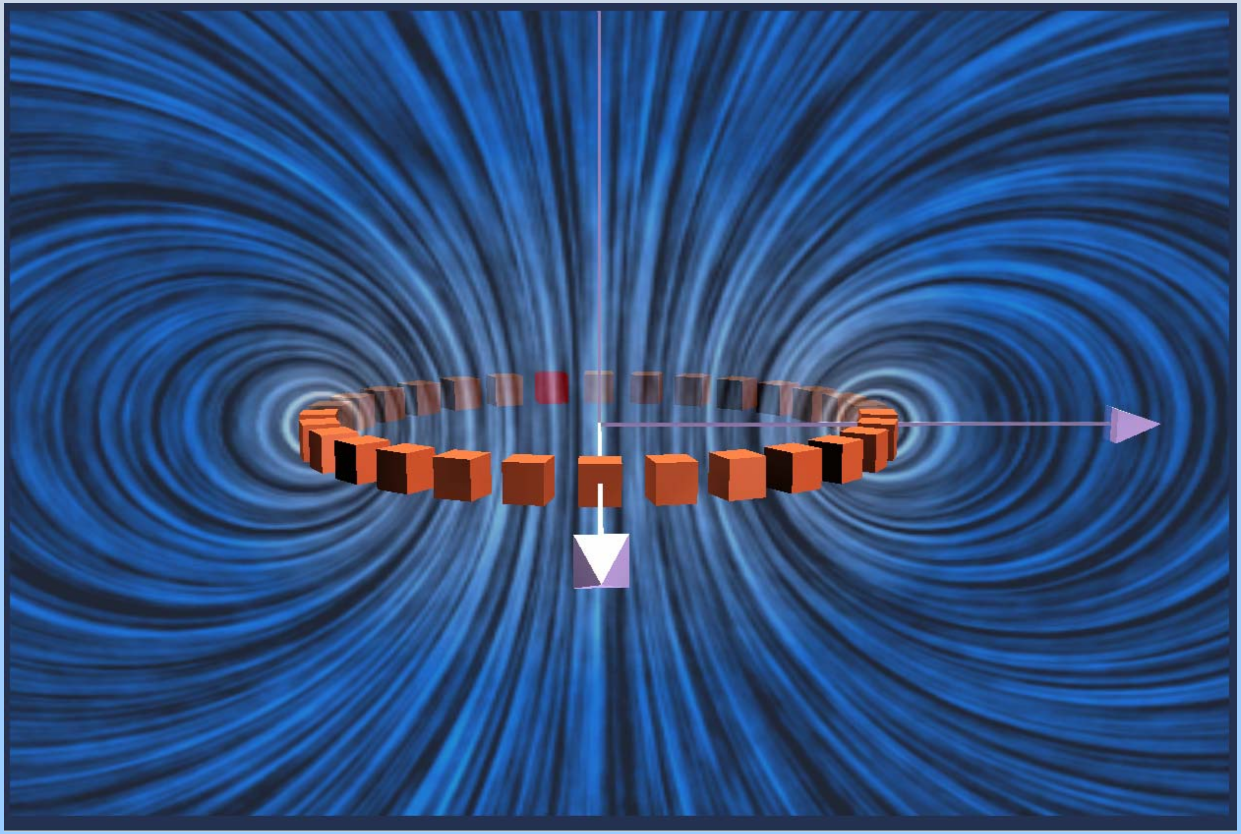
Field of wire connects to field of compass needle and “pulls” the dipole into alignment

Demonstration: Galvanometer

Magnetic Dipole Moment

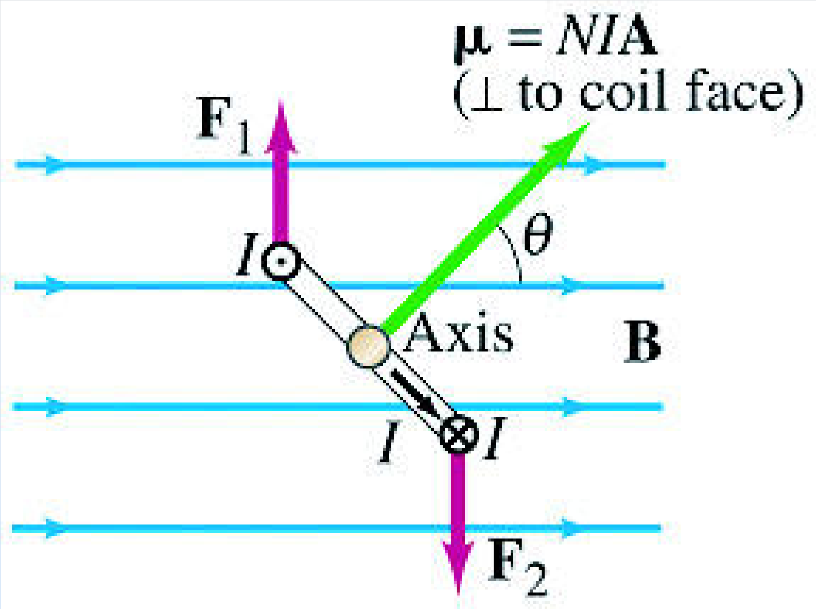


$$\vec{\boldsymbol{\mu}} \equiv IA\hat{\mathbf{n}} \equiv I\vec{\mathbf{A}}$$

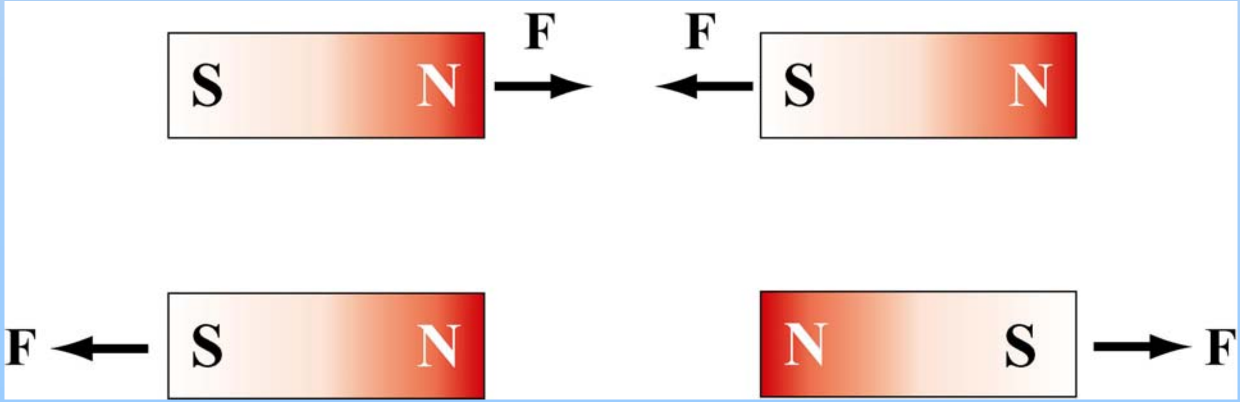


PRS Question:
Torque on Dipole in Uniform Field

Dipoles don't move???



This dipole rotates but doesn't feel a net force



But dipoles CAN feel force due to **B**. What's up?

PRS Question:
Force on Magnetic Dipole

Something New

Dipoles in Non-Uniform Fields: Force

Force on Magnetic Dipole

To determine force, we need to know energy:

$$U_{Dipole} = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$$

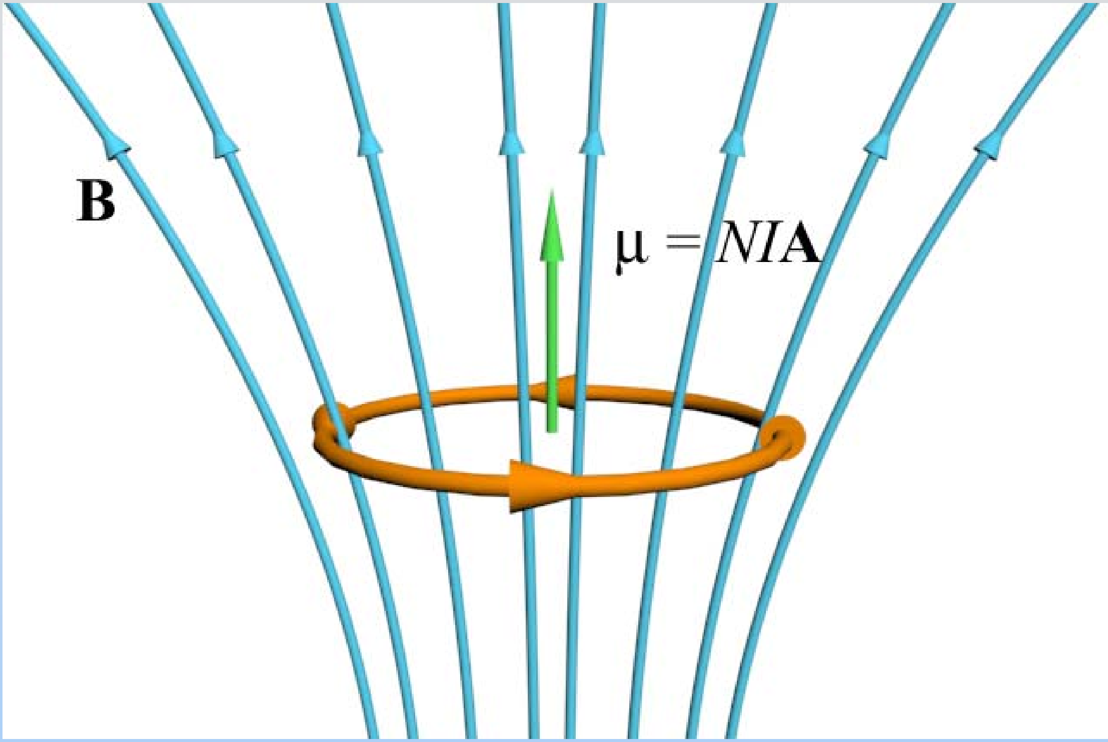
Force tells how the energy changes with position:

$$\vec{\mathbf{F}}_{Dipole} = -\vec{\nabla} U_{Dipole} = \vec{\nabla}\left(\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}\right)$$

$$\text{(after math)} = \left(\vec{\boldsymbol{\mu}} \cdot \vec{\nabla}\right)\vec{\mathbf{B}}$$

Dipoles only feel force in *non-uniform* field

Force on Magnetic Dipole

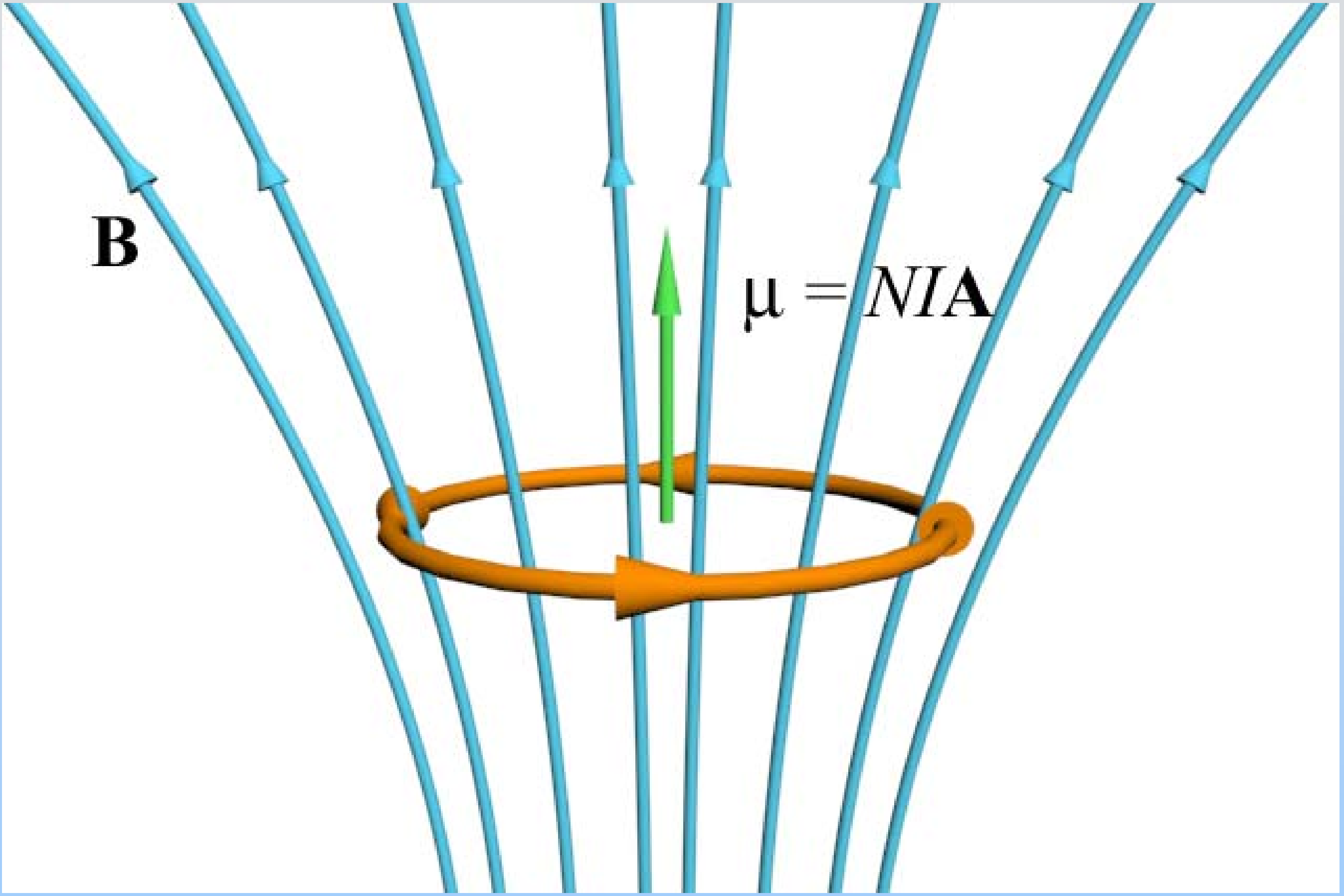


$$\frac{\partial \vec{\mathbf{B}}}{\partial z} \text{ negative}$$

Force down!

$$\vec{\mathbf{F}}_{Dipole} = \left(\vec{\boldsymbol{\mu}} \cdot \vec{\nabla}\right)\vec{\mathbf{B}} = \mu \frac{\partial \vec{\mathbf{B}}}{\partial z}$$

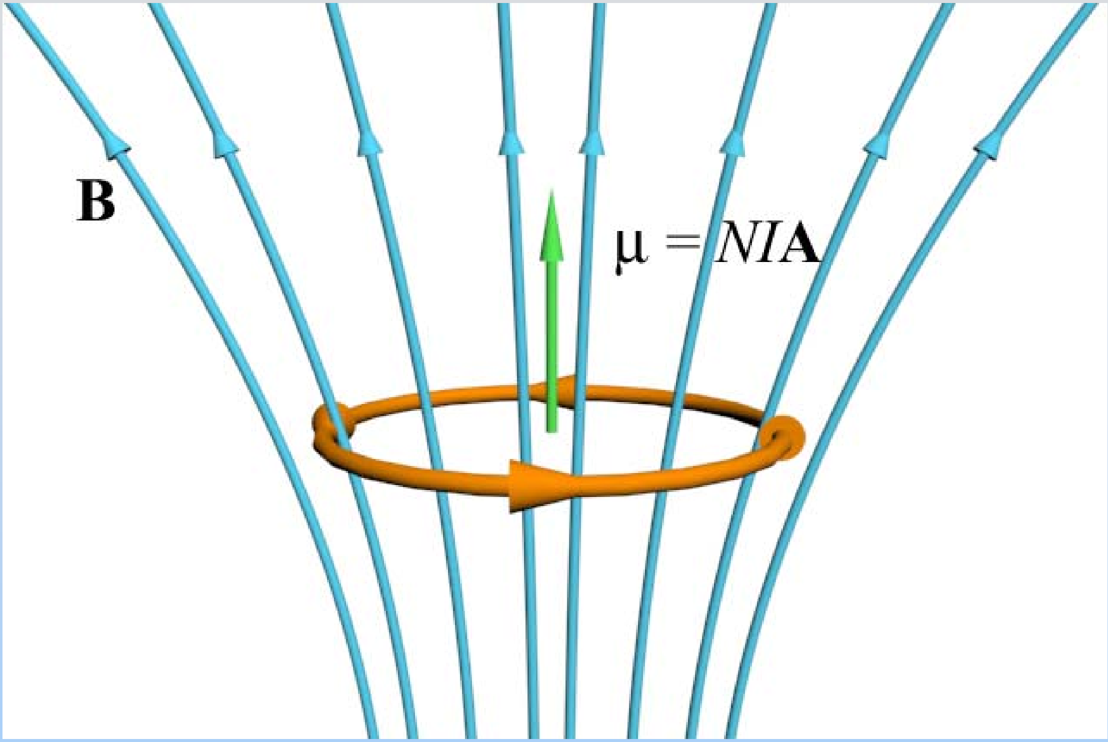
Force on Magnetic Dipole

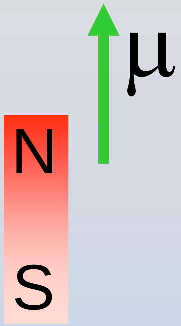


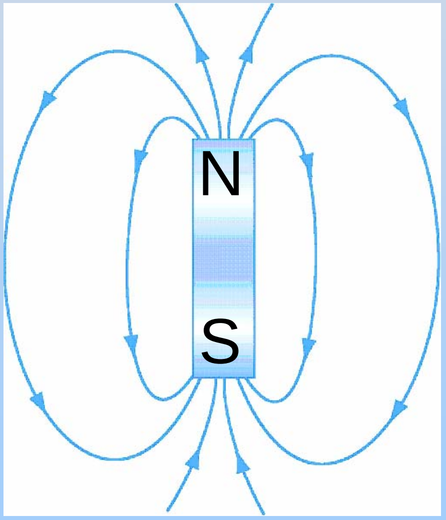
<u>Alternate Thought</u>

What makes the field pictured?

Force on Magnetic Dipole



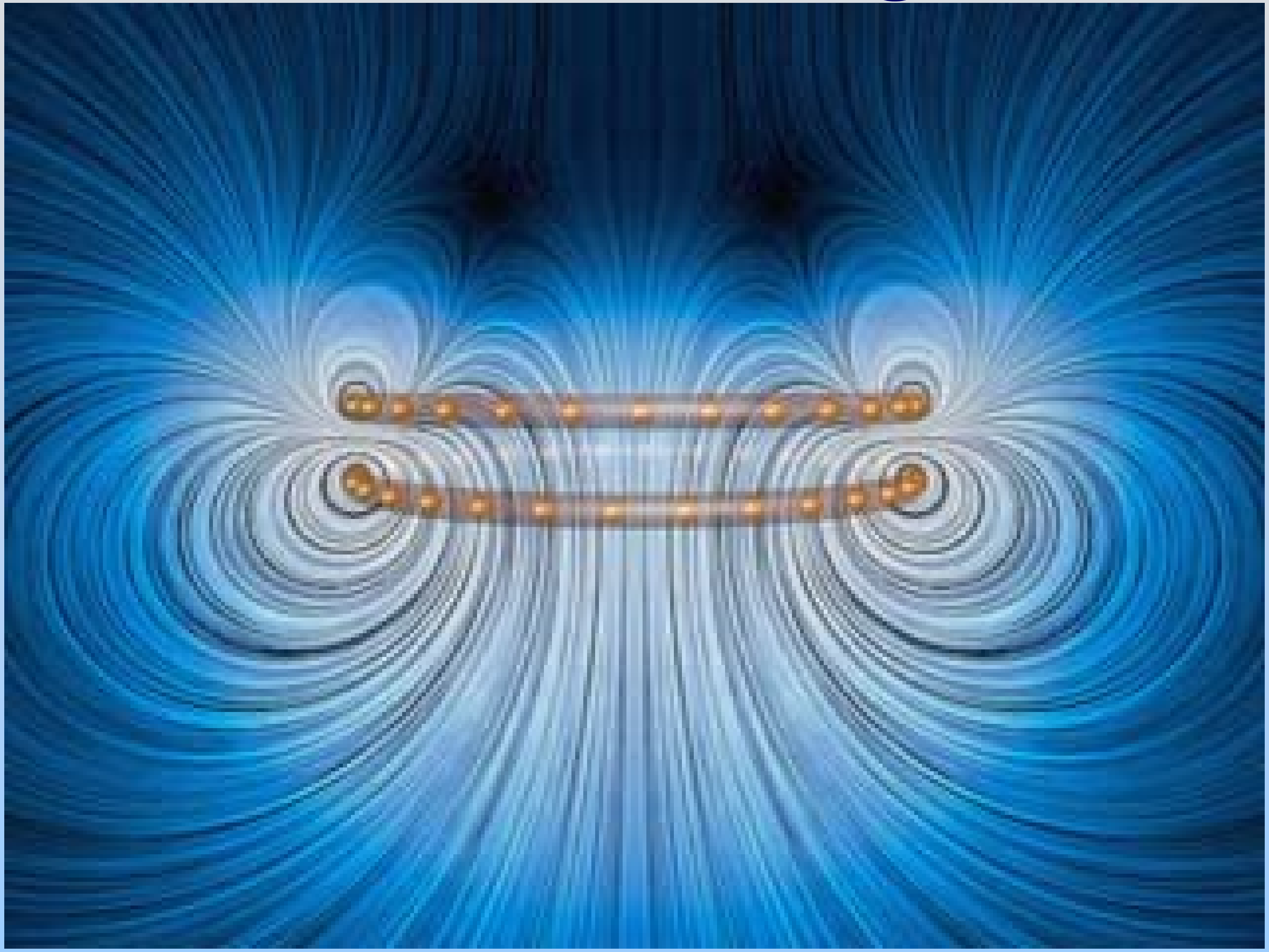




Bar magnet below dipole, with N pole on top
It is aligned with the dipole pictured, they attract!

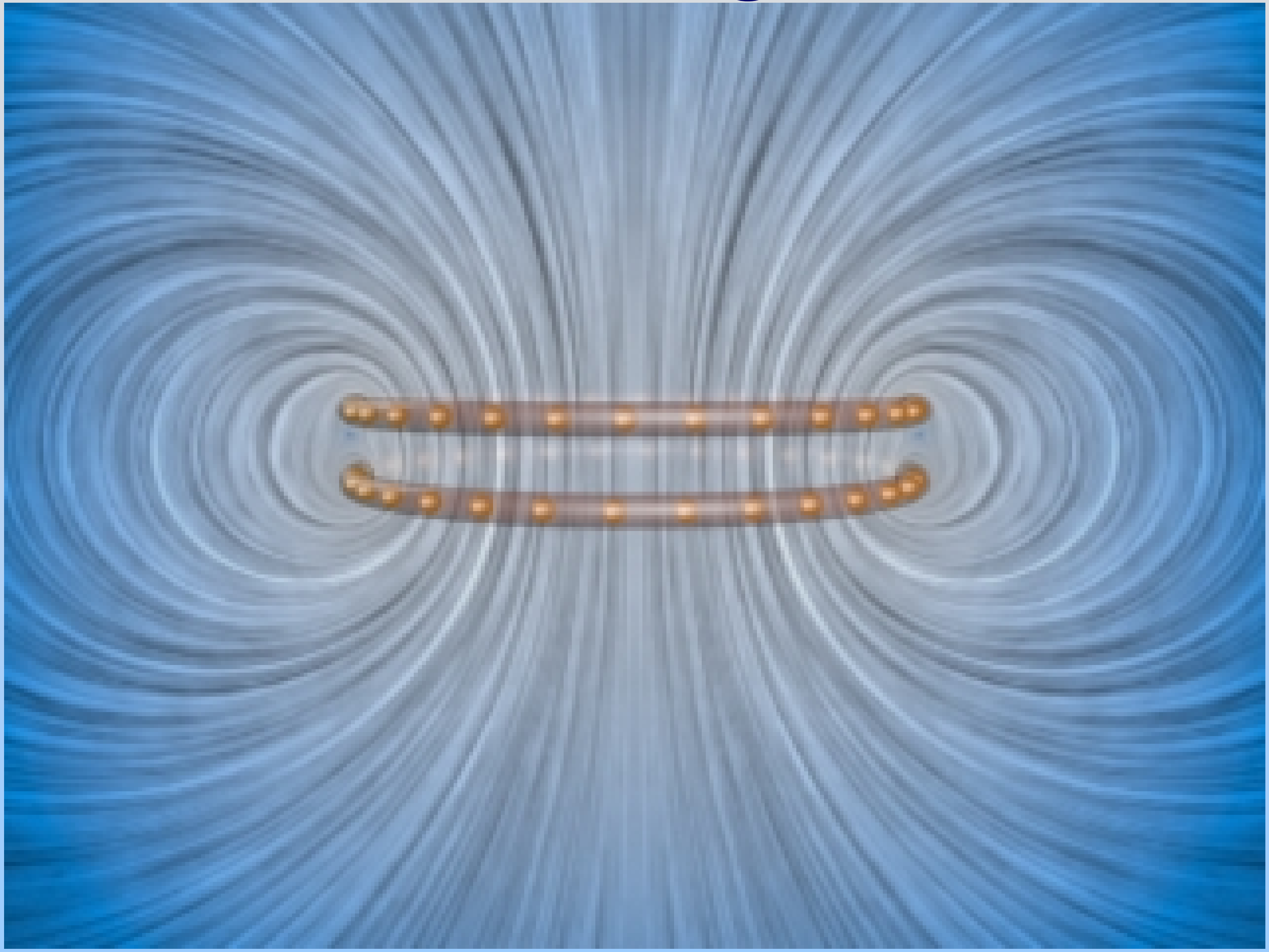
Experiment 7:
Magnetic Forces on Dipoles

Force on Dipole from Dipole: Anti-Parallel Alignment



http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/magnetostatics/16-MagneticForceRepel/16-MagForceRepel_f65_320.html

Force on Dipole from Dipole: Parallel Alignment



http://ocw.mit.edu/ans7870/8/8.02T/f04/visualizations/magnetostatics/15-MagneticForceAttract/15-MagForceAtt_f65_320.html

PRS Questions:
Force on Magnetic Dipole