Class 04: Outline

Hour 1:

Working In Groups

Expt. 1: Visualizations

Hour 2:

Electric Potential

Pick up Group Assignment at Back of Room

Groups

Advantages of Groups

- Three heads are better than one
- Don’t know? Ask your teammates
- Do know? Teaching reinforces knowledge

Leave no teammate behind!

- Practice for real life – science and engineering require teamwork; learn to work with others

What Groups Aren't

- A Free Ride

We do much group based work (labs & Friday problem solving). Each individual must contribute and sign name to work

If you don't contribute (e.g. aren't in class) you don't get credit

Group Isn't Working Well?

1. Diagnose problem and solve it yourself
   -- Most prevalent MIT problem: free rider.
2. Talk to Grad TA
3. Talk to the teamwork consultant

Don't wait: *Like most problems, teamwork problems get worse the longer you ignore them*

Introduce Yourselves

Please discuss:

- What is your experience in E&M?
- How do you see group working?
- What do you expect/want from class?
- What if someone doesn’t participate?
- What if someone doesn’t come to class?

Try to articulate solutions to foreseeable problems now (write them down)

Experiment 1: Visualizations

Need experiment write-up from course packet.

Turn in tear sheet at end of class

Each GROUP hands in ONE tear sheet signed by each member of group

Last Time:
Gravitational & Electric Fields

Gravity - Electricity

	Mass M	Charge q (±)
CREATE:	$\vec{\mathbf{g}} = -G\dfrac{M}{r^2}\hat{\mathbf{r}}$	$\vec{\mathbf{E}} = k_e\dfrac{q}{r^2}\hat{\mathbf{r}}$
FEEL:	$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$	$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$

This is easiest way to picture field

Potential Energy and Potential

Start with Gravity

Gravity: Force and Work

Gravitational Force on m due to M:

$$\vec{\mathbf{F}}_g = -G\frac{Mm}{r^2}\hat{\mathbf{r}}$$

Work done by gravity moving m from A to B:

$$W_g = \int_A^B \vec{\mathbf{F}}_{\mathrm{g}} \cdot d\,\vec{\mathbf{s}}$$

PATH INTEGRAL

Work Done by Earth's Gravity

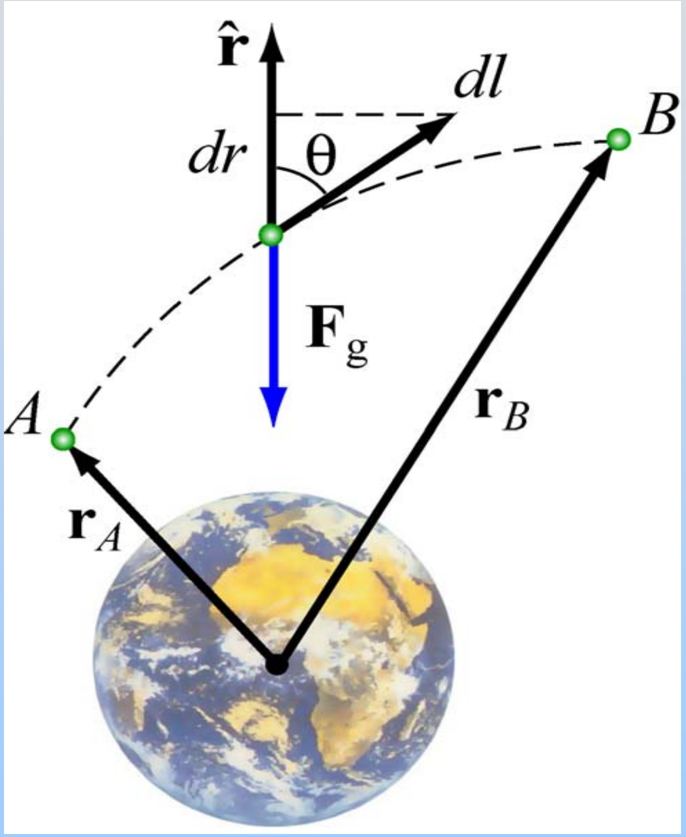
Work done by gravity moving m from *A* to *B:*

$$W_g = \int \vec{\mathbf{F}}_{\mathrm{g}} \cdot d\,\vec{\mathbf{s}}$$

$$= \int_A^B \left(-\frac{GMm}{r^2}\hat{\mathbf{r}} \right) \cdot \left(dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} \right)$$

$$= \int_{r_A}^{r_B} -\frac{GMm}{r^2} dr \quad = \left[\frac{GMm}{r} \right]_{r_A}^{r_B}$$

$$= GMm\left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

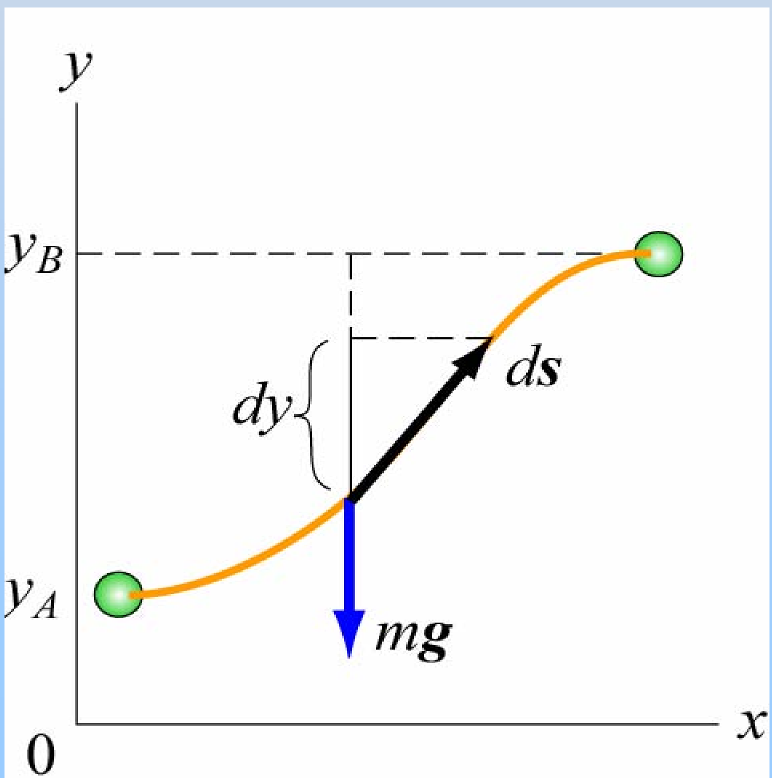


What is the sign moving from r_A to r_B?

Work Near Earth's Surface

G roughly constant: $\vec{\mathbf{g}} \approx -\frac{GM}{r_E^{\ 2}}\hat{\mathbf{y}} = -g\hat{\mathbf{y}}$

Work done by gravity moving m from *A* to *B*:



$$W_g = \int \vec{\mathbf{F}}_g \cdot d\,\vec{\mathbf{s}} = \int_A^B \left(-mg\hat{\mathbf{y}}\right) \cdot d\vec{\mathbf{s}}$$

$$= -\int_{yA}^{y_B} mg\,dy = -mg(y_B - y_A)$$

W_g depends ***only*** on endpoints

– ***not*** on path taken –

Conservative Force

Potential Energy (Joules)

$$\Delta U_g = U_B - U_A = -\int_A^B \vec{\mathbf{F}}_g \cdot d\,\vec{\mathbf{s}} = -W_g = +W_{ext}$$

$$(1) \quad \vec{\mathbf{F}}_g = -\frac{GMm}{r^2}\hat{\mathbf{r}} \quad \rightarrow \quad U_g = -\frac{GMm}{r} + U_0$$

$$(2) \quad \vec{\mathbf{F}}_g = -mg\hat{\mathbf{y}} \quad \rightarrow \quad U_g = mgy + U_0$$

- U_0: constant depending on reference point
- Only potential difference ΔU has physical significance

Gravitational Potential (Joules/kilogram)

Define gravitational potential difference:

$$\Delta V_g = \frac{\Delta U_g}{m} = -\int_A^B (\vec{\mathbf{F}}_{\mathrm{g}} / m) \cdot d\,\vec{\mathbf{s}} = -\int_A^B \vec{\mathbf{g}} \cdot d\,\vec{\mathbf{s}}$$

$$\text{Just as } \underbrace{\vec{\mathbf{F}}_{\mathrm{g}}}_{\text{Force}} \rightarrow \underbrace{\vec{\mathbf{g}}}_{\text{Field}},\ \underbrace{\Delta U_g}_{\text{Energy}} \rightarrow \underbrace{\Delta V_g}_{\text{Potential}}$$

That is, two particle interaction → single particle effect

PRS Question: Masses in Potentials

Move to Electrostatics

Gravity - Electrostatics

Mass M

$$\vec{\mathbf{g}} = -G\frac{M}{r^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$$

Charge q (±)

$$\vec{\mathbf{E}} = k_e\frac{q}{r^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$$

Both forces are conservative, so…

$$\Delta V_g = -\int_A^B \vec{\mathbf{g}} \cdot d\,\vec{\mathbf{s}}$$

$$\Delta U_g = -\int_A^B \vec{\mathbf{F}}_g \cdot d\,\vec{\mathbf{s}}$$

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$$

$$\Delta U = -\int_A^B \vec{\mathbf{F}}_E \cdot d\,\vec{\mathbf{s}}$$

Potential & Energy

$$\Delta V \equiv -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Units:
Joules/Coulomb
= Volts

Work done to move *q* from A to B:

$$W_{ext} = \Delta U = U_B - U_A$$

$$= q\Delta V$$ Joules

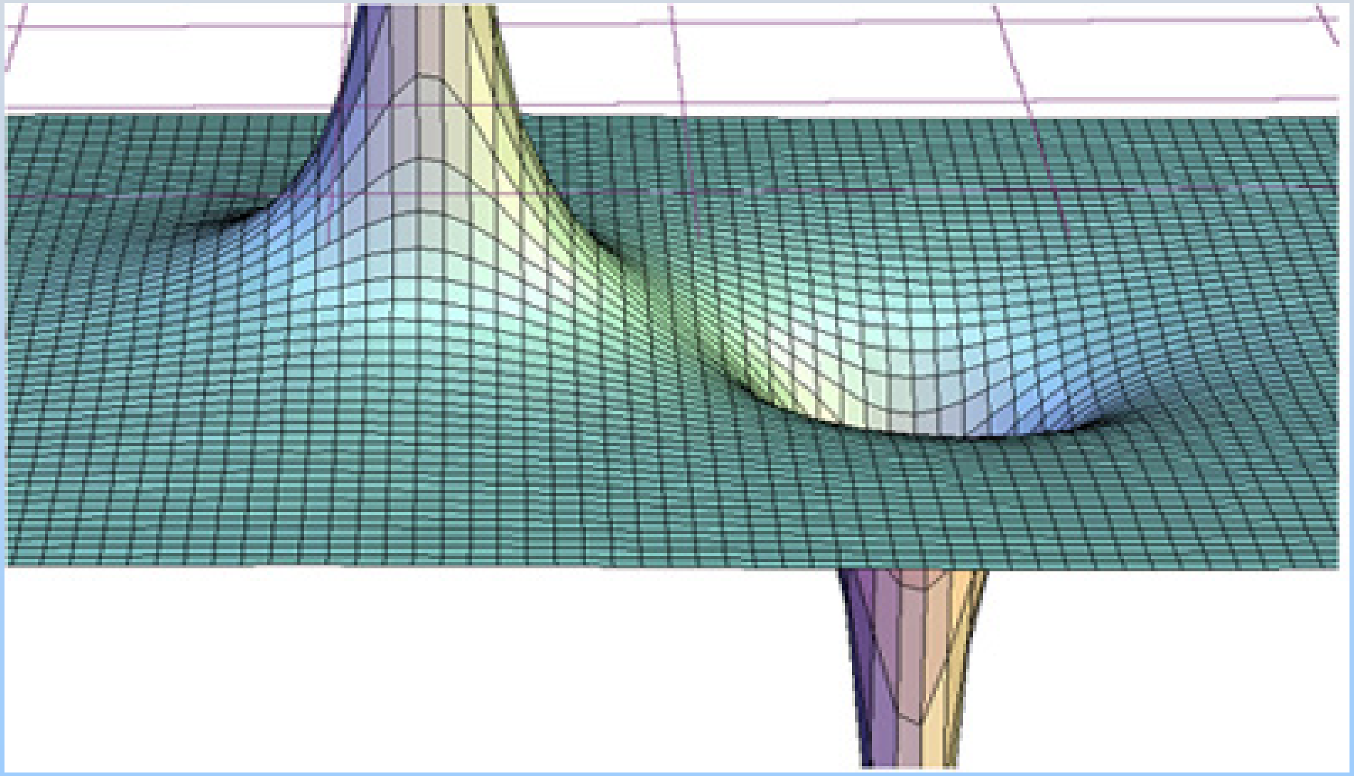
Potential: Summary Thus Far

Charges *CREATE* Potential Landscapes

$$V(\vec{\mathbf{r}}) = V_0 + \Delta V \equiv V_0 - \int_{"0"}^{\vec{\mathbf{r}}} \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$$

Potential Landscape

Positive Charge



Negative Charge

Potential: Summary Thus Far

Charges *CREATE* Potential Landscapes

$$V(\vec{\mathbf{r}}) = V_0 + \Delta V \equiv V_0 - \int_{"0"}^{\vec{\mathbf{r}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

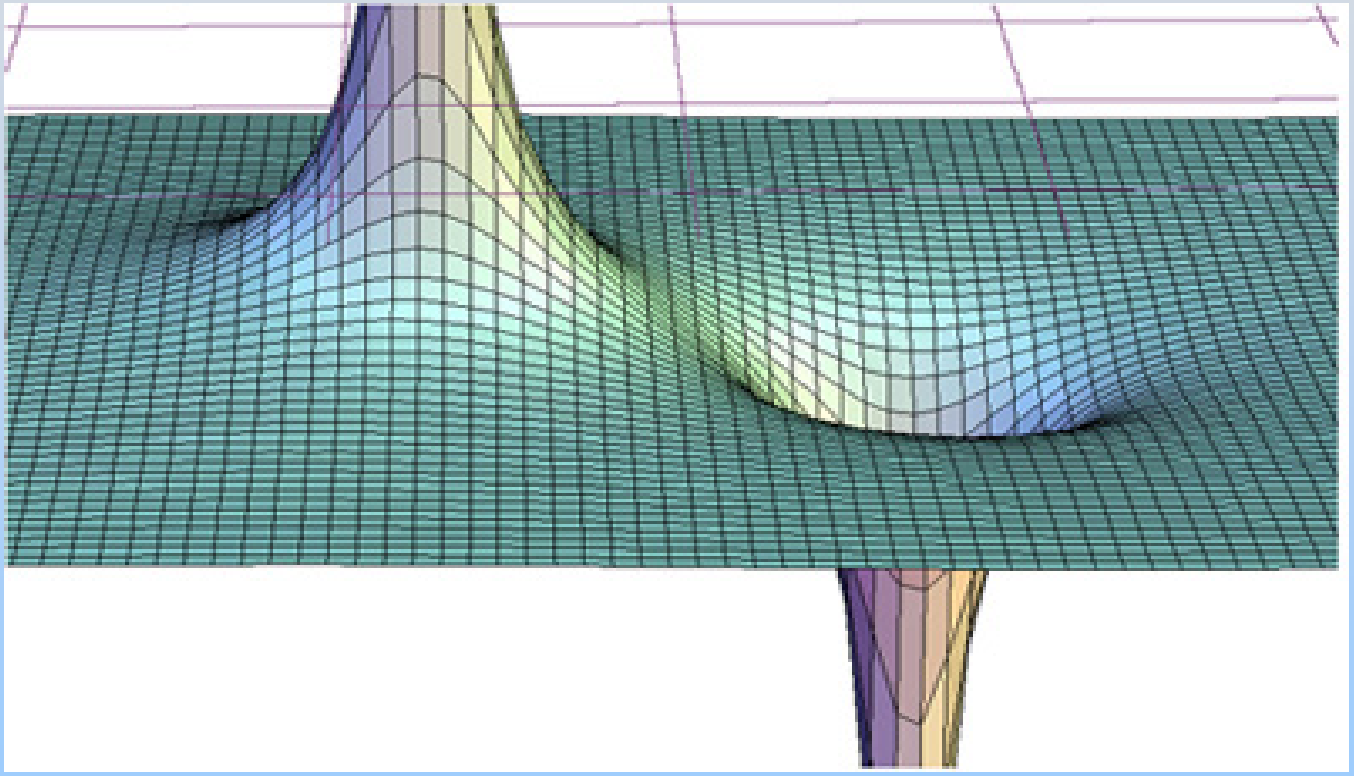
Charges *FEEL* Potential Landscapes

$$U(\vec{\mathbf{r}}) = qV(\vec{\mathbf{r}})$$

We work with ΔU (ΔV) because
only changes matter

Potential Landscape

Positive Charge



Negative Charge

3 PRS Questions:
Potential & Potential Energy

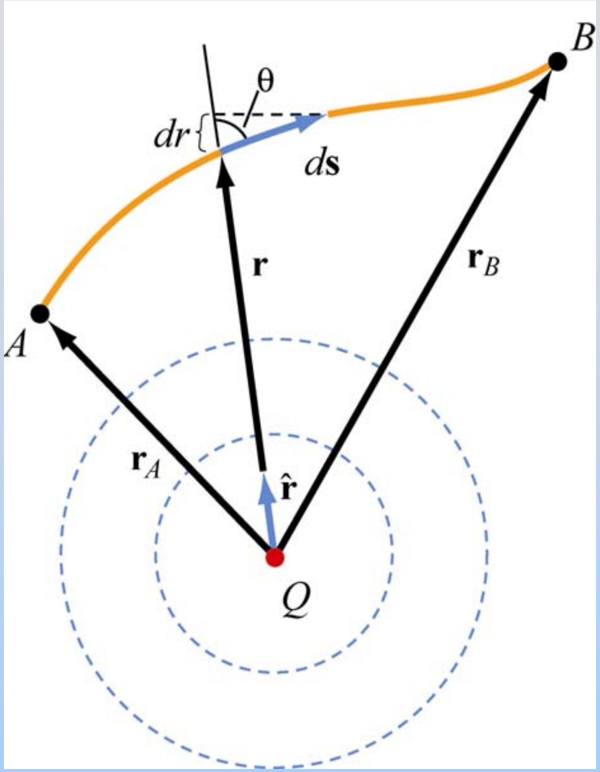
Creating Potentials: Two Examples

Potential Created by Pt Charge

$$\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$= -\int_A^B kQ \frac{\hat{\mathbf{r}}}{r^2} \cdot d\vec{\mathbf{s}} = -kQ \int_A^B \frac{dr}{r^2}$$

$$= kQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$



Take $V = 0$ at $r = \infty$:

$$V_{\text{Point Charge}}(r) = \frac{kQ}{r}$$

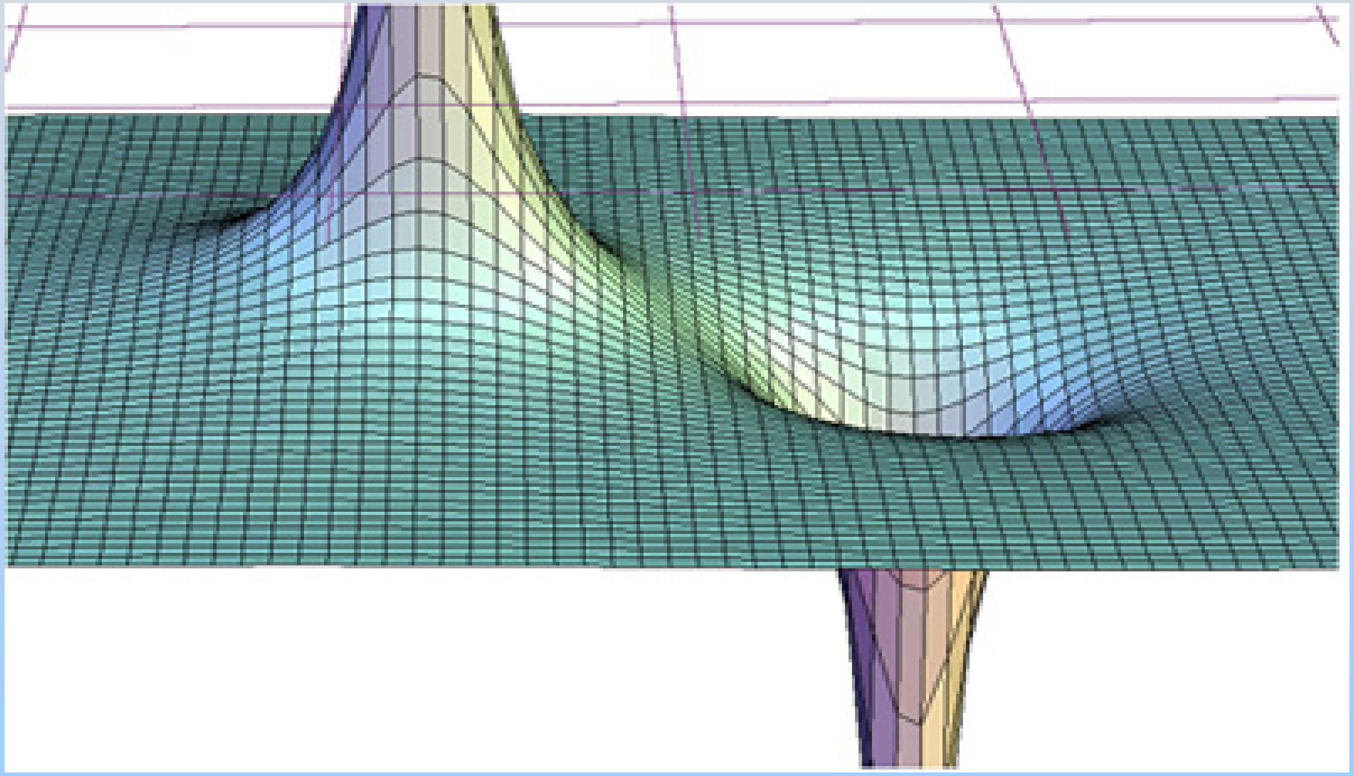
$$\vec{\mathbf{E}} = kQ \frac{\hat{\mathbf{r}}}{r^2}$$

$$\mathbf{d}\vec{\mathbf{s}} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}}$$

2 PRS Questions:
Point Charge Potential

Potential Landscape

Positive Charge



Negative Charge

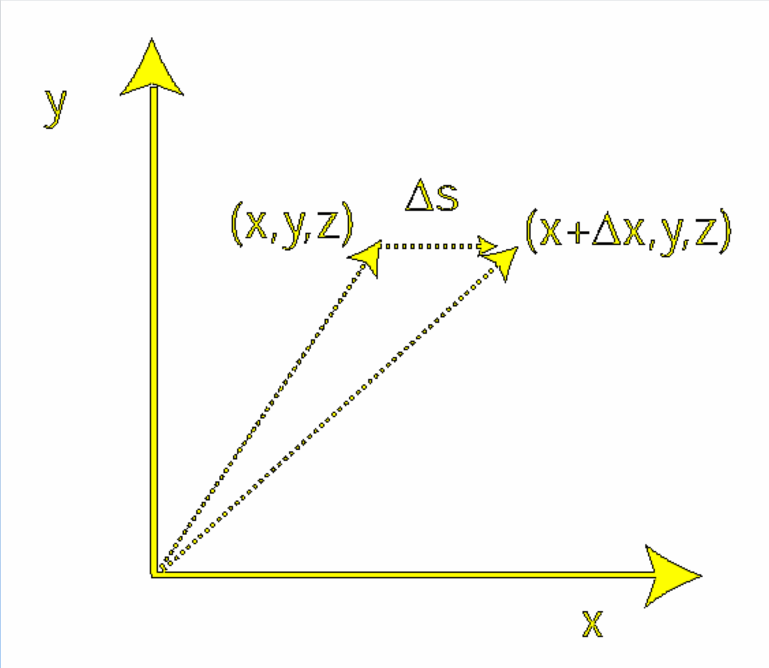
Deriving E from V

Deriving E from V

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

A = (x,y,z), B=(x+Δx,y,z)

$$\Delta \vec{\mathbf{s}} = \Delta x \,\hat{\mathbf{i}}$$



$$\Delta V = -\int_{(x,y,z)}^{(x+\Delta x,y,z)} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} \;\cong -\vec{\mathbf{E}} \cdot \Delta \vec{\mathbf{s}} = -\vec{\mathbf{E}} \cdot (\Delta x \hat{\mathbf{i}}) = -E_x \Delta x$$

$$\boxed{E_x \cong -\frac{\Delta V}{\Delta x} \to -\frac{\partial V}{\partial x}}$$

E_x = Rate of change in V with y and z held constant

Deriving E from V

If we do all coordinates:

$$\vec{\mathbf{E}} = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}} \right)$$

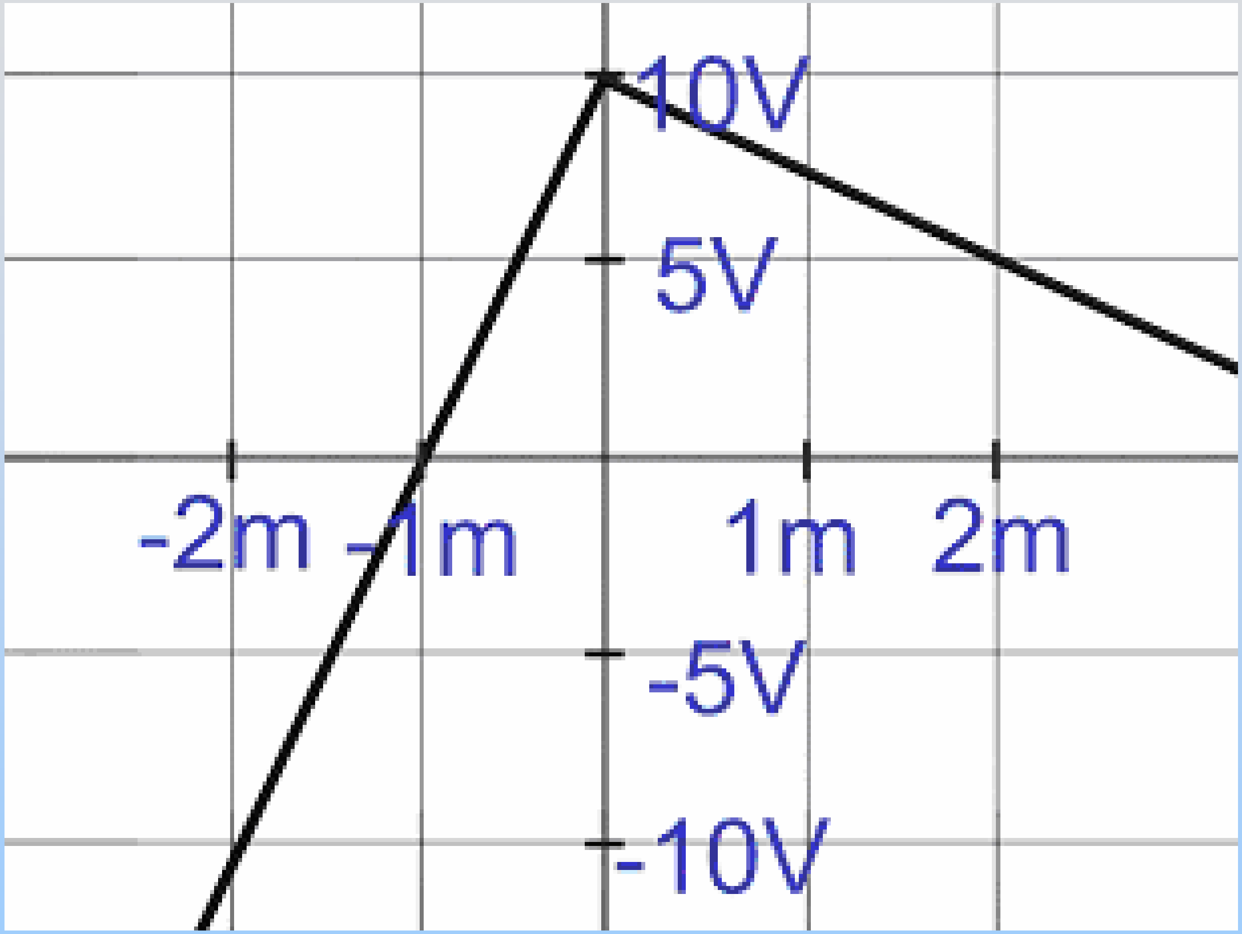
$$= -\underbrace{\left(\frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}} \right)} V$$

Gradient (del) operator:

$$\nabla \equiv \frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}$$

$$\boxed{\vec{\mathbf{E}} = -\nabla V}$$

In Class Problem



From this plot of potential vs. position, create a plot of electric field vs. position

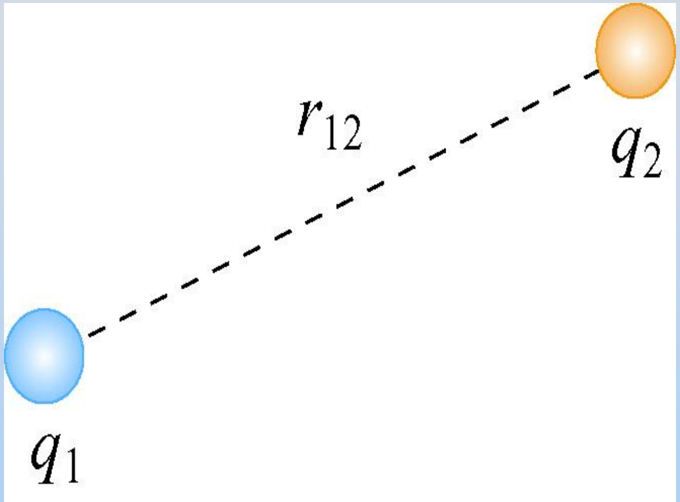
Bonus: Is there charge somewhere? Where?

Configuration Energy

Configuration Energy

How much energy to put two charges as pictured?

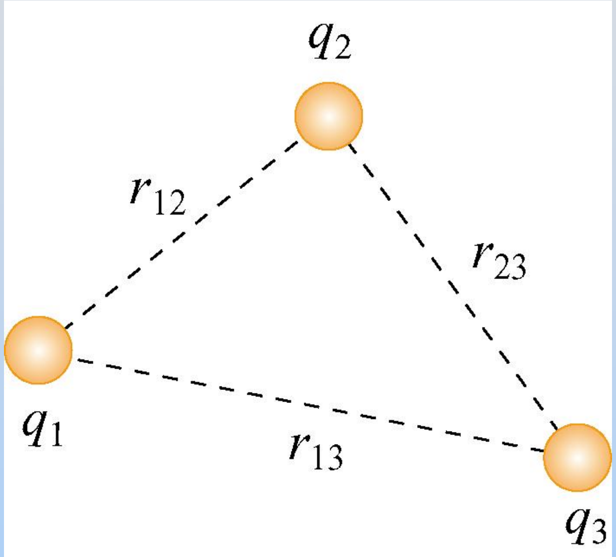
1) First charge is free
2) Second charge sees first:



$$U_{12} = W_2 = q_2 V_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}}$$

Configuration Energy

How much energy to put three charges as pictured?



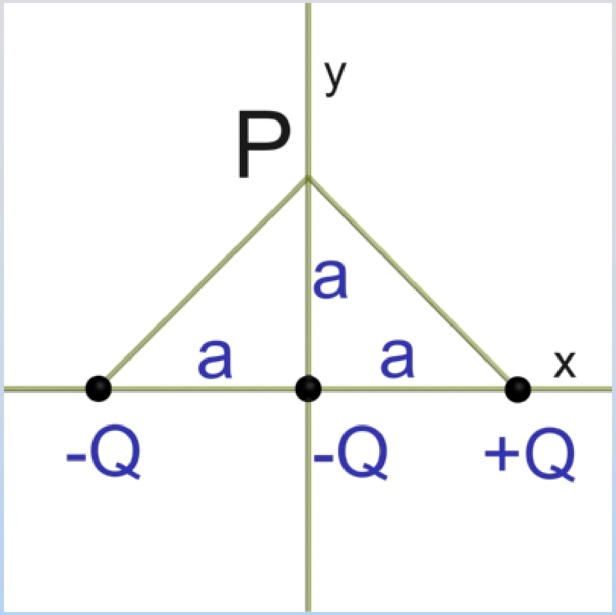
1) Know how to do first two
2) Bring in third:

$$W_3 = q_3 \left(V_1 + V_2 \right) = \frac{q_3}{4\pi\varepsilon_0} \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}} \right)$$

Total configuration energy:

$$U = W_2 + W_3 = \frac{1}{4\pi\varepsilon_0} \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) = U_{12} + U_{13} + U_{23}$$

In Class Problem



What is the electric potential in volts at point P?

How much energy in joules is required to put the three charges in the configuration pictured if they start out at infinity?

Suppose you move a fourth change +3Q from infinity in to point P. How much energy does that require (joules)?