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8.02 Electricity and Magnetism, Spring 2002
Transcript – Lecture 34

So last time we discussed the interference patterns due to two coherent light sources.

Today I will expand on this by exploring many, many light sources.

Suppose instead of having two slits through which I allow the light to go I have many.

I have N , capital N .

And let the separation between two adjacent ones be D , and so plane parallel waves come in and each one of these light sources is going to be a Huygens source, is going to produce spherical waves.

And so now we can ask ourselves the same question that we did before, and that is look at a long distance far away at certain angle θ .

Where will we see maxima and where will we see minima?

And then we can put up here a screen at a distance L and we will call this X equals 0, and then we can even ask the question where exactly on that screen will we see these maxima?

You will have constructive interference, exactly the same situation that we had with the double-slit interference pattern, when the sine of θ of N equals N lambda divided by D .

And if you're dealing with very small angle θ , you should all remember that the sine of an angle is the same as the angle itself, provided that you work in radians.

So for small angles, you can always use this approximation, if you remember that it is in radians.

And that's only in the small angle approximation.

And so the conclusion then is if we work in radians for now that theta of N for the maxima is then at $N\lambda$ divided by D, N being 0 right here, N being 1 right here, N being 2 right there.

And if you want to express that in terms of a linear displacement from 0, then X of N again for small angles is L times that number.

And so now you get displacement here in terms of centimeters or in terms of millimeters.

So you will say well big deal, it's the same result that we had for the double-slit interferometer.

We had exactly the same equation.

There was no difference.

And D now is the separation between two sources here.

It is obvious that it is the same because if these two are constructively interfering then these two will too and these two will too and these two will too so all of them will, so it's not too surprising that you get exactly the same result.

But now comes the big surprise.

We haven't discussed yet the issue where the locations are where light plus light gives darkness.

We haven't discussed the destructive interference.

And to derive that properly is very tricky.

In fact if you take 8.03 you will see a perfect derivation.

But I will give you the results.

What is not so intuitive, that if you have N sources, that between two major maxima, that means between this maximum at N equals 0 and a maximum at N equals 1, there are now N, capital N, minus 1 minima.

And minima means complete destructive interference.

So if capital N is 2, which we did last time, $2-1=1$, exactly, that was correct.

We had only one zero in between the two maxima.

But that's not the case anymore when capital N is much larger than 2.

And so let me now make you a -- a sketch whereby I plot the intensity of the light as a function of angle theta and this is the intensity, so that's in watts per square meter, remember that's the Poynting vector, and let this be 0, and let the angle theta 1 be here, and for small angles then that's lambda divided by D, and here you have theta 2, which is 2 lambda divided by D, and so on.

I take the small angle approximation.

So this angle is now in radians.

What you're going to see now is the following intensity, as a function of theta.

You see here a peak, and you're going to see here a peak, and you're going to see here one, and so on, and the same of course is true on the other side.

And here in between you're going to see now N-1 locations whereby you have total destructive interference.

And the same is the case here.

And this can be huge.

N can be a few hundred.

So we have many many locations where you have 100% destructive interference.

Now this point, this first location, where we hit the zero, that now is at the position lambda divided by D divided by capital N.

And I will call that angle from the maximum to that zero, from this maximum to this zero, I will call that angle for now delta theta.

Because that $\Delta\theta$ is a measure for the width of the line, here it is at maximum, here it is zero, and so that angle $\Delta\theta$ in terms of radians is λ divided by D times N , which then is approximately θ_1 divided by N , because θ_1 itself is λ divided by D .

And so you see that it is N times smaller than this distance.

And so if N is large, these lines become extremely narrow, and that's the big difference between two-slit interference and multiple-slit interference.

And the larger N is, the higher these peaks will be.

The height of these peaks, the intensity here, is proportional to N squared.

And you may say, "gee, why -- why not -- is why is it not linearly proportional to N ?" Well that's easy to see.

Suppose I increase capital N , the number of sources, by a factor of three.

Then the electric field vector where there are maxima is three times larger.

But if the electric field vector is three times larger the Poynting vector is nine times larger.

So you get nine times more light.

Now you may say, "gee, that's a violation of the conservation of energy."

Three times more sources, nine times more light, how can that be?" Well, you overlook then that if you make N go up by a factor of three that the lines get narrower by a factor of three, because of this N here, and so they get higher by a factor of nine, and they get narrower by a factor of three, and so you gain a factor of three in light.

Of course you gain a factor of three.

You have three times more sources.

You get three times more light.

So you see there's no violation of the conservation of energy here.

And I want to demonstrate this to you using a -- a red laser which we have used before.

And I will use what we call a grating, a grating is a plate which is specially prepared, a transparent plate, which has grooves in it, and the one that I will use has tw- 2500 grooves, we call them lines, per inch.

That means the separation D between two adjacent grooves in my case is about 2.16 microns.

A micron is 10 to the -6 meters.

And the wavelength that I'm going to use is our red laser, which is about 6.3 times 10 to the -7 meters.

And I'm going to put the whole thing there.

I'm going to make you see it there at a distance L .

Which is about 10 meters.

And so this allows me now to calculate where the zero order will fall, where the first order and where the second order will fall.

We call when N is 0, we call that zero order, so this is zero order, when N is 1 we call that first order, and when N is 2 we call that second order.

And you have of course the first order also on this side and the second order also on this side.

Everything that you have here you have to also think of it as being on the other side.

So I can predict now where the zero order will be when N is 0.

That is 0 degrees.

That's immediately obvious.

I use that equation.

If N is 0 the zero order is always right at the center, provided that all these sources are in phase.

And they will be in phase because I use plane waves.

So Huygens will tell you that they're going to oscillate exactly at the same time, they produce the same frequency, they produce the same wavelength, and they're all in phase with each other.

So there will be a maximum at $\theta_1 = 0$.

And then there will be a maximum which I calculated to be at 3.55 degrees.

I calculated that from this equation and then θ_2 will be at roughly 7.1 degrees.

If you want to know how wide the width of this peak is going to be, then you have to know how many lines of my grating I will be using.

Well, my grating is like so.

Here I have these lines not unlike the grating that you have in your optics kit.

There are 2500 of those lines per inch.

And my laser beam is roughly 2 millimeters in size.

So this is about 2 millimeters.

And that tells me then that I cover about 200 lines.

And if I have 200 lines I can now calculate how wide that line is going to be.

Because this factor of N enters into it here.

And if I express that in terms of that angle $\Delta\theta$, then the angle $\Delta\theta$, going back here, so $\Delta\theta$ is then the 3.55 degrees divided by 200, and that's an extremely small angle, that angle is

approximately one arc minute, which is 60 times smaller than one degree.

And if you want to translate that in terms of how wide that spot will be, if I see it on the screen 10 meters away from me, and if you want to call that ΔX , then you would naively predict that ΔX is something like 3 millimeters, and the reason why I say naively because you will not see that it is 3 millimeters, it will be extremely narrow, but it will be more than 3 millimeters, because the limiting factor is always the divergence of my laser beam.

And so the divergence of my laser beam is more than one arc minute, and so I don't get down to the one arc minute narrow beam.

I'm not too far away from it, though.

So this is what I want to show you first.

I will turn on the laser first, and then make it very dark because we do need darkness -- or this has to come off because that would obviously -- oh, I turned off the wrong laser, but that -- I turned on the wrong laser, but that's OK.

That's a second demonstration which I do with uh with the green laser, this is the one that I need, this is the red laser, will come on very quickly.

There it is.

Tom, if you can turn that off, maybe that will help, although everyone can see it but it would help.

So you see here very clearly the, the zero order is at the m- at the -- right in the middle.

This one, so θ is 0, and this θ_1 is my 3.5 degrees, this is also 3.5 degrees.

This is the 7.1 degrees and so on.

And so you see this whole pattern of -- of interference as a result of multiple slits.

And so this is a -- these are grooves in a piece of plastics.

And notice how small they are, how narrow they are compared with the double-slit interference.

So they don't have that theoretical minimum with this $1/N$, but they approach that, and the reason why they are not that narrow is because the divergence of the laser beam itself is larger than that one arc minute that I calculated.

And so you can never beat that of course.

We did this experiment with white, with red light, but keep in mind that if I take red light, here is a maximum, at zero order, here is a maximum, provided that this λ is λ for red, here is a maximum, provided that this is the λ for red.

But if I have white light, then of course I deal with other colors.

And if I have blue light in my white light, it will also have a maximum here, that's nonnegotiable, but it has its first order maximum here, because the wavelength is shorter.

And it will have its second order maximum here.

This will be the same distance.

And so when you do this with white light you're going to see always at zero order white light, because all the colors have their maximum at 0 order, but at first and second and higher orders, the colors -- the -- the colors uh walk at their own pace so to speak.

And then the smaller the wavelength is the closer it will be to the zero order and then the spacings between first and second will also be smaller than in the case of the long wavelength, in this case red.

And this is something that I also want to demonstrate to you.

It is not so easy to get a very strong powerful source of white light.

I'm using for this a -- a reflection grating.

You can also use gratings in reflection.

You take metal and then the -- the grooves are made on the metal.

And you get a reflection, which we will have there on the wall.

This reflection grating has a spacing which is four times smaller than the one we have here.

It's only 2.5 microns and so the angle of θ_1 will not be 3.5 degrees but it will be four times larger.

The main purpose wh- why I want to show you this is I use white light, that the zero order is white.

And then we will see also of course the first and the second order if we have good eyes because the f- the -- the whole thing is not so -- so very bright.

Make sure that I have the -- my light, flashlight, so your eyes may have to adjust a little bit to the darkness.

The effect is not overpowering because our light source is not very bright.

But you see the where is my laser pointer, this is the zero order maximum, and the zero order maximum is very wide, the reason is the divergence of the white light beam, it's not this factor of N that I gain, I gain much less, uh, I can turn on also a laser which I use at the same time, and you will see that the red laser will give its own, th- there they come, so the zero order of the red laser is of course also here, all colors are here, and then you see here first order of the red, that's a large angle, but the D is very small, you see here the first order of the red, second order of the red, you see here the first order of the blue, first order of the blue, you see a big difference, between the separation of the red and the separation from the zero order and the blue, it's a big difference.

There is actually a much better way that I can make you see all this and that is if I ask you which I think I'm going to do now to use your grating, but hold -- hold it for a second.

Before you get your -- your own gratings out.

Our equations that we have derived so far only hold if we look very far away.

These angles of theta are only true if you go very far away, for reasons that we discussed last time, because these surfaces of maxima are hyperbolas.

And it's only -- the angle theta is only an approximation if you're very far away.

We can however do something very clever.

We can use a lens, and if we have a lens we can bring the image very close without disturbing the angles.

If this is a grating, this is the number of sources that I have, and so the light comes in in this direction and if I put here a lens and this is the screen, the focal point of the lens, then if the angle theta -- if the angle theta for which I would have expected in this direction my first order maximum, and here of course my zero order maximum, then the lens will not change that angle.

We never discussed lenses so it may not be so obvious to you.

But the lens will always maintain the integrity of angles.

So the angle theta that we derived here is the correct angle, but of course in terms of X that is enormously reduced if this distance is very small.

So then we have the option that we don't have to allow for very large distances like now, 10 meters.

And so your eye is a perfect tool to use for that because you have a lens in your -- in your eye, that's the whole idea.

And so now I want you to get your gratings out.

And I want you to hold the gratings in front of your eyes and manipulate the gratings a little bit so that you get the lines vertical.

And you will easily be able to do that, this is your light source.

Your lines, your gratings, have 1000 lines per millimeter.

That means the spacing of your grating is one micron.

One micron is 10 times smaller than this number.

So the angles are huge in your case.

They're way larger than what we have there.

I will make it completely dark and then I want you to rotate your gratings such that you get the spectra on either side, on the left and on the right.

That means your grooves are then in vertical direction.

And what you see now, way better than what I could show you in my previous demonstration, you see the zero order is the lamp itself.

All the colors are right at the center.

That's the lamp.

That's your zero order maximum.

And then you see if you go to the right you see the blue coming in beautifully first because that's the smallest wavelength, you're going further to the right, you see the first order red, you're going further to the right, you see the second order blue, you go further to the right, and you see second order red, but since D is so amazingly small in your case, there may not be too many maxima in the red.

In fact I will ask you on one of the homework assignments, which is the optional one, how many maxima in the red you will see.

So essential here is that you see that the f - zero order maximum at the center is white light.

And so it's not until you get to first and second order that you begin to see the colors separate.

Now these gratings can be used to do atomic physics.

There are atoms and molecules which emit very discrete frequencies, very discrete wavelength.

And when you look at them with a grating you can see very distinctly where these lines fall.

Where these wavelengths fall.

And that's the next thing that I want to do.

I will show you now, I will turn off the white light source and I will turn on for you neon.

And so I want you to look now at the neon and if you give yourself some time you will see that the neon is not a continuous spectrum like you saw with the white light bulb but you see very distinct locations where you see maxima.

You see many in the red, and I think you see several in the orange, and I noticed this morning that I see two lines in the green.

You have to look very carefully because these lines in the green are very faint.

And so the whole purpose then is that with these gratings you can not only find out which wavelengths are emitted by these atoms and these molecules but you can also find the relative strength, and that is of course entering the domain of atomic physics.

And these gratings are extremely powerful to do that.

And I would advise you to carry these gratings with you at least for the next few weeks and when you're outside and you get a chance to see some bright lights out on highways or on the street, to look through the grating and see whether you can see these emission lines, uh mercury, if you get mercury lamps, they are very beautiful.

You see many, many different colors, very discrete frequencies, very discrete wavelengths, are emitted by mercury in the same way, the same kind of physics that you see that here with, with neon.

So now comes something that may come as a surprise to you.

Because now I would like to discuss with you, um, the interference pattern if we have only one slit.

We discussed 2, we discussed capital N is a large number, but what now if we have only one slit?

Even if you have only one slit there will be directions in space whereby light plus light gives darkness, and there will be directions whereby the light constructively interferes with each other.

And strangely enough this is given a different name.

We call this diffraction.

It's exactly the same physics.

There is no difference.

It should have been called interference, but it's-- in the literature you will see it under the name diffraction.

It all comes down again to Huygens' principle.

So let me here now have a single opening and this opening is a slit perpendicular to the blackboard and the opening is A in size.

A single one.

And the plane waves come in and Mr. Huygens says that all these sources here are all going to be emitting the light at the same frequency, the same wavelength, and they will all be in phase with each other.

Because these plane waves arrive here all at the same time.

And so I could now ask myself the question at what angle of θ will I see maximum, what angle of θ will I see minimum, and you can also put a screen then at large distance L , you can call X equals 0 here and you can ask yourself where will I see these maxima and where will I see these minima?

To derive this in this case is not easy.

Again I refer to 8.03.

It's as difficult as deriving in this case this whole structure in between the maxima.

One thing is obvious and that is that you got to get a maximum that is nonnegotiable when θ equals 0.

That's simply a matter of symmetry of the problem.

If all these sources are in phase, clearly you're going to get a maximum here.

No one will question that.

The minima is very tricky.

And the minima will fall at the following locations.

The sine of theta of N equals N times lambda divided by A.

And for small angle approximation, this is the same as theta N in radians.

And when you see that equation your first reaction should be that maybe I goofed by a factor of two.

Because your first reaction will be that's the same equation that we have there and then we have D there, and so how can we have minimum here where we have maxima there?

Well, the situation is different.

The best way that you can see that that equation is not wrong is perhaps the following.

Suppose you take the angle theta 1.

So that's for N equals 1.

Then the relation that you see there will tell you that this source here, this Huygens source, and this Huygens source have a difference in path length of lambda.

And so you will then say, "aha, that's constructive interference." That's true, but that means this Huygens source and this Huygens source will then have a difference in path of half lambda.

So they will kill each other.

And that means this Huygens source and this Huygens source will have a difference in path length of one-half lambda.

So they will also kill each other.

And so in this upper half there is always one Huygens source which will kill the one at the bottom.

And you can do a similar reasoning for the angle theta 2.

And so that is indeed the correct equation.

You will find complete minima when theta 1 in terms of radians is lambda divided by A and theta 2 is 2 lambda divided by A and so on.

That's where you find your minima.

And if you convert that into X, where they actually fall on a screen, well then X1 will be, for small angle approximations, L times lambda divided by A.

That's no different.

And so now what I owe you is a pattern.

What will the pattern look like?

When I look on the screen there, what will I really see?

Well, it's looking, it's going to look very different from what you may think.

I will plot it now in terms of X.

I could have plotted it in terms of theta.

But I decided to plot it in terms of X.

So here at X equals 0 there is unmistakable, unnegotiable, there is a maximum, that is -- that coincides with theta equals 0.

And then here at lambda divided by A, completely destructive interference, that's that angle theta 1 times L.

And then here $L\lambda$ divided by A , complete destructive interference and the same is true of course on the other side.

And what you're going to see now in terms of the intensity that I showed you there, watts per square meters, is a curve that looks like this.

You get an enormously broad maximum, absolutely 0 here, very small maximum, absolutely 0 there, very small maximum and 0 there, and this continues for a long time.

And this is very different from anything we've seen with multiple sources.

If the intensity here is I_0 , then the maximum here which I didn't even calculate where it is, it is somewhere in between these two minima, but I didn't calculate precisely where it is, that maximum is very low, it's only 4.5 percent, in strength, of I_0 .

And this is even lower.

This is only 1.6 percent of I_0 .

So when you look at a diffraction pattern, we call this a diffraction pattern like this, you will see a very broad center maximum and then you see these dark spots on either side, and you see light coming up again in between them, sort of submaxima.

And so the width of this center maximum, this width, which is really X_1 , that width is then $L\lambda$ divided by A , and if you want to be picky and you say well the center maximum is really twice that much, fine, be my guest, but this is clearly a measure for how wide that center spot will be when you see it on a screen.

And now there comes something that is completely not intuitive for you as well as for me.

And that is if you make A very small, that means you let the light go through an extremely narrow slit, then this -- what you will see on the wall is extremely wide.

The smaller A is the wider it will be.

It's exactly opposed to what you would predict.

You would think if you make the opening through which you put light very small, you would think that what you see on the screen is also very small.

It's exactly the opposite.

And that's is what I want to demonstrate to you.

I have here a demonstration with a variable slit.

I can vary A .

And we will use the brightest laser that we have, a beam of green laser light, about 5400 Angstroms.

And what I will do is I will make this opening narrower and narrower and narrower.

And as I make the opening I start very large.

I start with a large opening of maybe 5 millimeters.

At a large opening, A is so large that this is negligibly small, because A is very large, then this is not very large, this is very small.

But as I make A smaller and smaller and smaller and smaller there comes a time that the diffraction width is going to be dominating the whole scene and what you will see then on the screen there that the bright spot will get wider, wider, wider, wider.

And that for me is always so enormously fascinating because it goes so strongly against our intuition.

And so this is what I have next on the menu.

I have here this variable slit.

I must make sure that I have my flashlight and I'll turn on this laser that I earlier accidentally turned on.

I hope it's coming on.

Yes it is.

And so you see there the slit is now very wide.

And so the size of that slit that you see there -- the size of the -- of the bright spot is now entirely dictated by the divergence of my laser beam.

The diffraction width is negligibly small because A is very large.

But now watch.

Now I'm going to tighten, make A smaller.

And I'm doing that now.

I'm making it smaller and smaller.

The diffraction width is still negligible.

Making it smaller.

Ah.

I'm beginning to see the dark lines.

Ah.

The diffraction width is taking over.

Look at the center maximum.

Right there.

It's getting wider.

It's getting wider.

And I make the slit narrower.

It's getting wider.

It's getting wider.

Look how beautiful, you see these destructive interferences.

Where light plus light gives darkness.

Wider.

Wider.

Right now it is at least 10 times wider than it was before.

And now it is 20 times wider.

And it gets very faint.

Of course it gets very faint because if I make the slit very narrow not much light goes through, there's nothing I can do about it.

But notice how incredibly impressive this is.

How wide that center maximum becomes.

And that is very characteristic for diffraction.

The narrower the slit the wider the diffraction pattern at the center maximum.

Now I did this in monochromatic light, monochromatic light means that you have practically only one wavelength.

And there is a way that I can make you see this from your own seats.

And that is what we're going to do with the, um, with the cards that you have.

So if you can get the cards out now, we did this in one color, right, in green light, almost one color, almost monochromatic, so you see a beautiful pattern here, very well-defined dark lines.

You now can use this little slit that you have, put it in front of your eye, and I'm going to make you see this white light and you would see all the features that you're supposed to see but it's even more interesting in white light because with white light you have a little red, you have a little blue, you have a little yellow, and so these minima will fall at different locations of course.

And so you don't see it as beautiful as I showed you, as distinct as I showed you in green, but you see very distinctly the center maximum and you see the dark lines on either side.

But the main reason why I want you to see this is that if you manage to manipulate the size of the slit, the size of A , if you manage to manipulate that, when you make it narrower, notice that the diffraction pattern gets broader and not the other way around.

So first make sure that you get it, that you begin to see the dark areas, and then try to make it a little narrower and then you see that it opens up.

This is precisely what I did with the variable slit here.

So give it a d- b- bit of time.

What helps me that actually you don't have to pull it open but yet you can move one piece of the card behind the other card.

So your one thumb goes to the back and the other your thumb comes forward.

That works very well for me.

Who can see clearly the diffraction pattern?

Unmistakably.

Very good.

Take the card with you, impress your parents.

And look at home at very bright street lights and you will still see the same diffraction pattern.

Although not as ideal as you see it here because our source is a line source and that helps of course if your slit is vertical.

OK.

If I don't have a slit as an opening but if I have a circular opening then the pattern that you would expect is the same that you see here but

you have to rotate it about this line because you now have axial symmetry.

So you don't have a -- a long slit, but you have a circular opening.

And indeed that is approximately correct.

If you had a circular opening you would see a center maximum which would be very bright and then you would see rings around it of zeroes.

And so if I try to make you see it this would be the center maximum.

It would be a ring around it here.

Complete darkness and then again a little bit of light, not very much, because remember that this maximum is only 4.5 percent of that one.

And then you would see again a ring with complete darkness and so on.

So you have a little pinhole and this is the image that you would get on a screen.

From that pinhole.

And you would think now that this angle from here to here, this is $\theta = 0$ and we call that θ_1 .

This is the θ_1 where you see your first zero, you would think that that is λ divided by A if A is the diameter of your pinhole.

Well, it's almost that.

It is a little larger.

Because a pinhole, a circular geometry is different from a line.

And so take my word for it that it comes not at λ divided by A but it comes at roughly 1.2λ divided by A .

If you want to be picky it's really 1.22λ divided by A and this of course is in radians again.

I work now exclusively in terms of radians, small angle approximation.

And so this raises the issue of what we call in physics angular resolution.

Suppose I have a pinhole and I look at the images of two light sources.

One light comes in from there and the other comes in from there, could be the headlights of a car, could be two stars in the sky, well, each one of them will give a diffraction pattern, that's nonnegotiable, you can never bypass diffraction.

So one star will give a diffraction pattern here, or one headlight, and the other will give a diffraction pattern here.

You would have no problem to say oh, yeah, they're all, there are two light sources, there's one star and there's another star.

OK.

Now make the angle between the two sources smaller and smaller and smaller and smaller so that these two diffraction patterns come closer and closer and closer and closer.

How close can you now bring them so that we, you and I, will still say, yeah, there are still two sources?

We call that angular resolution.

And so how we define angular resolution is that both light sources have exactly the same strength and let's assume for now for simplicity that they're monochromatic, so that there's only one wavelength that they emit.

Then the criterion that is generally accepted so that we can still decide that there are two light sources, that this one, the center of this one, is no closer than the location where this one has complete darkness.

In other words, the spot of the second star should fall right where the other one has darkness.

If you bring them closer your brains will say no.

No, no, that's not two sources.

That's really only one source.

And we call this the Rayleigh criterion of resolution.

And that Rayleigh criterion of resolution therefore is that the separation between the two light beams, stars or the headlights from a car, the separation has to be larger than this angle.

And it is a function of A .

If A is larger, then that angle can be substantially smaller.

And this is what we call the diffraction limitation on angular resolution.

It doesn't matter whether you have a pinhole or whether you have a lens, a c- circular lens that we use with telescopes.

Or whether you have concave mirrors, which we use with telescopes.

In all cases are you always stuck with the idea at best, that's the best you can ever do, that is the angular separation that θ_1 -- I call it here the θ_1 , is that 1.2λ divided by A , and that is then in radians.

If you take a lens which has a diameter A of about 20 centimeters then that translates into a θ_1 of about half an arc second, for 5000 Angstroms.

So I take λ 5000 Angstroms, remember an Angstrom is 10^{-10} meters.

So θ_1 then becomes half an arc second.

0.5 arc seconds.

An arc minute is one-sixtieth of a d- arc degree, and an arc second is 60 times lower than that.

And so if A were 2.4 meters, telescope, 2.4 meter telescope, that's a real biggie, then θ_1 would be approximately one twenty-fifth of an arc second.

So the larger you make your telescope the better angular resolution you would have.

This angular resolution is 12 times better than this one.

Because A is 12 times larger.

So now you may think that if you take a 2.4 meter telescope on the ground and you look at stars, at two stars equally bright, you would be able to tell them apart if they are farther away from each other than one twenty-fifth of an arc second.

That is not true.

The contrary.

It is very bad.

You can't even tell them apart often when they are half a second apart.

And the reason for that is not that the diffraction limitation is going to kill you, but the reason is that the air is always turbulent.

And it is the turbulence on the air that makes the image, the diffraction-limited image, which itself is very small, move around on your photographic plates, it moves it around in a matter of ten seconds over an area which itself could be as large as one second.

Astronomers call that seeing.

And so when you look at your picture, the whole star is smeared out over an area which is in angular size one arc second or maybe half an arc second at best.

So you can never achieve this from the ground.

So all telescopes from the ground without exception can do at best half an arc second.

They cannot do much better because of the air turbulence.

And this is now the great thing about the Hubble space telescope.

Hubble is up there, or maybe down there, whichever it is, I don't know where it is, maybe Jeffrey knows where it is, but it is somewhere.

And Hubble has no air.

And so Hubble doesn't suffer of the air turbulence.

And so Hubble's mirrors are indeed diffraction-limited.

And Hubble has a mirror which is 2.4 meters diameter.

And indeed at 5000 Angstrom, I checked that yesterday with people at Hubble space telescope, indeed Hubble is diffraction-limited, and Hubble has an angular resolution at 5000 Angstroms, which is about one twenty-fifth of an arc second.

And at shorter wavelengths, it's even better, and at longer wavelengths, it is a little worse.

And so I would like to show you at least one picture of Hubble without going into the details of what you are seeing of the astro- of the astronomy.

And that's the one that is coming up.

It's a very famous picture that Hubble made several years ago.

It is a picture of a supernova explosion.

John, if we can have the slide, there it is.

You're looking here at an explosion, it's called Supernova 1987 A, which occurred in February of 1987.

This object is 150000 light-years away from us.

That means the explosion really took place -- took place a 150000 years ago.

But we saw it for the first time in February '87.

And without going into the details of what you're looking at, which is of course very fascinating, but that's not the objective today, I want you to appreciate that this distance here is one arc second.

And look at the incredible detail that Hubble can show you.

If you took a picture like this with a ground-based observatory, this whole part would just be one smudge.

You would not be able to resolve that.

And that is the power that you see in front of you now, of a diffraction-limited telescope which -- which has a diameter of 2.4 meters.

You get an angular resolution which is very close to four hundredths of an arc second.

The amount of detail that you see is incredible.

That's the big power, the big reason why this telescope was put in orbit, to do away with the air turbulence, what the astronomers call as seeing, which is always the limitation of your angular resolution.

So in the remaining five minutes, I want to address the issue of the angular resolution of your own eye.

You can now calculate what the ultimate angular resolution is of your own eye.

Because you can estimate what the diameter is of the pupil.

The opening of your eye.

Three millimeters, maybe five millimeters, a little bit larger at night when it is dark.

Pupil opens up.

But we can calculate what this is.

Uh, if I take four millimeters, so I put in for A four millimeters, and if I take λ 5000 Angstroms, it's not an unreasonable value, then I find that the best angular resolution of a human eye is half an arc minute.

Cannot be any better.

There's just no way around it.

You're always stuck with the diffraction limitation.

I think though, that most of you will not be able to see with an angular resolution of one-half arc minute.

Most of you are probably in the domain of one arc minute.

It's a little larger than diffraction-limited.

But it's very close to that.

And that is something that I would like to test.

Not to see how good your eyes are, but for yourself to get a feeling for angular resolution.

And the way I'm going to do that is as follows.

We have prepared a box which Marcos is going to wheel in very shortly which has two pinholes at the top.

And these two pinholes are 2.5 millimeters apart.

And then there are two pinholes which are 5 millimeters apart.

And then there are two pinholes which are 10 millimeters apart.

And then there are two pinholes which are 15 millimeters apart.

So maybe we can take a look at that.

There it comes.

Thank you Marcos.

Here are two pinholes which are 2.5 millimeters apart.

We repeat the whole thing three times.

And the reason why we do that is so that the different angles in the audience, you can probably always see two pinholes well.

What am I going to do now to test the angular resolution of your eyes?

If I make the assumption that your angular resolution is one arc minute, no worse, no better, remember it can never be better than half an arc minute, that's nonnegotiable.

But it could be worse than one arc minute.

That would mean that all students who are closer than nine meters from me should be able to see this as two independent light sources.

Those who are farther than nine meters away from me will not see these as two sources.

If they did, their angular resolution would be better than one arc minute.

And all students which are closer than 17 meters will be able to say yeah, I see these as two sources, and all students which are closer than 34 meters should be able to say yeah, I see these as two sources.

And so we're now going to make it a little darker.

And you don't need your gratings, you don't need this card, I just want you to look at the lights, these light sources, and then tell me which you're going to see as two separate light sources.

And that then allows me to tell you very roughly what your angular resolution is.

So try to look at them.

And so now my first question is who can see the upper, either here or there or there, they're the same, who can clearly see those as two light sources?

Come on.

Come on.

You don't see them as two?

Something wrong, is the lights not on?

You must be kidding.

Are all of you sick or something?

Oh man, I have no difficulties at all.

The upper one is two light sources.

You've all got to see an eye doctor.

OK, who sees the second line as two?

Who sees the third line as two but not the second?

You see how interesting, just look around you, you see we're moving back in the audience.

Who sees the -- who can see none of them at two -- two light sources?

OK, so maybe not -- no eye doctors are needed then.

Who can only see the third line as two sources, only the third line?

OK, well, I expect that, you see, no -- yeah, maybe your angular resolution is not very -- do you wear glasses?

So you -- yeah, I'm asking you, so you see the third line as double?

And not the second line?

Yeah, there's nothing to worry about, maybe two arc minute resolution.

So now you have tested your angular resolution.

When you think of diffraction it's really an incredibly fascinating thing, because what does diffraction actually means?

That it is a limitation that is put upon us, on everyone, also God, no one can bypass diffraction.

No matter how hard we try we can never undo our chains and handcuffs that are imposed upon us by diffraction.

And remember it's all Huygens' fault.

But let's forgive Huygens because after all he was Dutch.