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8.02 Electricity and Magnetism, Spring 2002
Transcript – Lecture 32

These are the subjects will be covered during our third exam.

There's no way I can cover all during this review.

Nor can I cover all of them of course during the exam.

I can only touch upon a few of them.

And what I cannot cover today, what I will not cover today, can and will be on the exam.

Let's first look at magnetic materials.

Magnetic materials come in dia-, para- and ferromagnetic materials.

The molecules and the atoms in para- and ferromagnetic materials have intrinsic magnetic dipole moments.

These have always -- they're always a multiple of the Bohr magneton.

Has to do with quantum mechanics.

It's not part of 8.02.

And they are going to be aligned by the external field, I call that um the vacuum field.

And the degree of success depends on the temperature and on the strength of that external field.

The lower the temperature, the easier it is to align them, to overcome the thermal agitation.

And above a certain temperature which we call the Curie temperature, ferromagnet- magnetic material loses all its qualities and becomes paramagnetic and I have demonstrated that during my lectures.

Suppose we have a solenoid and the solenoid has N windings, and the length of the solenoid is N.

And the current I is flowing through the solenoid.

Then the magnetic field generated by that solenoid which I have called the vacuum field, that magnetic field can be derived using Ampere's law, which you see down -- down there.

That magnetic field is approximately mu 0 times I times N divided by L.

If now I put in here ferromagnetic material then I have to include this factor kappa of M or K of M, whatever you want to call it.

The magnetic permeability, and this can be huge.

This can be 10, 100, even up to 1000 and higher.

So you get an enormous increase in magnetic field strength.

Self-inductance is defined as magnetic flux divided by the current I.

That's just the definition of self-inductance.

If the magnetic field goes up by a factor of kappa M then of course the magnetic flux will go up by the same factor and so the self-inductance will go up.

And you may remember a demonstration that I did when I had an iron core which I moved inside the solenoid and depending upon how far I moved it in could we see that the self-inductance went up and when I pulled it out self-inductance went down again.

We have an interesting problem.

I think it is assignment seven, whereby we have iron core here and then we have somewhere an air gap and you may want to revisit that to refresh your memory.

Let's now turn to transformers.

A transformer often comes in this shape.

Let me move it a little bit to the right.

Often comes in this shape which is then ferromagnetic material, to give perfect coupling between the left and the right sides, also increases the magnetic field.

This is the -- let's call this primary side.

N1 windings, index, self-inductance L1.

And here I put in a voltmeter to always monitor that value, I call that V1.

And this is the secondary side.

N2 windings.

Self-inductance L2.

And I put here a voltmeter which always monitors that voltage and I call that one V2.

You can show with Faraday's law as I did in class, in lectures, that V2 divided by V1, let's not worry about plus or minus signs, is N2 divided by N1.

That's a good approximation.

Depends on how well the coupling goes.

It depends on several factors, but you can come very close to this and this means then that if you make N2 larger than N1 then you can step up in voltage, we call that a step-up transformer.

But you can also step down if you make N2 smaller than N1.

Under very special conditions will the power generated on the primary side be all consumed for 100% or nearly 100% on the secondary side.

That is very, very special.

If that's the case then the time averaged power here V1 I1 is the same as V2 I2, here time averaged.

And so as a logical consequence of that you'll find that I2 divided by I1, let's not worry about minus signs, is that N1 divided by N2.

That, however, is not so easy as you may think.

It only can work approximately and I mentioned that on the side in my lectures.

If the resistance here and the resistance there is way, way smaller than the value for omega L.

And we did try to achieve that during one of the demonstrations that I gave on this.

I remember we had the induction oven whereby N2 was 1 and N1 was very large, I don't remember what it was anymore but it was of the order of several hundred, maybe a thousand, and we managed to get a current in the secondary which was huge, which was close to 1000 amperes.

It was enough to melt that iron nail.

And we made every effort then to make sure that the resistance was much, much smaller than omega L.

I think problem 7-1 of our assignments deals with that, and very naively assumes that this is all true.

But you should realize that it is not always so easy to achieve the conditions for that.

So let's now go to RLC circuits there.

Let's take an, uh, system which has a resistor R, it has a self-inductor, a pure self-inductor, L, and a capacitance, C.

AC.

And this driving power supply provides with a voltage, V, which is V0 cosine omega T.

Keep in mind that this can be always be sine omega T of course.

There is nothing special about cosine in life.

The steady state solution, that is not when you turn the thing on but if you wait awhile, you get a steady state solution for the current.

And the current that is going to flow now is V0 divided by the square root of R squared plus omega L minus one over omega C squared times the cosine of omega T minus phi.

And the tangent of phi is omega L minus one / omega C divided by R.

We call this the reactance.

The upstairs.

For which we give often the symbol X.

And so this is also X then divided by R.

And this whole square root that we have here, we call that the impedance.

The units are ohms.

And we call that Z.

And so the maximum current that you can have, the current is of course oscillating with angular frequency omega, the maximum value that you can have for the current, which I call I max, is then V0 divided by Z.

Then the cosine term is either plus or minus 1.

I can plot now this I max as a function of frequency.

So here is frequency and here is I max.

If the frequency is very low or near 0, then this term here becomes infinitely high because the impedance is infinitely high and so the current is 0.

I max is 0.

There's no current flowing at all.

When we go to very high frequencies it is the omega L term that goes to infinity.

And so again Z goes to infinity so again I max goes to 0.

And for other values of omega you get an I max which is not zero and so you get a curve like this which has the name of resonance curve.

This I max reaches a maximum value when the system is at resonance, that's what we call resonance.

And that's the case clearly when the reactance is 0.

Because when the reactance is zero this part vanishes.

And if the reactance is not 0 then the maximum current can only be lower, can never be higher.

And so when X equals 0 you'll find that omega L is 1/omega C and so the -- the frequency for which that happens, I call that omega 0, reminds me that it is the -- the resonance, is one divided by the square root of LC.

When I am at resonance, phi becomes 0.

So there is no phase delay between current and the driving voltage.

They are in phase with each other.

And the value for I max now simply becomes V0 divided by R.

Because the impedance itself becomes R.

Very boring, very simple, you're looking here at Ohm's law.

When the system is at resonance, forget the self-inductance, forget the capacitor, they are not there, they annihilate each other, and so the system behaves as if there were only a resistor, and that's exactly what you see here.

I have here some numbers which you have seen before.

During my lectures.

You can download this from the Web but you have to go back to the lecture when I discussed that.

And you see here s- some numbers for R, L and C and also for V0.

And I calculate for you here the resonance frequency.

I calculate the frequency also in terms of kilohertz.

And here you see the impedance and here you see the reactance.

If I'm 10% below resonance notice that the 1 / omega C term is always larger than omega L.

So your reactance in this case becomes -86 ohms.

The minus sign has of course no consequence for the current because you have an X squared here.

But notice that Z is now almost exclusively determined by X and not by R anymore.

Because the 10 ohms of the R here play no role, almost no role, in comparison with the 86.

Z becomes 87 and the maximum current is one-tenth of an ampere.

When you're on resonance, and that is characteristic for on resonance, the 2 omega L and 1/ omega C eat each other up.

They annihilate each other and so the reactance becomes 0.

So Z now is just pure R.

X is 0.

And so the maximum current in this case is 1.

Because I chose V0 at 10 and I chose R 10.

And then when I'm 10% above resonance then the omega L term is larger than the reactance of the capacitor and accordingly you get a lower current again, about one-eighth of the -- of an ampere.

And so you see this curve being formed in a very natural way and that's quantitative, you see there some numbers.

So now comes the question which of course in practice is very important.

And that has to do with the power that is generated by the power supply.

That power comes out in the form of heat.

Heat in the resistor and so if you time average the power, then the time average value, you can take the -- the voltage of the power supply, multiply that by the current.

You could also take the time average value of I squared R.

Because all that energy will ultimately come out in the form of heat of the resistor.

Either one will be fine.

I've decided to take this one.

So I will get then V0 cosine omega T -- the I becomes V0 divided by Z times the cosine (omega T - phi).

This is the power at any moment in time.

I will do the time averaging a little later.

When I see cosine (omega T - phi), that reminds me of my high school days, cosine alpha minus cosine -- no, cosine alpha minus beta is cosine alpha cosine beta + sine alpha sine beta.

That was drilled into my memory here.

I will never forget that, I think.

And so I will write down here -- my math teacher will be proud of me -- cosine omega T cosine phi plus sine omega T sine phi.

So this is this term.

If I'm going to time average it, I have a cosine omega T multiplied by sine omega T, that time average is 0.

So this term vanishes.

So the time average value of the power, I get a V0 squared, I get a Z here, and now I have here cosine omega T times cosine omega T.

The time average value of cosine squared omega T is one-half.

So I get a two here.

And then I still have my cosine phi there.

And I'm done.

If you like to get rid of this cosine phi, you can do that.

Because remember, the way that phi is defined, the tangent of phi is the reactance divided by R.

You still see it there.

So that means if this angle is 90 degrees that this side must be Z.

That's the square root of X squared plus R squared.

That's this part.

And so the cosine of phi is also R divided by Z.

And so if you prefer that -- there's no particular advantage but if you prefer that you can write down for cosine phi R divided by Z.

And so you get V0 squared times R divided by 2 and now you get Z squared.

And so there you see the power, time averaged power in an RLC circuit.

So now we can look at resonance.

It's always a very special situation.

When we are at resonance, Z equals R.

So you replace this capital Z by R.

And then you find V0 squared divided by 2R.

That's utterly trivial.

You could have predicted that.

It's really Ohm's law staring you in the face.

There is no self-inductance and there is no capacitor at resonance.

So you might as well have treated it as a simple system only with R.

And you find immediately then that answer.

At any other frequency than omega 0, Z would always be larger than R.

You see that immediately here.

And so that means that the average power would always be lower.

So it's only at resonance that you generate the highest power possible.

All right.

Let's go back to our subjects and see what's next.

We did LRC circuits.

Oh yes, we're getting now to traveling waves and standing waves.

Let's start with a traveling wave in a string.

That's always very nice to do that because the parallel with electromagnetic waves is nearly 100%.

I have a string that oscillates in the Y direction.

And let's say it propagates in the X direction.

Y0 cosine (K X - omega T).

When I see this traveling wave, so the traveling wave in the Y direction and it propagates in the X direction, I see immediately that it goes into the plus X direction, because I have a minus sign here, it tells me in the plus X direction.

K gives me all the information on the wavelength.

It's 2 pi divided by lambda.

Omega equals 2 pi times F, F being the frequency in hertz.

It would also be 2 pi divided by capital T, capital T now being the period of one complete oscillation.

The speed of propagation, the disturbance propagates in the X direction, is omega divided by K.

And so if I draw here at a particular moment in time this string, and this would be Y0, that would also be Y0, and it would propagate in this direction with that velocity, and this would be the wavelength, lambda, and that lambda is V times T.

It's immediately obvious if something propagates with a speed V and it has T seconds to go for one oscillation it moves over a distance lambda.

That's 8.01.

Do not confuse this speed of propagation with which the disturbance moves with the actual speed of the atoms, of the particles in the string.

If you were a particle in the string and you were sitting here, you never move in this direction.

That's the same with water waves.

If a water wave comes by all you do is this.

You go up and down.

Your motion is only in the Y direction.

You go from here to there and you oscillate back and forth with that frequency, angular frequency omega.

And so if you're really interested in the speed with which you are moving up and down, that of course, that speed which we call the transverse speed is dY/dT, and you have to do that then for a particular location X, which you can choose, wherever you want to sit on that string.

And I think we had a problem once where we asked you in the homework what the transverse speed was.

So if now move to electromagnetic waves then very little changes.

Uh, take an electromagnetic wave, a plane wave, whereby the E vector is in the Y direction, so E is E0, there's the unit vector in the Y direction, cosine (K X - omega T).

This could be a sine of course.

There is nothing special in life with a cosine.

I come to the same conclusion.

The wave is traveling in the plus X direction and the velocity, speed of propagation, is omega divided by K and if this is vacuum which will I assume for now, then that is C and as we have seen from Maxwell's equations C is one divided by the square root of epsilon 0 mu 0.

Surprising as that is that comes out of Maxwell's equations.

And so if now you were asked, which would be a natural thing for me to ask you, what is the associated magnetic field, well, the magnetic field is perpendicular to the direction of propagation.

It's also perpendicular to E.

And B0 is E0 divided by C in vacuum.

And so if I make a coordinate system, this is X, this is Y and this is Z, notice that my coordinate system always is chosen so that X roof cross Y roof is Z roof, this is called the right-handed coordinate system.

If you choose any other system you're an idiot.

You always get yourself into trouble.

Make sure that you always choose this as a coordinate system.

And notice I have X cross Y is Z coming out of the blackboard.

At a moment, a particular moment in time, let's say the E vector is in this direction, in the Y direction, I pick a random moment in time and I pick a random location for X.

Now I must be sure that E cross B is in the direction of propagation.

So E cross B must be in this case in the direction of X.

Because it's going in the plus X direction.

And so my problem is solved.

I know that that can only happen if B is in this direction.

At this moment in time at this location.

And that's all I need.

Because now I can write down that the B vector that is associated with that electric field is E0 divided by C, that is the largest value that the magnetic field can have, times the same cosine (K X - omega T).

And now I must have here Z roof and now I'm in business.

So now I have the E vector that goes with the -- the B vector that goes with the E vector.

We would often be interested in energy, how much energy per unit area per unit time is in the plane wave.

That is given by the Poynting vector S which is E cross B divided by mu 0.

We're still in vacuum.

I still assume that it's all vacuum, so this is watts per square meter.

Of course we are in general not interested in the instantaneous value of the Poynting vector, who cares about that, it oscillates like mad, I'm more interested in a time average value.

And so the time average value, this has a cosine omega T, this has a cosine omega T, the product average out to be one-half of cosine square omega T.

I get an E0.

I get a B0.

And I get a mu 0.

And if now you want to get rid of your B0 because you want to get everything in terms of E0, you can replace B0 by E0 divided by C.

And so this would also be fine.

E0 squared divided by mu 0 C.

And so now we have the time average value of the Poynting vector.

The moment that you move to vacuum -- from vacuum to -- to matter, so we now go from vacuum, we move to matter, and the matter has dielectric constant K and it has magnetic permeability K of M.

That's only important if we deal with ferromagnetism because with paramagnetism and diamagnetism K of M is always practically, for all practical purposes 1.

But kappa can vary a great deal.

From the various substances.

And so now all you have to do, if you go to Maxwell's equations, if they were given only in vacuum, then you have to replace epsilon 0 by kappa epsilon 0.

That's already done there.

And you have to replace mu 0 by kappa M times mu 0.

And therefore you have to place C by V because the velocity is different, in matter, of electromagnetic radiation.

And you see immediately how it changes because epsilon 0 and mu 0 have to be replaced by epsilon 0 K and mu 0 kappa of M and so you see here that the velocity, following my recipe, becomes epsilon 0 mu 0 kappa kappa M.

And so my B0 becomes now E0 divided by V and no longer divided by C.

So this B0 becomes E0 divided by V.

Kappa can be a very strong function of frequency.

So can kappa M, but kappa M of course is only important for ferromagnetic materials.

I've shown you an example before that kappa, the dielectric constant for water, was 80 at low frequencies, even at 100 megahertz.

Radio frequencies.

It was still 80.

But at the visible light where you're dealing with frequencies of a few times 10 to the 14 hertz, that value for kappa was way lower, was 1.77.

And so kappa is a small function, a strong function of um of frequency, and we introduce index of refraction, which is C divided by V.

And since V itself is a strong function of frequency, the index of refraction can also be a very strong function of frequency.

Was 1.3 roughly for water.

But it is a little different from -- for red light from blue light.

If not, we wouldn't be able to see rainbows.

OK.

Let's now talk about standing waves.

Let's start with strings again.

This is a string with length L and I generate in this string a standing wave.

Standing waves can only be generated at very discrete frequencies for very special wavelengths.

It's a resonance phenomenon.

And the lowest frequency for which this occurs will make the string oscillate in this fashion.

The string will do this.

This is called the fundamental.

Also the first harmonic.

Then there is a second harmonic which is the next frequency up for which it will resonate, which adds an extra node, there is already a node here and a node there, and the system will then oscillate like so.

Fweet fweet fweet fweet.

And then so this is the second harmonic.

And then I can go to the third harmonic and I can continue forever, not quite, but I can continue quite a bit, and this now would be the third harmonic.

And the frequencies that are the resonant frequencies are given by F of N, N being Nancy, that means either one or two or three or four, one being the fundamental, two being the second harmonic, this frequency is given N times V divided by 2 L whereby V is the speed of propagation of a disturbance along the direction of the string.

And L in this case is the length.

And the asso- associated wavelength lambda for that particular harmonic is 2 L / N.

If you put in N equals 1 you see that the wavelength is indeed twice the length and if you put in N equals 2 you see that the wavelength is exactly L.

So let us write down for the fundamental the equation for a standing wave.

Y1, 1 refers to the fundamental, would be Y01 times the cosine of omega 1 * T times the sine of K1 X.

This is very different from a traveling wave.

All the time information now here is decoupled from the spatial information.

K1 is again as before -- let me write down a nicer K1 -- 2 pi / lambda.

And omega 1 is 2 pi times F1.

What is so special here is that there are points here, values for X, for which the sine becomes 0.

In the case of the fundamental put in X equals 0 and you find that that sine is always 0.

But if you put in X equals L, you can check that, it's also 0.

And if you go to the second harmonic you will find that the sine is 0 there as well.

And so there are points now which never move, we call those nodes.

And that's very characteristic for a standing wave.

I could go to the second harmonic and all I would have to do is put here a 2 and here a 2 and here a 2 and here a 2.

It would have its own little amplitude, which would be this.

It would have its own frequency.

But that frequency omega 2 is nonnegotiable.

That must be 2 omega 1.

And if I go to the third harmonic, then omega 3 is going to be 3 omega 1.

But it has its own wavelength, you'll get K of 2 would be 2 pi divided by lambda 2, and the lambdas are -- the lambdas are given by this relationship.

I want you to get a few minutes' rest now so that you can digest this and then we'll go to electromagnetic standing waves.

And I want you to see this again.

You've seen that before but I want you to see it in a way that you have not seen it yet.

We have here a -- a rubber hose which we can oscillate and we're going to oscillate it in such a way, at least that's our goal, to get the third harmonic, and it's not so easy to get exactly on resonance, but we will try that, and Marcos decided to make it very beautiful for you, so he's going to put up a -- a black screen there because once we are at resonance we're going to strobe the s- string so that you will in effect be able to follow the motion.

Your eyes can't see what's going up and what's going down.

It goes too fast.

But when we strobe it we can slow that down, make the thing actually stand still.

And that's our objective, so you'll be able to see that.

And so let me turn on the strobe, and this black background will help, the strobe frequency is not exactly the same as the frequency of the -- of the oscillating string, so that allows you to actually see the slow motion.

And no one is arguing with me, right, when the center portion is down, the left and the right portion are up, and vice versa.

So you can really see now the characteristics of this standing wave.

Notice you see in this case four nodes.

One at either end, and you see two nodes in the middle.

That is where the s- sine of that curve is always 0.

I can add another strobe light at a slightly different frequency.

I collaborated for a few years with an artist, his name was Tsai, he worked at the Center for Advanced Visual Studies here at MIT, and he actually made art by oscillating objects at resonance frequency, rods and strings, and he strobed them in a way that I'm doing, so I actually learned this from him.

Quite pretty but also very instructive.

You can really see what's going on.

In green you see the s- string at a s- different moment in time than you see it in red.

And as I said I purposely made the frequencies of the strobe a little different.

Thanks, Marcos.

As you perhaps remember, you can also generate a standing wave in air itself.

Wind instruments are nothing but air columns which go into resonance.

If I have here a wind instrument which is open and open on both sides, this is the length of that instrument, then the frequencies that I can generate are given by this equation.

The only fundamental difference between the strings and the wind instruments is that with strings you can change V at will more or less, you can choose different materials through which the speed of propagation is different, and you can also change the tension.

If you increase the tension this V goes up.

So you can give a s- violin four different strings, they'll give you four different fundamentals.

With a wind instrument you cannot manipulate V because V is the speed of sound.

And so that V of air at room temperature is 340 meters per second.

That's nonnegotiable.

And so you can with wind instruments, you can very easily predict the frequencies that you're going to hear.

I have here a pipe which is open and open, open on both sides.

1.5 meters long.

And if I apply that equation I will find that the fundamental would be at 113 hertz.

Perhaps you remember that I told you that resonances can occur when you sometimes least expect them.

Just by blowing air you can get resonances.

If you take a wind instrument and you start to blow air it excites resonances.

Remember the Tacoma Bridge.

There was wind and it went into resonance.

It was very destructive.

Well here we have a system that we can also make go into resonance when you least expect it.

We have a -- a copper grid here.

And I'm going to heat that grid.

And when I heat the grid -- get an air flow going through there.

That all by itself will not make it go into resonance.

But when I take my heat source away and that grid starts to cool, it goes into resonance.

So I'm going to heat it now.

Will take a while.

If I heat it too long, the copper grid will melt.

Many years ago when I was doing this with a very short burner the molten copper came down on my hands, which was not funny, believe me.

If I don't do it long enough, it won't go into resonance.

So I'm sort of guessing this a little.

113 hertz.

All right.

Standing electromagnetic waves.

When we go to standing electromagnetic waves, let me stay on the center board here.

The situation is almost identical to standing waves on a string.

Again we have -- we have nodes.

We have locations where the electric field is always zero.

Very, very different from a traveling wave.

I refer you to problem 9-4 where you will see a standing electromagnetic wave which just like we had with the -- with the string, it has exactly this form.

The time domain is decoupled from the spatial domain.

The only complication that you have with a standing wave, electromagnetic, is that it is not so easy to find the associated magnetic field.

And therefore I refer you to that problem 9-4 if you want to revisit that.

Polarization.

Oh, we want to see the subjects again.

See where we are on the list.

But I think the time has come to talk a little bit about polarization.

That's right.

Polarization.

Let's take electromagnetic waves that come straight out of the blackboard to you.

And I call this the Y direction and I call this the Z direc- the -- the X direction.

And Z is to you.

Notice X cross Y is Z always in my case, right-handed coordinate system.

And so the electric field vector of the plane wave that is coming out of the blackboard, let us assume that the electric vector is oscillating like so.

Fweet fweet fweet fweet fweet fweet fweet fweet.

With angular frequency omega.

If this is a straight line we call that linearly polarized radiation.

Could be radio emission as we did with the 75 megahertz transmitter.

It can also be visible light.

As long as the E vector stays along a straight line, we call that linearly polarized electromagnetic radiation.

Electromagnetic radiation, including radio waves, including visible light, can be circularly polarized.

In which case the electric vector doesn't oscillate like you see here but always has the same strength and is rotating around in a circle, either in this direction or in this direction.

And it's very easy to make, actually.

I could have done that here in lectures but I never did.

Suppose we have an antenna in the Y direction and we have another one in the X direction, like our 75 megahertz transmitter was a s- copper bar in this direction.

And suppose each one radiates with exactly the same value for E0, with exactly the same frequency but they're 90 degrees out of phase.

That's not so difficult to arrange.

Then I would get an EX which would be E0.

And if I pick my value for Z -- let's take Z equals 0, who cares where you are in this line, so there is no K Z term, so we simply have a cosine omega T here.

So this is the component of the electric vector in the direction of X.

And let the one in the Y direction -- must be 90 degrees out of phase, but must have exactly the same amplitude, so 90 degrees out of phase would for instance be sine omega T.

Omegas must be the same.

To get circularly polarized radiation.

And so the net electric field vector, the one that you will experience sitting here on the Z axis, will be EX in the X direction + EY in the Y direction.

And so the magnitude of this vector will be the square root of EX squared + EY squared, and that is E0.

Because sine squared omega T + cosine square omega T is 1.

And so you get under the square root E0 squared times 1, and so that's E0.

And so you see that the amplitude is always E0.

You see that here in front of you, and so it's going to rotate around, when it's maximum in Y, then it is zero in X, and when it is maximum in X it is zero in Y, and so therefore you get this rotation, either this way or that way.

Depending upon how the phase delay is arranged.

And you can turn this into elliptical polarized radiation by simply for instance putting a 2 here.

If you put a 2 there, or let me a put a 2 in the X direction, because I have more room in the X direction on the blackboard, so if I put a 2 here that means that in the X direction I can go twice as far as I can go in the Y direction and so now the E vector will go like this.

You see this now is twice as much as this and so now I have elliptically polarized radiation.

OK, so far is for the polarization is concerned.

Let's talk about Snell's law.

Snell's law was discovered 250 years before Maxwell.

It's quite an amazing accomplishment, even though you can derive it from Maxwell's equations, of course.

But it was derived by this Dutchman 250 years earlier.

And what Maxwell's e- what Snell's law tells us is that if we have light going from a medium 1 with index of refraction N1 going to medium 2 with index of refraction N2, and this angle of incidence is theta 1, this is the normal to the surface, and we get some reflection, this angle is also theta 1, and then some of that light will enter into this medium and this angle will then be theta 2.

And Snell's law says that the sine of theta 1 divided by the sine of theta 2 is N1 divided by N2.

Always the medium where you're going is up, the medium where -- oh, it's the other way around.

Ha, good that I caught that, this is N2 divided by N1, so the medium that you're going is up and the medium where you came from is down.

And so it's immediately obvious that if N2 is larger than N1 that the angle of theta 2 is always smaller than theta 1.

Let us assume now that you go from air to glass but that somehow you come out in the air again.

So right here, this is your angle of incidence now, I call it I, it's clear that I is theta 2.

And here you're coming out of the medium in air again, and I call this angle R, and I'm going to ask you the question, what is that angle R, now some of you may have great insight and immediately say, "oh well it's obvious that it's going to be the same as theta 1." And that is indeed correct.

You can easily see that because the sine of theta 1 divided by the sine of theta 2 at this transition at point A -- so this is at point A -- would be N of glass divided by N of air, and now we come here at this point B, so at point B we get the sine of this angle of incidence I, which we know is theta 2, that's obvious, divided by the sine of R is now the one where we're going to which is air divided by the one where we were, so this is N air divided by N glass.

And we already agreed that I is theta 2.

And so when I multiply these two equations, on the right side I get exactly 1.

Independent of the color of the light.

If blue light has a different index of refraction than red light, that doesn't matter, because I have the same one here that I have there.

And so you get exactly 1 on the right side, so you must get exactly 1 on the left side, and so the consequence is that theta 1 must be R, and so this angle here is the same as that angle, which is perhaps not so surprising.

Because these two planes are parallel to each other.

If they were not parallel, as they were with one of your problems where we had a prism, then you would get a separation of the colors here, and then the red and the blue would come out in different directions.

In this case red and blue and green and yellow all come out in the same direction and so you see white light when you look through plane parallel glass.

When it comes to total reflection, I refer you to problem 9-8 if you can spare the time.

You're going to have five problems.

Two of them have one question.

Two of them have two questions.

And one has four true-false questions.

For each correct answer you get four points.

For each wrong answer I have to subtract four points.

However, you don't have to answer.

If you don't answer you don't gain, you don't lose points.

Now before you hate me for subtracting four points, think about this for a minute.

If you give true-false questions to a class of five-year-olds they will have half on average correct and half wrong.

Yet they deserve 0.

So clearly the only reasonable thing is that for a wrong answer you must subtract points.

But you don't have to answer.

So if you know the answer to two for sure out of four you could consider not to answer the other two.

That is your choice.

I'll give you an example.

"The Benham top consists of several colors.

When you rotate it fast you see white light." That's wrong.

That's false.

Because the Benham top did not have several colors and when we rotated it we didn't see white light.

I'll give you another example.

"One of two tails of comets is due to radiation pressure and the other is due to the solar winds." That's correct.

We discussed that in lectures.

Let me end with some fatherly advice.

Read each problem at least twice and do those problems first that you like the best.

Those that suit you the best.

Never spend more than 10 minutes on one problem.

Then move to another.

There is another review tomorrow evening for three hours by Ali Nayeri.

You may want to attend that.

And we also will provide you with tutoring this Sunday.

Look at the Web.

Because we will update it as the time comes.

See you Monday.

Have a good weekend.