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8.02 Electricity and Magnetism, Spring 2002
Transcript – Lecture 25

So we have covered RC circuits and RL circuits, and today, we will spend the entire lecture on LRC circuits.

We will only discuss them in series so that you get the basic idea.

I have here a driving power supply, alternating, and here I have a capacitor C, self-inductor L, and a resistance R, this is AC, and let the driving voltage be V0 cosine omega T.

We have to set up the differential equation for this, and I want to remind you that Kirchhoff's Loop Rule does not hold.

So the closed loop integral of E dot dL, in spite of what the author of your book wants you to believe, that is not 0.

So how do we set it up?

There are various ways that you can do that, I have my own discipline.

I, in my mind, I think of this first being a, a battery -- by this is the plus side, and this is the minus side -- a current is going to flow, capacitor is going to charge up, electric field inside the capacitor is in this direction, the electric field in the self-inductor is always 0, because the self-inductor has no resistance.

There's no electric field inside the self-inductor, no matter what some of your books want you to believe.

Then, the electric field in the resistor is in this direction, and the electric field inside the power supply goes from plus to minus, would be in this direction.

So if I set up the differential equation, I start here, I always go in the same direction as I, because only then is the closed loop integral -L dI/dt.

So I go over this capacitor, that is V of C, then I go through the wire of the self-inductor.

There is no electric field, so the integral E dot dL there is 0.

Then I go through the resistor, so I get IR, and then I have here my power supply, so I get -V0 cosine omega T, and that, now, according to Faraday's Law, equals L, minus L dI/dt.

The current equals dQ/dt.

If the current is positive -- this is my positive direction -- then the charge of the capacitor will increase.

And I also know that V of C, the potential difference over the capacitor is the charge on each one of the capacitor plates, divided by C.

And so I substitute that in this equation, and I bring the L dI/dt to the left side.

That is conventionally done.

You don't have to do that, but that's often done.

So I get a plus L, dI/dt now becomes d2Q/dt squared -- my goal is to get everything in terms of Q -- then my IR become R dQ/dt, and my V of C becomes Q / C -- notice that I ranked them in order, D2Q/dt squared, dQ/dt, and then Q, you don't have to do that, but there is nothing wrong with doing that -- and then we get here, equals V0 cosine omega T.

And this is the form in which most books would present this differential equation.

And they arrive that in various ways, most books arrive at this equation in a completely wrong way, but they get -- anyhow, they end up with this equation.

And so, you have to solve this equation, which is really beyond your present abilities, it's second-order differential equation, it's really part of 18.03, so I will give you the solution.

The basic idea being that you find a solution for Q as a function of time, and once you know Q as a function of time, you have, of course, the current, because then you take the derivative of your solution, and you get the current.

I will give you the current as a function of time.

So I, that satisfies that differential equation, is the V0 divided by [whistles] R squared + omega L - one over omega C squared, and the whole thing times cosine omega T minus phi.

And the tangent of phi equals omega L minus one over omega C divided by R.

We give this upstairs here a name, we call that the reactance.

The reactance, and that X, or sometimes it's called chi, is omega L minus one over omega C.

And the units are also ohms.

We call the entire square root that you see here, we call that capital Z, which is called the impedance, so the square root of R squared plus that X squared equals Z, that also has units of ohm, and that is called the impedance.

And so Z is an effective resistance, because this whole thing behaves like a resistance.

But the resistance depends not only on R, L and C, but also on the values of omega.

This solution is what we call a steady-state solution, it is the solution that you get if you wait a certain amount of time.

If you turn the instrument on, so you all of a sudden start this experiment, then in the beginning, you get a different solution, which is more complicated, you get transient phenomenon, but these transient phenomenon die out, and you end up with this solution.

Now, there are several interesting things that you can see in this solution.

We have to start digesting, this whole hour, this solution.

It has very interesting aspects.

For one thing, you can see that the current can be delayed over the driving voltage when phi is positive.

Then the current comes later than the voltage.

And that's the result of the inductor, we've discussed that before.

But now, that's also possible that the current is leading the voltage, which is very hard to understand intuitively.

That is the case when this term dominates over this one, then phi becomes negative, and so minus phi becomes positive.

If minus phi is positive, the current is leading the voltage.

Now you may say, "How can it possibly be?

Does that meant that before I switch the instrument on, that I already have a current?" Of course it doesn't mean that.

But that's the transient solution, remember?

When you turn something on, when you switch it on, this solution doesn't hold yet.

This is the steady-state solution.

So the value for I Max, we have always called what is front of the cosine term, we've always called that I Max, that value for I Max is a function of omega itself -- as we will analyze in detail today -- and of course, also, of R, L and C.

And there is one particular value for Z, and therefore for omega, whereby this value reaches a maximum, and that's what we call resonance.

There is no value for omega for which the current is any higher.

And so I will call here, the situation, at resonance.

It is at resonance when X equals 0, so when omega L is one over omega C, so when omega is one over the square root of L C.

And we call that the resonance frequency, and we often give a little subscript 0 there to remind you that you're dealing with the resonance frequency.

And Z is then just R, because when X is 0, the omega L and the one over omega C eat each other up.

They are not there any more, it's gone.

And so the system behaves as if there were only a resistor.

And so you also see that the maximum current that you get is then, simply, V0 divided by that value for R, because Z, the impedance, is now R.

And in addition, if you're interested in phi, phi then becomes 0, so the driving voltage is then in phase with the current that follows.

And so the signal that you will see is a cosinusoidal variation in the current, so if I have here the current as a function of time, and you get a signal like so, and this here, this period T equals your 2 pi / omega.

So that is the -- directly connected to your driving frequency.

And if the impedance Z is very low, then this maximum value of the current, this is what we call the maximum value -- and, of course, the maximum value is also here, except that the cosine is -1 here, and the cosine is +1 here -- so if Z is very low, then this will be high.

If Z is very high, this will be low.

And there is only one and one value of Z for which the system is at resonance, and that is when the self-inductance and the capacitor eat each other up, and then you get the maximum possible value for the current at maximum, which is V0 over R.

And that's the highest value that you could ever get them.

Imagine that we have an LRC circuit, and we have L and R and C fixed, but we change the driving frequency.

So we move over various values of Z by changing omega from a very low value to a very high value.

If you start at a very low value for omega, let's say it approaches 0, then notice that Z goes to infinity, and so the maximum current becomes 0.

And the person responsible for that is the capacitor, because if omega goes to 0, this goes to infinity.

And that's intuitively pleasing, because omega 0 really means you have no AC any more, you have DC.

And with DC, what you're doing is, you charge up the capacitor when it's fully charged, no current can flow any more.

So that's intuitively pleasing.

When omega becomes very high, let's call it infinity, then Z, again, goes to infinity.

So again, the maximum current, again goes to 0.

And the person responsible for that is the self-inductor, because when omega goes to infinity, again, Z goes to infinity.

So again, you get 0 here.

And that's also intuitively pleasing, because if you have an infinitely high frequency, that means the self-inductance puts up an enormous fight.

It's ideal for a self-inductor to fight currents if the time over which the changes occur go to 0.

And so, then, again, it says, "Sorry, you can't have any current." So that's also intuitively pleasing, that the self-inductance, then, becomes the dominant factor.

And so what I can do now, I can plot the I Max as a function of omega.

So here is omega, and here is I Max, and we already agreed that when omega is 0, then I Max is 0.

But when omega is very high, it's also 0.

But when omega is at resonance, omega 0, which is one over the square root of L C -- notice that R has nothing to do with the resonant frequency, it's really determined by L and C, because it's the chi, it's the X that you want to make 0, and X is only a function of L and C -- at this frequency, we have a value here which is V0 divided by R.

And so the curve that you're going to see, which we call the resonance curve, is something like this.

You start out with an extremely small current, you go through resonance, we have a high current, and then at high frequencies, again, you go down to 0.

And so the left part, when you are below resonance, it's really the capacitance which is the dominant guy in the whole game -- and phi, by the way, is here, less than 0 -- here it is the inductor that plays the key role, and here phi equals larger than 0, and right here, phi, and only there, phi is 0, only when you're exactly at resonance.

I'd like to show you some numerical results, and for that I have a transparency -- it's also on the web, so you don't have to copy the numbers, uh, you can download them -- these are just some numerical numbers which I want to digest with you, so that you get a feeling for the effect, that you see it in front of your own eyes, what is happening, how this curve evolves.

We have here a given R, L, and C: ten, 5 times 10 to the -2 henry, and 3 times 10 to the -7 farads.

The resonant frequency is a little over 8000 radians per second, you see it here in kilohertz, and you see here the impedance -- and what I do here, I have a driving frequency which is 10% below the resonance frequency.

And I calculate for you, the omega L, which is 367 ohms, and one over omega C, which is 453 ohms.

You are a little bit below resonance, and so C dominates.

And you can see, indeed, that this ohm value is larger than this one.

And so out of that pops a value for X, out of that pops a value for Z.

Notice that X is 86, and Z is only a hair larger than 86, because this R almost doesn't add to Z, because you get here the square root of 10 squared plus 86 squared, that is almost 86.

It becomes 87.

And then you see that the current, the maximum current, which is this value for V0 divided by, uh, the Z, by 87, becomes 0.11 amperes.

And now, the system is driven at resonance, and notice that it's exactly characteristic for resonance that omega L and one over omega C have the same value.

They are not there any more, they're gone.

And so X becomes 0, so the impedance becomes ohm -- 10 ohms, which is the resistance, and so the maximum current is now V0 divided by R, which is 1 amperes.

And when you're 10% over resonance, then the self-inductor becomes to be more powerful than the capacitor, and again, your current is substantially down, in this, case, 8 times lower than at resonance.

We define, at a height of 0.7 times the value at resonance, we define a width of this curve.

And this width is given in terms of delta omega.

And that width -- and I will give you the answer without mathematical proof, it's not so difficult, but it's a little bit of a headache -- that value is R divided by L.

So the larger R is, the broader it becomes.

So if we look at delta omega, for the numbers that we have there, the numbers of the transparency -- so this is for, for the numbers that we have there, we have delta omega, would be R, which is 10 ohms, divided by 5 times 10 to the -2, and that is about 200 radians per second.

We define Q not as charge -- don't never confuse that with charge -- we call that the quality of the resonance, and the quality is defined as omega 0 divided by delta omega.

Now, omega 0 itself is one over the square root of L C, and delta omega is R divided by L.

And so that makes the quality 1/R times the square root of L/C.

And the quality is the measure for omega 0, which is this, what I'm pointing at now, divided by delta omega, which is this.

So if the quality is high, this peak is relatively narrow, and if the quality is low, it's relatively wide.

You may ask yourself the question, why do we define delta omega at 70% of the maximum current at resonance?

Why not at half?

There's a good reason for that, because, in practice, we are more interested in power than that we are in currents.

And power is proportional with I squared.

And so when you square this, you get 0.5.

And 0.5 means, then, that this is really the width at half-power.

And so that's the reason why we chose the 0.7 times the maximum current at resonance.

It's really the half-power width.

Resonance can be destructive.

Uh, imagine, if you have a very high-Q system, if you're slightly off-resonance, there's almost no current, no power dissipated in your resistor, and now, you come, all of a sudden, on the resonance, you can an enormous current, and that means there's an enormous power dissipation in your resistor, and you can burn out your resistor.

You can destroy your circuits, if you're not careful.

And next lecture and Monday, I will also discuss with you some med-mechanical resonances.

Mechanical systems can also go into [unintelligible] can also be destructive.

At certain frequencies, the systems behave -- call it k- violently, they respond extremely strongly to their input frequency, and things can break.

Humans also have resonance frequencies, you can call them, if you want, emotional resonances.

All have sensitive nerves.

Someone makes a particular remark, go through the roof.

Also, falling in love, when you think about it, is a resonance phenomenon, and that, too, can be rather destructive.

As many of us know.

But now I would like to demonstrate to you the resonance curves -- I'm going to choose particular values of, um, R, L, and C, which I can change, and then I will show you the current as a function of frequency.

And these are the values that I have chosen.

Again, this is on the Web, you can download it, so you don't have to copy it now.

And I will change the -- the light setting so that we can also enjoy the demonstration.

The idea being that, for these values that I have there, in the first line you see R, 60 ohms, and the self-inductance is 50 millihenry, and the capacitance is 0.3 microfarads.

So that's a given there.

And I give you here the resonance frequency, 8000, in terms of omega radians per second, this is the resonance frequency in Hertz -- and just in case you're interested, I gave you the Q value there as well.

And what I'm going to do now for you, is I'm going to sweep the input frequency from 0 to 16000 radians per second.

So my omega can go from 0 to 16000.

And I leave the values as they are, here.

So I'm going to sweep, sweep over this 8000.

And so you're going to see that curve.

Except that I'm show -- I'm going to show you I as a function of frequency, not I Max.

And I is oscillating, because there's a cosine term.

And so, for instance, if I were here, with this value for omega, you would see then that it goes up, it goes down, it goes up, it goes down, it goes up, and it goes down.

And when I'm here, you will see this.

And keep that in mind when you look at the curve that you're going to see there -- and so it's only the envelope, then, that is the I Max.

But you actually see the entire current as a function of frequency.

And I am going to do that, then, for all these four values that you see there.

So, let's first change the light so that we get an optimum situation for you.

And now, I will show you.

Already, the results of the first line -- so these are the values that you see there.

And I go -- I sl- I go very slowly.

Now omega is 0 here, omega goes up, I go through resonance, and omega is here, the value would be 16000 radians per second that we have here.

And it sweeps back and forth between 0 and 16000.

So you see a dramatic increase at resonance.

So now what I'm going to do, I'm going to double the self-inductance.

And when you double the self-inductance, this one goes up, omega 0 goes down.

So all I want you to see, that the resonance frequency, which is here -- that's the maximum -- that the resonance frequency will shift.

Because if L goes up by a factor of 2, the resonant frequency will come down by the square root of 2.

And so I am going to increase L by a factor of 2 -- let me make sure that I have the right knob here -- and this is my L.

Notice that the f- the resonant frequency is now at a lower value, it is here.

Also notice that the peak value at resonance has not changed.

Because the peak value at resonance, you see on the blackboard here, is V0 over R.

So as long as I don't change R, that doesn't change.

It's only the frequency that changes.

Omega 0 is one over the square root of LC.

That changes.

It is now here.

So I have increased L by a factor of 2, I can bring it back to my original resonance frequency by now changing C.

If you increase L by a factor of 2, all you have to do is decrease C by a factor of 2, and you're back at the same resonance.

So I'm going to make C down -- C lower by a factor of 2, which I'm doing now, and if you look, now, here, and you have a good memory, you will see that the resonant frequency is back where it was.

Again, V0 over R is not changed, and the resonance frequency is back here, even though L is now twice is higher, and C is twice lower.

To show you the effect of R, I will double R now, and I will leave everything else alone, so the resonance frequency will stay here, but of course the maximum current -- this high value will now come down, because, you see, at resonance, this value is V0 divided by R, and since R goes up by a factor of 2, you will see that the maximum current will go down by a factor of 2.

And so, I go now from 60 -- from 50 -- oh, no, I go from 60 to 100, I don't double it -- I can't go any further than 100.

But you'll see a substantial reduction.

So if you're ready for this -- remember this height -- and now you see 100 ohms, and it is much lower.

It was this high before, and now it's only here.

But notice the resonance frequency has not changed.

So this is an extremely interesting behavior, and every time, you have to think through what is happening, you have not much intuition for it, you're not alone, I don't have much intuition for that either.

But with these rather simple equations -- they're really not that difficult -- here, this is really the heart of the equation, and then, of course, your tangent phi, in case you're interested in the phase lags -- that they're really not that difficult.

I will not hold you responsible for being able to derive the r- solution.

I give you the solution.

But with that solution, you can do a lot, and you can understand the behavior of these circuits quite well.

Now I'm going to do a demonstration, and I warn you, you have to very closely follow, step-by-step, what I'm going to do.

Because if you miss one small step, you're lost for the next 12 minutes.

Completely lost.

You will just see nice things, but you don't know what you're looking at.

So follow me closely.

I have an LRC circuit.

And the LRC circuit is right here.

This is my L, 0.1 Henry, these are my Cs, and this is my R, it's a 200 watt light bulb.

I have an LRC circuit.

And I will give you the values for L, for R, and for C.

I'm going to drive it at 60 hertz.

So omega equals 377 radians per second.

And whatever you're going to see, for the next 12 minutes, that is not going to change.

That frequency is a given.

I simply plug it into the wall, and so my driving voltage is 110 times the square root of 2, times the cosine of omega T, and it is this omega.

So it's the 110 volts that comes out of the wall, so to speak.

My light bulb R is 60 ohms when it is hot, and it is a 200 watt bulb.

So if it's bright, 200 watt bulb.

The self-inductance L is 0.1 Henry, and the capacitance is 8 microfarads.

So this is God-given.

With these three values, I can calculate what Z is, and Z is 300 ohms.

In case you're interested, I can also give you omega L, that is 38 ohms, and I can give you one over omega C, that is 332 ohms.

So notice that this is, by far, the dominant player in the game, compared to omega L.

In case you're interested in the resonance frequency, omega 0 is one over the square root of L C, and that is about 1120 radians per second.

In other words, my driver is nowhere near resonance.

I am way below resonance.

When you're way below resonance, it is the C that dominates.

Look at the 332 ohms, and compare that with the 38.

I can calculate, now, what the maximum current is, that we get, I Max.

That is V0, divided by Z, that impedance, so that is 110 square root 2, divided by 300 ohms, and that is 0.5 amperes.

And now I can tell you how much power is dissipated in the light bulb, which is I square R.

So in the light bulb, which is the only component which has a resistance -- well, maybe the L has, also, a little bit of resistance, but we will ignore that for now -- so in the light bulb, the average power over one full cycle of my oscillations equals the mean value between I squared R, time averaged, the time average value of cosine omega T squared is always one-half, so this one-half that comes here is the result of the fact that the I there has a cosine omega T, and you square that, you get one half, if you average it over oscillations, so now I get I Max squared times R.

So this is one-half times my 0.5 squared, and the R of the light bulb is 60, and this turns out to be 7.5 watts.

Nothing.

Why is it nothing?

Well, that's the way we designed this system.

We're way below resonance.

So if I plug the system in, which I will do for you, this 200 watt light bulb will only dissipate 7.5 watts, and you won't see anything.

You ready for this?

Three, two, one, zero.

Physics works.

You see nothing.

Light bulb, not working.

Great.

So now comes the question, what could we do to get the system back on resonance?

Well, we know that our driving frequency is 377 radians per second.

And so what we could do, we could change the C in the circuit, and we could change L in the circuit.

If we increase C, or we increase L, then the resonance frequency will shift from 1120 down, make L larger, or make C larger, then, obviously, you shift your resonance frequency down.

And my goal is to shift it down to 377 radians per second.

I first want to make a curve for you here, very roughly, of I Max as a function of frequency, omega, and that's the de- demonstration I just did.

When this was 1120, we had about 0.5 amperes at R value of 377.

So this is one point of our I Max versus omega curve.

I can now ask the question, if we drive the system at resonance, what then, would have been the maximum current?

We're not driving it at resonance.

But suppose we had driven it at resonance.

Well, at resonance -- so at resonance, I Max is V0 divided by R.

There is no resi- there is no self-inductor, and there is no capacitance at resonance.

And so that would then be 110, square root of 2, divided by 60.

But that is 2.6 amperes.

That is substantially larger than one-half.

So this value here is 2.6 amperes.

It is too bad that I can't go there, because I am stuck to my 377, and I'm not going to change that.

Throughout the demonstration, I stick to my driver here.

So keep in mind that the driver is always here, I don't change that.

So the resonance curve -- we call this the resonance curve -- is very broad, something like this.

And I believe the Q is around 2, you can check that for yourself.

I can calculate what the power is, that the bulb is dissipating at resonance.

Well, that is my one-half, which is the time-average cosine squared -- my I Max is now point -- 2.6 amperes, so now I get 2.6 squared -- and my resistance is still 60 ohms -- and this is 200 watts.

What a coincidence, that's the way we designed the experiment.

So if we could be at resonance, the light bulb would be happy like a clam at high tide, it's exactly 200 watts.

It wants 2.6 amperes, that's what it wants.

Then it is a 200 watt light bulb.

And so we want to make the light bulb happy.

And how can we do that?

Well, we can either increase L, or we can increase C.

And I want to catch two birds with one stone during this demonstration, I'm going to teach you a way that you can increase L, something that we have never discussed before.

I have here, ferromagnetic material, which has a value for kappa somewhere around 10, 12 -- let's say it is 10 for now -- so that means that if I bring that iron core inside, that I could make this go up to 1 henry.

Because remember, the meaning of self-inductance is the flux divided by the current in the solenoid.

And so if you bring in ferromagnetic material with a kappa M of 10, then the flux goes up by a factor of 10.

The current remains the same, and therefore your self-inductance goes up by a factor of 10.

So you see that you can make a variable self-inductance by bringing in ferromagnetic material, and by removing it again.

And so the question, now, is, how high should L be to get on resonance?

Well, that's very easy, because we know that one over the square root of LC, we want to make that 377, that's our driver.

But we know that C is 8 microfarads, that's a God-given, we're not going to change that.

And so you can easily show now that if only we could make L 0.88 Henry, we should be back on resonance.

Then we have 377 radians per second as our resonance.

And you shouldn't be surprised that omega L is then 332 ohms.

Of course it is 332 ohms, because it's going to eat up that 1 / omega C, they're going to cancel each other.

That's another way you could have calculated the value of L, by simply saying omega L has to be 332.

So there are various ways to get your resonance frequency.

And so what I have done, if I make L 0.88 Henry, I shift the resonance frequency to this value, and my resonance curve now will look like this.

But this height is going to be the same, because the height is only determined by V0 and R.

And so what I'm going to do now is I'm going to turn on the instrument where it was before, when this was the resonance curve, you won't see any light.

And I'm going to move the bar in, I probably don't have to move it in all the way, and then the light bulb will go through resonance, the system will go through resonance, you will get your 2.6 amperes, and the light bulb will be happy.

And so let's do that now.

All right?

So here, system is now -- current is going.

60 Hertz, 110 volts.

root mean square as we call that, and it's way off resonance, 7.5 watts, the light bulb doesn't even feel warm.

And now I bring the -- this enormous piece of iron in there, and when I shove it in, the self-inductance will slowly go up, the resonance frequency will shift, and the current through the system will increase.

And there we go.

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There it is.

I'm on resonance now.

So the light bulb is quite happy.

I want to do one more thing.

I'm going to double the capacitance.

And what happens when I double the capacitance?

The frequency, the resonance frequency goes further down that it was already, because remember, when you increase L or C, the resonance frequency goes down.

So I'm going to double the capacitance.

So that curve that you see on the left there, has shifted, now, even further to the left.

And the light bulb is not very happy.

The light bulb is -- the system is off resonance again.

But if I increase C -- I doubled it -- I can decrease L by a factor of 2, all I have to do is pull the iron slowly out.

And then the resonance frequency comes back at 377 radians per second.

Watch it.

Back on resonance.

Amazing, isn't it?

Physics works.

So I showed you this demonstration, and I spent so much time on it because I wanted you to be able to appreciate -- I want you to be able to see through these equations.

The answer, in physics, in general, lies in the equations, but it only works if you build up a certain amount of understanding.

And that is not always easy, and the demonstrations help that.

Seeing is believing.

And you may have to go over this at home in order to digest it a little bit.

I don't expect you to get all this just in one lecture, of course not.

We have a new impedance here, we have the reactance here, we have the idea that omega L eats up omega C, then we get into resonance, all these phenomenon take time to digest.

There are many key practical applications in LRC circuits, more than you may think.

Your radio and your TV are systems that you, without realizing it, you tune them to resonance.

There's an antenna, and that receives many, many stations at all different frequencies, and you are changing -- in general, you change the capacitor.

And when you change the capacitor in the LRC circuit, you are changing the resonance frequency, and at that resonance frequency, the system is very sensitive, drives a very high current, and it selects, then, a particular station at that frequency.

That's what you're doing with your TV, and that's what you're doing with your radio.

You're changing a variable capacitor.

Another application which is quite common are metal detectors.

Metal detectors are resonant circuits, they are set at resonance, and then, when you bring metal nearby, you bring them off resonance, and then the alarm goes.

In general, they have two coils -- this is one coil, and this is another coil.

Inductance L1, resistor R1, capacitance C1, L2, R2, C2.

And so there's a current, I1, running through this one, a current I2 running through that one.

To set up the differential equations for this system is not so easy.

You get two differential equations, of course, one for this system, and one for this system.

The problem -- but at the same time the trick -- is that there is a mutual inductance between the two, because when this one produces a current I2, there is a magnetic flux going through this one.

That's why we call it mutual inductance, which we have never really dealt with in any detail in 8.02, and we won't.

But I want to mention that this differential equation here is an equation in I1, L1, R1, C1, but also contains a term N times dI2/dt.

And this differential equation is an equation in L2, R2, C2, I2, but also contains a term, N times dI1/dt.

And so now you get two coupled differential equations.

It's not just one in I1, and one in I2.

It is one in I1 and I2, and it is another one in I1 and I2.

And that is a pain in the neck, and it's not easy to solve.

But there are mathematicians who can do all that, and they solved that for us physicists.

In any case, what is important that clearly, since you have two coils, you get two resonance frequencies, not just one, but you have two resonance frequencies.

And at least one of those resonance frequencies depends very strongly on the mutual inductance.

And so the system is tuned to set at one of these two resonance frequencies.

And now someone brings in metal.

You bring in a chu- chunk of metal.

And in this metal -- the metal will experience the magnetic field, flux changes, AC of course, always, in the metal, it will start building up eddy currents.

And so the eddy currents will change the magnetic coupling between these two loops.

And so M changes.

And when M changes, you go off resonance, and bingo, your alarm goes off.

Clearly, and very unfortunately, this never works for plastic bombs.

You must have conducting material that you bring in, so it must be a metal.

These systems have very high Q, very high Q means they are extremely sensitive.

If you have very high Q, that curve is extremely narrow, and so the slightest change in M, you go off resonance, and you get your alarm.

And so they come in various forms, and in various applications.

At the airport, it is very simple.

You walk through these two coils.

You may not have realized that, but you simply walk through.

You walk through, like, there's one coil here, one coil there, and that's how they detect whether you have any, uh, metal on you.

The one -- the metal detectors that are used to search your body, including the ones that are used to search the ground for coins -- uh, when I was a kid, I went often to the beach in the Netherlands, and there were often people who were having these metal detectors, they were looking for coins.

And those metal detectors, basically the same idea, always two coils -- this is this coil, and this is this coil, and when metal comes nearby, you have a system that goes off resonance, you get a difference -- magnetic coupling between the two coils, and the system goes into alarm.

And I'd like to demonstrate that, for wh- for which I need a student.

Any student who wants to volunteer, I have here a metal detector, and, thank goodness, we don't have to go through metal detectors yet, at MIT -- you want to volunteer?

Boy, you're brave, you're all the way at the end of the -- can't wait what you have in store for us.

Oh, you look like you're made of steel.

OK, you -- I hope you left all your secret weapons up there, did you?

[unintelligible].

So this is a -- this is a metal detector, and if you opened this up here, you would see two coils, one in the center, and one near the edge.

[beep].

And we have to -- I have to calibrate it first -- OK.

Oh, you left your cigarettes out there?

They -- they won't record.

What did you leave out there?

I am just getting curious.

Oh, you're afraid they will get damaged?

Oh, we hear nothing.

That's nice.

Student: Definitely not working.

No, that's probably not working.

Well, just a second, may have to recalibrate this.

.

Pretty quiet.

Doesn't look like you have any metal there.

I'm pr- fairly sure I turned it on.

They don't always work, do they?

Marcos, you have any wisdom on that?

[beep] Uh-uh.

It's working.

No, it's not working very well.

It's working well here.

[laughter].

Wonder what you got there.

Are you metal -- is this not made of metal, all these things in here?

Student: I have lots of metal on me, and [unintelligible] I'm surprised it's only going off at the belt buckle, it's weird.

Oh, you may be a weird person, that may be [laughter].

[beeping].

Can you turn around?

Oh, that's your watch.

[beeping].

[laughter].

You'd better turn around again.

Ah, there are the keys.

You're not made of steel, but you're a great guy, thank you very much.

[applause].

Today is Wednesday, right?

See you Friday.