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8.02 Electricity and Magnetism, Spring 2002
Transcript – Lecture 23

Here are the topics the way I see them.

They're on the web.

Look on the lecture supplements of today and you can download them.

I want to point out that, uh, the discount is very reasonable because these magnets are broken, and when you break a magnet, you end up with two monopoles, so you get 50 percent off.

You should know better by looking at the key equation there, that magnetic monopoles don't exist, but that's a detail.

I want you to appreciate that I cannot possibly cover today these topics in any depth, nor can I cover all of these during a 50 minute exam.

So please understand that this review is highly incomplete, and what is not covered today can and will be on the exam.

I'm interested in concepts.

I'm not interested in math.

There will be seven problems.

Five of the seven problems have only one question.

Two problems have two questions.

I don't think that time, that the length of the exam is going to be an issue.

This exam was taken by several instructors.

It took them 15 to 18 minutes, and that's normally my objective.

Courtesy of Professor Belcher, we have old tests on the website.

I don't have the solutions.

There's only so much I can do.

I'm very grateful to Professor Belcher that he made these exams available.

If you can't do some of the problems, I would suggest you see your tutors or you see your instructor.

I will also be available all day afternoon, not tomorrow, because tomorrow afternoon I'll be in 26-100 all afternoon to work on demonstrations for the next week.

Many of these problems are straightforward.

You may also want to consult your study guides.

All right.

Let's first start with Biot-Savart.

There are not too many problems that one can do with Biot-Savart.

\vec{dB} equals μ_0 divided by 4π times the current dL cross \vec{R} / R^3 .

That's the formalism.

A classic problem that you probably have done.

We have point P here at a distance D from a wire, and the current through the wire is I.

If you want to know what the magnetic field at P is, you can use Biot-Savart.

It would be a stupid thing to do, but you can do it.

You take then a small element dL here of the wire.

This distance is R.

This is the unit vector \hat{r} , which you have in this equation.

And you can calculate, then, what the contribution to the magnetic field right here is -- it comes out of the blackboard -- due to this section dL .

This angle is θ , and the sine of θ is D divided by R .

And then you have to do an integral over the whole wire, θ 0 to π .

And then you get the magnetic field here.

Not very smart thing to do.

A waste of time.

Because clearly the way to do this is to use Ampere's Law, which is the one at the bottom there.

In which case, you would construct a closed loop with radius D .

This loop is perpendicular to the blackboard.

I'll try to make you see it three dimensionally.

You have to attach an open surface to that closed loop.

Any open surface will do.

Let's make the open surface flat.

And then we apply Ampere's Law, which is the one at the bottom there.

We don't have the second portion because there is no changing electric flux.

We don't deal with κM at all.

So we simply have that B times $2\pi D$, that is going around in this circle, because I know that B is tangentially to that circle.

I also even know the direction according to the right-hand corkscrew rule, coming out of the blackboard here.

So as I go around the circle, B and the dL that you see there at the bottom are in the same direction.

So I get $2\pi B$ times D equals μ_0 times the current that penetrates that surface, and that is I .

And so the answer is very simple, $\mu_0 I$ divided by $2\pi D$.

That's the way you would do this problem, and you would stay away from Biot-Savart.

There is one particular problem whereby Ampere's Law will fail.

Of course, Ampere's Law in general works when we have cylindrical symmetry.

This is cylindrical symmetry.

There is one problem where Ampere's Law bitterly fails and where Biot-Savart is highly superior.

I have here a conducting loop.

It's a circle.

And you're being asked, it runs a certain current I , and you're being asked, what is the magnetic field right at the center?

It only works for the center.

You could not find what the magnetic field is here.

We did that in class.

You probably also did that for your homework.

Biot-Savart will immediately give you the answer.

And I will leave you with that.

And Ampere's Law won't work.

So let's now turn to Ampere's Law and do a few problems with Ampere's Law.

We need cylindrical symmetry, with very few exceptions.

I have a hollow cylinder here, radius R_1 .

Concentric another cylinder with radius R_2 .

These are very long cylinders.

And assume that there is a current flowing, I , in this direction on the surface of the inner cylinder, and the current I is returning on the surface of the outer cylinder, and the two currents are the same in magnitude.

And I want to know what is the magnetic field everywhere in space.

I will make a drawing whereby we only see the cross-section.

So this is R_1 and this is R_2 .

And let's first calculate the magnetic field for R being larger than R_2 .

It's immediately obvious that your closed loop that you choose itself is going to be a circle.

That is a must.

Radius R .

We use a symmetry argument.

Whatever the magnetic field is at that distance R , little R , it must be the same everywhere.

It cannot be any different here in terms of magnitude than there because of the symmetry of the problem.

We have cylindrical symmetry.

We also know that if there is any magnetic field, that it is going to be tangential, either in this direction or in that direction.

And so we go around.

We make the closed loop integral, use the equation that we have there at the bottom, and so you get $B \text{ times } 2 \pi \text{ little } R \text{ equals } \mu_0 I$.

And now I have to attach an open surface to this loop.

I will use the surface in the blackboard.

I could use any surface, but I might as well use a flat surface.

And now I have to know what is the current going through that surface.

Right here, on this surface, the current is going into the blackboard, and right here, on this surface, the current is coming out of the blackboard.

The two magnitudes are the same, so the net current is 0.

And so the magnetic field outside the second cylinder is 0.

So let's now look at the area in between the two cylinders.

These are hollow cylinders, now, this is completely open here and this is completely open.

They are thin, thin material, thin shells, are both cylinders.

Now of course the closed loop is going to be one inside the opening between the two cylinders.

And again, I pick radius R .

And here I go.

I get $B \text{ times } 2 \pi R \text{ equals } \mu_0 I$, but now there is current going through this surface, and the current that is going through is I .

Not this one, but this one.

And so we get I .

And so now we get that B is μ_0 times I divided by $2\pi R$.

One over R field.

The direction you will find from the right-hand corkscrew rule.

That's the way I normally do it.

You take a corkscrew and you turn it clockwise, it goes into the blackboard, so the magnetic field here is in this direction.

If you're not used to turning corkscrews in corks, think about when you try to tighten a screw with a screwdriver.

If you go clockwise, the screw goes in.

At least, that is the case with 99.99 percent of all screws that you find in this country.

They have a right-handed thread.

You could make one that has a left-handed thread, but that's not done in general.

So now we can go to the area $R < R_1$.

So that's inside this thin shell cylinder, and again, you would take a surface, a closed loop, that is a circle with radius little R , and this is your surface attached, open surface attached to that closed loop.

And you will find for the same reason that you found here that B is 0, because there is no current going through that surface.

And so if you now make a plot of the magnetic field B as a function of R , here is R_1 , here is R_2 , then it is 0 here, it's 0 there, it has some value here, which is this value if you substitute for little R R_1 and right here it is this value, if you substitute for little R R_2 .

And this is a curve that is proportional to $1/R$.

And so this is the magnetic field, and inside, seen from where you were sitting, it is clockwise.

There is one case, and we did that in lectures, whereby we wanted to calculate the magnetic field inside a solenoid, where Ampere's Law works very well, even though we don't have then closed loops which are circles, we chose a rectangle.

I want you to revisit that.

It's undoubtedly in your book, and I covered it during my lectures.

We assumed that the magnetic field was uniform inside the solenoid and 0 outside the solenoid, which was not a bad assumption, and that allows you then, with Ampere's Law, even though you go rectangular and not circular, to get the magnetic field inside a solenoid.

Very classic.

Check your lecture notes or watch my lecture again, which you can do on the web.

So now I want to turn to Lorentz force.

The Lorentz force, F , is $Q(E + V \text{ cross } B)$.

This is the charge which is moving with velocity V in a magnetic field B and at the location of that charge there happens to be also an electric field E .

Let's take a situation that I have an electron.

I put a minus sign there to remind you that the charge is negative.

And let this be the velocity of that electron.

And let the magnetic field be uniform, perpendicular to the blackboard in this direction, going into the blackboard.

There is no electric field, so we only deal with Q times $V \text{ cross } B$.

$V \text{ cross } B$ -- you should be able to do that cross product -- is in this direction, but since the charge is negative, the force on this charge is in this direction.

And so the electron is going to turn over in this direction and you're going to get a circle if the magnetic field is uniform.

And the force is right at the center of that circle.

A little later in time, when the electron is here, going with the same speed, the force is like so.

The speed cannot change.

The force cannot do any work, cannot change the kinetic energy, because F is always perpendicular to the plane through V and B , because it's a cross product.

And since the force is always perpendicular to the velocity, you cannot change the kinetic energy, because you're never doing any work.

The only thing that the force is doing is make it go around.

It changes the direction of the velocity.

It doesn't change the magnitude of the velocity.

It doesn't change the speed.

All right?

So let this radius be little R .

Combining 8.02 with 8.01, we now have that $M V^2$ over R -- M being the mass of the electron -- is that force.

And since V is perpendicular to B , the sine of the angle between them is one, and so I simply get here $Q V B$.

This is the magnitude of the force.

We already know the direction.

And so one V goes, and so I get that the radius of that circle is $M V / Q B$.

This being the momentum of the electron.

So V is the velocity of the electron, speed.

Q is the charge.

B is the magnetic field.

M is the mass of the electron.

And as long as this velocity is reasonably smaller than the speed of light, we don't have to make a relativistic correction here.

If it's not much smaller than the speed of light, we have to make a relativistic correction.

I discussed the relativistic correction in my lectures, but I also mentioned that that would not be part of exams, so I will not further elaborate on that.

So we will just assume that this is completely non-relativistic.

The time for this electron to go around, capital T, is $2\pi R$ divided by V.

But I know R, so I get 2π times M V divided by Q B.

And then I have this V, and I lose this V, and that is not so intuitive, but you see that the time is independent of the velocity of the electron, assuming that it's non-relativistic.

And we discussed that in the framework of cyclotrons, time for these particles to go around is independent of the velocity.

Suppose I have a magnetic field B which is 7.8×10^{-4} tesla.

Now I know what you're thinking.

You're thinking, why the hell do you take this crazy value?

There is a reason for that, because I have a demonstration here.

And the magnetic field is 7.8×10^{-4} tesla.

If that is the field, I can calculate how long it takes to go around, because I know the mass of the electron, I know the charge of the electron, I know the magnetic field.

So in that case, T is about 46 nanoseconds.

And so the number of times that it goes around, F , which is the frequency in hertz, is one over the period, is about 22 megahertz.

So it goes around, this electron, 22 million times per second.

The demonstration that I will show you.

We have a, a glass container, spherical.

It's about yea big.

You can see it there.

There is low pressure gas in there, so that some of the electrons can go around in a circle before they ionize the gas.

We will accelerate the electrons over a potential difference, which I can vary from 0 to 300 volts.

And so I can give them a certain velocity.

Suppose I gave them a potential difference of 100 volts, then Q times ΔV , which is the potential difference, would then be one half $M V$ squared, and so if I make this 100 volts, I can calculate what V is, and I've found that the velocity of the electrons then is about 5.9 times 10^6 meters per second.

So that's the velocity if I have a potential difference of 100 volts.

And so I can use that velocity, substitute that in this equation, and then I get the radius of these electrons in that magnetic field, and that radius turns out then to be about 4.3 centimeters.

And that is a little smaller than the sphere that I have.

How can I create a uniform magnetic field of that magnitude, that's indeed perpendicular to the orbit of the electrons?

We do that using Helmholtz coils- uh, coils.

We never discussed that in detail during our lectures.

You see here coils, and you see here coils.

And when you have two sets of coils like that, and you have a proper distance between them, you can create between them a magnetic field which is almost constant.

In this case, this is the field, fifteen times larger than the Earth's magnetic field.

Perhaps some of you remember that I linked you on the 8.02 website a few weeks ago to a very nice program that I received from Professor Belcher which allowed you to calculate magnetic field configurations with current loops.

And one of the things I asked you on the website is, try to create what we call a magnetic bottle.

This is a magnetic bottle.

It's also referred to as Helmholtz coils, whereby you can create between two coils a near-uniform magnetic field.

And so that's what we have here, and I'm going to show you now how beautiful these electrons go around in circles.

They ionize the gas inside, and then through de-excitation of the ionization, you will see light.

Just the same idea that you see from Aurora.

So in that sense, you're seeing an artificial Aurora inside a glass tube.

I have to make it very dark for this, because the light is not very bright.

So we have to turn this off.

And I have to turn all this off.

And I have to turn this off.

And some of you who are close can actually see the light immediately in the, in the sphere right here in front of me.

But if not, then you will see it there on the screen.

So this is the -- let me make sure that I can have my -- so this is the electron gun here.

It has a potential difference which I set roughly at 100 volts.

And here you see these electrons going around.

And they go around roughly 22 million times per second.

And the gas pressure is low so that some of them make it all the way before they ionize the gas.

And I can change the delta V.

I now make the speed lower by making the potential difference over which I accelerate the electrons, by making that lower.

The radius is proportional to the square root of that potential difference.

So now you see here, when I increase the potential difference, the radius goes up.

See how beautiful that circle is?

It's absolutely amazing.

Very, very nice demonstration.

And then we reach the edge of the -- we are now very close to the diameter of the, of the glass sphere, so we can't go much further.

So now I want to discuss with you Faraday's Law.

Faraday's Law comes in many different ways.

Faraday's Law -- I will get up the, get the Maxwell's equations up again.

Faraday's Law -- Faraday's Law tells you that the induced EMF is $-d\phi/dt$.

$d\phi/dt$ is the magnetic flux change through an open surface attached to a loop.

The loop could be somewhere in your brains or it could be a real loop.

You can imagine any loop in space.

That is always correct, that statement.

So the EMF induced equals $-d\phi/dt$, $-d/dt$ of the integral of $B \cdot dA$, open surface.

You can think of many ways of changing the magnetic flux.

One way is a stationary loop and having a changing magnetic field.

Another way is of having B constant but changing the geometry so that the magnetic flux is changing through the surface, if you rotate it around or you move things around.

Let's look at both today.

Let's first take the stationary loop.

I spent a major part of one lecture on that.

Say this is a conducting wire and it has a resistance capital R .

This is net resistance of the entire wire.

And suppose here I have a changing magnetic field, and this area right here is A .

And the magnetic field, let's say, is perpendicular to the blackboard.

And the strength of the field is B , and so the magnetic flux ϕ of B is then simply A times B .

And so $d\phi/dt$ is then A times dB/dt .

Remember we were going to keep the geometry constant.

It's a s- it's a stationary system, but we're going to change the magnetic field.

So this magnetic field is changing.

Perhaps it's coming out of the blackboard now, and maybe it's increasing.

And so you induce then in this conducting wire an EMF, and the EMF has this magnitude.

This is the magnitude of the induced EMF.

And so the current that is going to flow, that is the induced current, is the induced EMF divided by the resistance of that whole conducting loop.

The direction is never an issue, because you all can handle Lenz's Law.

Notice I didn't even put a minus sign here, because the EMF is of course $-d\phi/dt$.

That's not important for me if you put a minus sign here, but I put these bars there, so get rid of minus signs.

Because if the magnetic field is coming out of the board and if it were increasing, then Lenz's Law will run a current in this direction to oppose that increase.

And so minus signs in general are for the birds.

You can always reason in which direction the current is going.

So that's a case whereby we have a stationary loop and whereby the magnetic field is changing but not the geometry.

Now we'll have a case whereby the magnetic field, say, is constant.

So we have here a conducting wire and we have a magnetic field, for instance, coming out of the blackboard, uniform -- uniform is always nice when we do integration to get this flux.

It's always nice to have B uniform.

And we have a bar here which has length L .

This is length L here.

And we move the bar with velocity V to the right.

So this is a very simple case.

V is here perpendicular to the bar.

That makes it always easier.

And B is perpendicular to the plane through V and L , so that makes all sines of theta that we may have, or cosines of theta, all one.

Magnetic field is constant, and so the magnetic flux, ϕ of B , is the area times the magnetic field.

The angles are just wonderful, so that is B times X times L if this distance is X .

So the magnetic flux change in time, $d\phi_B/dt$ -- notice I don't care about minus signs.

I don't need n - minus signs.

That $d\phi/dt$ is there for L times B times dX/dt , and dX/dt is the velocity.

So the magnitude of the EMF is $L B V$.

And so the current that is going to flow, the induced current, is that value, $L B V$, divided by the resistance R , and that is the resistance, then, that is in this entire loop.

Whatever there is and where it is I don't care, but that is the magnitude of the current.

The direction is again easy, it's non-negotiable.

If I move this bar to the right, the magnetic flux is increasing.

It's pointing in this direction, the magnetic field, so the current is going to flow in such a direction that it opposes that change, and so the current will flow in this direction.

That's the induced current, and this is the magnitude.

In 1996, NASA attached a twenty kilometer conducting wire called a tether to the shuttle, so L was twenty kilometers.

The magnetic field of the Earth is about half a gauss.

Even at a distance of 200 miles, that is not much different from here.

So that is 5 times 10^{-5} tesla.

And the shuttle, as you should know from 8.01, like any near-Earth satellite, they all fly with a speed of about 8 kilometers per second.

If they go much faster, then they will leave the gravitational field of the Earth.

So V is about, in circular orbit, is about 8 kilometers per second.

So here you have it in meters per second.

If I calculate for this tether, moved around, if I calculate $L B V$, I would get 8 kilovolts.

However, keep in mind that that is only correct if B were perpendicular to the plane through $L V$, and also if the velocity of the shuttle were perpendicular to the direction of the wire.

None of that is the case.

The magnetic field where they were is not exactly perpendicular to the plane through V and L .

And so what they observed was 3.5 kilovolts, about half of the maximum that you could achieve.

You may say, gee, this is strange, because if you just drag a conducting wire through space, you don't have a closed loop circuit, so you can't talk about the idea of a magnetic flux, because you have no surface.

Well, at the altitude where the shuttle is flying, there is still a teeny-weeny little bit of air, very little but some, and that is highly ionized because of the ultraviolet light from the sun.

We call that the ionosphere.

It's a plasma.

And so in the surrounding of this wire, here and here, you have a conducting medium.

And so current can flow through that medium, this way or this way, and that's exactly what will happen.

So that you don't know precisely the path.

Current will flow, so you do have closed loops.

And so it is meaningful to talk about magnetic flux change and about the EMF, the induced EMF that is generated as a result of that.

The thing that NASA could not predict very well was the current, because you do not quite know what the resistance is of that closed loop.

You know what the resistance is of the cable, but you don't know what the resistance is of the currents as they flow through the ionosphere.

But the net result was that they had a current I in their wire of about one ampere.

It was very tragic, because this current was so high that the conducting wire melted, and the tether broke off, and very early on in that experiment, the tether separated from the shuttle.

But this is a marvelous example of where you have motional EMF in space.

I want you to think about it at home.

Where does the energy come from?

Because you're generating current.

You could have lit a light bulb.

The energy must come from somewhere.

Think about that.

Conceptually interesting question.

By the way, I linked you on the website to the tether.

All you have to do is click on it and you will get some more information on this incredible experiment.

All right.

Let's continue with the most important contribution, Faraday to our economy.

And that is the situation whereby we rotate a loop, a current loop, conducting loop -- could be rectangular, could be circular -- and we rotate it round with angular frequency ω .

And for simplicity, let's have the magnetic field just straight up.

This is supposed to be three dimensional.

That's my idea.

And let this side be length A and this B .

So if you look at it from this direction, you will only see the line.

B here is rotating like this, with angular velocity ω .

And a little later in time would be here.

And so this length here is B .

And so this length is $B \cos \theta$.

And let's assume -- we call this angle θ -- let's assume that it is here at T equals 0 when θ is 0.

I can choose that arbitrarily any way I want to, of course.

And there could be a resistance R in this entire loop.

And so the magnetic flux, ϕ of B , through a surface attached to this loop is $B \cdot A \cos \theta$ -- that's constant, that's not changing -- and we have A times B , but remember since it is a dot product between B dot dA then you get a cosine θ in there, so you get A times B times cosine θ .

So that's the flux.

But cosine θ is changing with time.

θ is ωT , constant angular frequency ω , so the magnetic flux is $A B \cos \omega T$.

So $d\phi/dT$, Take the derivative, I get an ω , I get A , I get B , I get B , I get sine of ωT .

If you're really interested in that minus sign, you get that automatically, there's nothing I can do about it.

You get a minus sign there, but I'm not too much interested in minus signs.

But since I get it, I can't shove it under the rug.

So I put a minus sign here and I change that into a $+1$.

The advantage is then that I have immediately the induced EMF.

Notice that the induced EMF itself is linearly proportional with ω .

And so my induced current that is going to run in that loop is the induced EMF divided by the resistance of that loop.

And so if the induced EMF is proportional to ω , so is the induced current.

So if you go faster around, you get a higher current for the same resistor.

You built motors, and during the prize ceremony I told you that the faster your motor rotates, the larger an induced EMF is which is generated in the loop.

You had a magnetic field which let's assume is more or less constant.

Of course, it wasn't in your case, but let's assume for the sake of argument.

And as your motor was going to rotate, there was an induced current generated by Mr. Faraday, which is proportional to ω , so the current is larger when your motor goes faster.

And that induced current opposes the current which was produced by your battery.

And perhaps you remember that I did a demonstration with the winning motor.

I blocked the rotor and I showed you that the current through the rotor when it was blocked, when ω was 0, was 1.6 amperes.

Simply Ohm's Law.

Battery, voltage V , resistance R , the current is V divided by R .

Then we ran the motor, and the current went down enormously, by a factor of 40.

It was only 40 milliamperes, the time-averaged current, when the motor was running.

The reason is what you see here.

You get an induced EMF, an induced current, which opposes the current from the battery.

L R circuits.

I prefer to stay on this center board, although I could go there, but I don't think we need these anymore.

Can I cover this?

Most of you are happy with that.

I could work there, but I prefer to stay at the center.

L R circuits.

With L R circuits, we have the embarrassment -- not for us but for some others -- that almost all textbooks, college physics, do not understand Faraday's Law, and therefore they treat the subject incorrectly.

Embarrassing, but of course since they know the answer, their answers are correct.

But their physics is totally wrong.

I will address a problem which I found on, on the website when I looked at Professor Belcher's exams.

He has one very nice problem of an L R circuit.

And I will go through that with you.

We have an AC power supply, V equals $V_0 \cos \omega T$.

The frequency is 60 hertz.

It just comes out of the wall.

And so ω , which is $2\pi F$, is about 377 radians per second.

And V_0 in this case, 100 volts.

And we have here a self-inductor, and here we have a light bulb.

And the light bulb has a resistance which is 100 ohms.

And this self-inductor is variable.

We don't know yet how you make a variable self-inductor, but we will learn that very shortly, either Friday or next week, I forgot.

This self-inductance can be increased over a certain range.

And the first question that Professor Belcher is asking is, what is the energy dissipation in this light bulb?

The closed loop integral of $\mathbf{E} \cdot d\mathbf{L}$ is not 0, because Kirchhoff's loop rule does not apply here.

There is a self-inductor.

If you attach an open surface to this closed loop, there is a magnetic flux going through there, so you must deal with the third equation there.

You must deal with Faraday's Law, no matter what your textbooks tell you.

Kirchhoff's loop rule does not hold.

And if you do that correctly, you end up with the right differential equation, which happens to be the same one that those people find who do not understand the physics, but they get the same equation.

They massage things in such a way that they get the same answer.

And the I- answer then is that the current that is going to flow as a result of this voltage, variable voltage, that current I is a maximum value times the cosine $\omega T - \phi$.

That the maximum current itself is V_0 divided by the square root of $R^2 + \omega L^2$.

And that the tangent of ϕ is ωL divided by R .

I spent quite some time on this during one of my lectures.

And so the maximum current that will flow depends on ω and depends on L , on the self-inductor -- of course on the resistor as well.

In Professor Belcher's problem, he starts off with $L = 0$, and he asks you what is the power dissipated in this light bulb.

If $L = 0$, this term is not there, and so you simply have Ohm's Law, $I = V_0$ divided by R .

So I_{\max} is $100/100$, is 1 ampere.

But then he asks you something that would give you the hiccups, and that is what is now the time-averaged power dissipated in that light bulb?

And you will remember, or should remember, that one half $I^2 R$ is the power dissipated in the light bulb if I is the current through the light bulb and R is the resistance of the light bulb.

And we will assume that the resistance is independent of the current, independent of temperature.

But I is changing with time, in a sinusoidal fashion.

And so now you have to be able to evaluate the time average of a cosine squared ωT function.

And the time average of a cosine squared or sine squared ωT function is always one half.

You'd like to remember that, rather than spending five minutes on deriving that.

And so this, time averaged, is then going to be one half times I_{max}^2 times R -- and by the way, sorry, the -- I was, I was ahead of myself with my, my -- $I^2 R$ is the energy that is dissipated, right, in the, in the resistor, not one half $I^2 R$.

It is $I^2 R$.

And because of the time-average of the cosine squared function I get my one half there.

And so it's very easy now.

I_{max} is 1 ampere, R is 100, and this factor of one half, which comes from the time average of the square of this function -- the ϕ has nothing to do with that -- the average of \cos^2 of this function is one half, and you get 50 watts.

And now he calls this device a light dimmer.

Now he is going to increase L to 300 millihenries.

So L now becomes 300 millihenry.

At 300 millihenry, ωL is 113 ohms.

I multiply 300 by the 377.

And now I find that this time-averaged value, which is now one half times this, I find that I_{max} -- I put in for ωL 113 squared, 100 squared, I_{max} is now 0.67 amperes.

I put that in this equation and I find that the power is now about 22 watts.

So that's what a light dimmer is doing for you.

So you turn up the self-inductance, and now your light is dimmer.

Now a very deep, very deep, maybe nasty, conceptual question.

Why would one want to build a light dimmer with a self-inductor?

Why not put in here a variable resistor and then turning up the resistor so that the current, which, when the resistor is 0, the current is one ampere, so you get you 50 watts, but then you increase that resistor so that the current goes down to 0.67 amperes, and then the light bulb will dissipate 22 joules per second.

Why would you not put here a variable resistor, but why a variable self-inductor?

If you can answer that question, that shows that you have a deep insight already in 8.02.

I promise you I will not ask this question Wednesday, but I may ask it on the final.

Displacement current.

Displacement current is always a little bit problematic in the sense that there are not too many problems that you can do with displacement current.

This is here this displacement current term, that Amp- that, uh, Maxwell added to Ampere's Law.

The only application at this stage in 8.02 that I can think of is the one that I hit very hard during my lectures.

You have plate capacitor disks.

They are circular plates.

And you charge the capacitor or discharge the capacitor, and you can calculate now, using this law, what the magnetic field is in between the capacitor plates.

I advise you to check your notes from that lecture or watch the lecture again on the web.

The lecture is on the web.

Then some fatherly advice when it comes to the exam itself.

I would advise you to read a problem at least twice to make sure that you fully understand the text.

If you read fast -- at least that happens to me -- you sometimes misread.

At least, I often misread.

I would then advise you to do the easiest problems first.

What may be the easiest for you may not be, be the easiest for you.

But do the easiest for you first.

If you get stuck on a problem, never spend more than ten minutes on one problem.

Immediately abandon it then, when you see that it's grinding yourself into a hole, and go to another problem.

Between now and Wednesday, I would suggest you see tutors if you need them.

You can see your instructor.

I have office hours this afternoon.

I'll make myself as much available today as I can.

Tomorrow afternoon I will be in 26-100 working on the demonstrations for you for next week, so I'm not available tomorrow.

I wish you luck, and I'll see you on Wednesday.