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8.02 Electricity and Magnetism, Spring 2002
Transcript – Lecture 12

You see here the topics the way I see them, you will get three problems to -- on the exam, and not all subjects can, of course, be represented on the exam.

Nor can I cover all of them in 50 minutes.

I will test some very basic ideas, the math will be utterly trivial, and if it becomes complicated, then you just know that you're on the wrong track.

If you get stuck, somehow, on a problem, my advice is, move on, don't stay with the problem, but move on and try some others first.

There is a reason why Gauss's Law there is in red, because Gauss's Law is, of course, extremely important in the early part of the course, the closed surface integral of E dot dA is the sum of the enclosed charge, divided by epsilon 0.

And that is so important that you can be sure that there will be one problem dealing with Gauss' Law.

Now, when you have Gauss' Law problems, there's always one of three.

You must have symmetry, you must have a special distribution of charges, because otherwise, Gauss' Law doesn't get you anywhere.

So we have spherical symmetry, we have cylindrical symmetry, and we have plane symmetry, and that's all there is.

So you're going to get one of those three.

I will do one now, you may choose.

We're going to have a vote.

One is a possibility, I do one on spherical symmetry, another one I do on cylindrical symmetry, or I do one on slab symmetry.

Who wants the spherical symmetry?

Hands.

Who wants the cylindrical symmetry?

Way more hands.

Who wants plane?

I think the cylinders have it.

But if you're clever, you can stay for the next lecture, and then you can try to get the other one.

We need a little bit of fun today.

And therefore, I want to introduce you first to something very special, which is close to my heart, it is a secret top, you're going to see it there, and that secret top, I'm going to spin, and if you're a believer in 8.01, which you should be by now, then we will -- should be able to predict that that stop cannot spin for very long, there is friction with the air and friction with the surface, and so chances are it will soon fall over.

We'll take a look at it later, again.

So let's now start our first problem, and that is a cylindrical symmetry.

Well, we have a cylinder -- and here is the cylinder -- it's very long, has radius R, and I have uniform charge distribution throughout the whole cylinder, and the density is rho coulombs per cubic meter.

Uniformly distributed through the cylinder.

I want to know what the electric field inside the cylinder is and outside the cylinder.

Let's first do outside the cylinder.

The gauss surface, clearly, is going to be itself a cylinder, there it goes -- you can give it any random length, L, cannot have any effect on the answer -- and so the end is flat, perpendicular to the axis of symmetry, and this front part is flat, and this is curved.

I give this a radius little r, and so I know that everywhere on the surface of that cylinder outside, that the electric field must be the same everywhere because the distance is the same, that's the symmetry argument.

Electric field cannot be any stronger here than it is there, if I'm on that surface.

Symmetry argument number one.

Symmetry argument number two is, given the fact that this is a cylinder, the electric field must everywhere be perpendicular to this axis, coming out -- I call it radially, but, of course, it is not radially, like a sphere -- it's radially coming out of this surface, always perpendicular to this axis of symmetry.

Nature could not decide any other way.

That's the second symmetry argument.

One you recognize that argument, the electric flux through this flat surface and through that flat surface must be 0.

Because then, the electric field and the local dA vector, which is the perpendicular to the surface, make angles of 90 degrees with each other, because E would be like this here, but dA is in the direction of the axis of symmetry.

So no flux can, therefore, get out here and get out here.

But only through this curved surface.

But on this curved surface, if it is a positive charge, then the E vector and the dA are in the same direction, if it is a negative charge, they are in opposite directions.

Later, you can change the sign of rho, let's just make it positive for now.

So if, now, I apply Gauss' Law, then I only have to take this surface into account and not these two end pieces.

And so I need to know, now, what this surface is, because E and dA are always in the same direction everywhere, thus the cosine of the angle between them is plus 1.

And so what is the surface area?

That is going to be L times 2 pi little r, and then the electric vector is everywhere, the same there, this was our symmetry argument, and that is now the charge inside this cylinder, divided by epsilon 0.

But, of course, the charge inside the cylinder, that's only the portion that is in this inner cylinder, and so that has also, then, length L.

The cross-section here is pi R squared, so this is the volume of the charge that I have inside my Gaussian surface, I must multiply by rho, that gives it a charge, and I divide by epsilon 0.

And of course, the L cancels, as it always does, and the pi cancels here, too, and so I find that the electric field equals R squared times rho divided by 2 epsilon 0 r, and if you want to see it vectorially, you can put an r roof there, r roof, then, would be a vector which is perpendicular to the axis -- I mentioned earlier, I called that radially outwards.

So this is the electric field outside the cylinder.

R squared rho 2 epsilon 0 r.

So it falls off as 1 over r.

So now I want to know what it is inside the cylinder.

So now I go to r, less than equal to R.

So it's clear that what I do now, I'm going to have a Gaussian surface which, again, is a cylinder, has length L, and it has, again, two flat pieces at the end, so no flux will go through those two pieces, so my first term of Gauss' Law is going to be the same, I have L times 2 pi little r, because the radius of this inner circle is also r, L 2 pi r times the electric field, the arguments are identical -- but now, there is less charge inside my Gaussian surface.

Uh, the volume is L, now times pi little r squared, and then I get rho to convert it to charge, divided by epsilon 0.

I lose my L, as I always do, my pi goes, and so now I get E equals -- there is an r here, and there is an r squared here, an so I only end up with one r, divided by 2 epsilon 0, and if you like that vector notation, you can always do this.

And of course, if rho were negative, then automatically, you see, if you put a negative charge density in here, then the E field flips over, so that's automatically taken into account both here and there.

So let's take a look at it, I'm quite happy with that.

If you substitute little r equals capital R, you are right at the surface of your cylinder, then you get the same answer in both cases.

Substitute R, capital R in here, then the magnitude of E -- don't worry about the direction now -- is rho capital R divided by 2 epsilon 0, and if you put in here for this little r, capital R, you find exactly the same answer.

So we can now make a plot of the electric field as a function of distance R, here being capital R, and here being the electric field strength.

It's a linear line, 0 here, it goes up to a certain maximum, and then it falls off as 1 over r.

And this value here is this value.

That's where little r is capital R.

It is obvious and pleasing that the electric field, on the axis itself, where little r is 0, that that electric field is 0, that is something that you could have predicted almost without any knowledge, because you have symmetry all around it, there is charge on the left, there is charge on the -- on the right, there's charge on north and south, and the electric fields right at the center, of course, all pair each other out, so you get no electric field right at the center.

If the charge, for some reason, would all be at the outer surface, if it were a solid conductor that would be the case, then the electric field would be 0 everywhere inside, and this would be unchanged, assuming then, that you have the same amount of charge on the outside per unit length as you now have on the inside.

So that is cylindrical symmetry.

I am dying to take a look at my top.

I am really -- and much to my shock, do I see that this top -- is still rotating.

So maybe I have to come to the conclusion that there is something wrong with 8.01.

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There must be a layer deeper than 8.01 -- there is friction, and yet, this top doesn't come to a halt.

And so that layer deeper -- maybe that layer deeper is 8.02.

Give that some thought, it may add to your sleepless nights.

We'll visit it later, because maybe it will come to a stop.

Very well.

Let's now do something very different.

I have two conducting flat plates for the plane plate capacitor, and it has a certain thickness, the material is a conducting material, has a certain thickness, it's very large in size.

Way bigger is the plane linearly than the separation.

And let this separation be little d, and I charge the upper plate with a positive charge so I get here surface charge density plus sigma, and here minus sigma, area is A, of both plates.

I know that in the conductor itself, there is no current, and therefore, in the conductor itself, the electric field must be 0.

I have an electric field between the two plates, which can be derived using Gauss's Law, but we have already used up our time on Gauss's Law, which is sigma divided by epsilon 0.

And the electric field outside these two plates is very close to 0 here, and also very close to 0 here.

We find that from the superposition principle -- because this plate, which is negatively charged, will add to the electric field pointing down, and as you perhaps remember, that that is independent of distance.

Well, provided that you are not so far away from the plates that the dimension of the plates is going to interfere.

The -- if the plate is 10 by 10 meters, then it's fine as long as you are, say, within a few meters.

But it you go 100 meters away, then it's not true anymore.

So if we assume that we don't go too far out, then the electric field due to this one pointing down, due to this one is pointing up, and they have equal strength, they are independent of distance, so they cancel each other here, and they cancel each other there, superposition argument.

Let this point be P inside the conductor, and this point be S, and the first thing I would ask you, for instance, is what is VP minus VS?

It's the potential difference over this capacitor, if you want to call it a capacitor.

That is the integral, in going from P to S, of E dot dL.

For reasons that I have never understood, your book will switch these around, and put here a minus sign, which is, of course, exactly the same thing.

And so now, we can calculate the potential difference.

If I am here, and I'm going to walk down -- suppose I walk down upon a straight line, so dL is in the same direction as E, then it's immediately obvious that this is simply E times that distance D, because E and dL are in the same direction.

So the cosine of the angle between them is plus 1.

And so I find that this is E, which is, um, sigma divided by epsilon 0, times that distance D.

If I had chosen another route, I would have found the same answer, because we're dealing here with conservative fields, so the path does not matter.

So as long as you go from this plate to this plate, that integral is always E times D.

What is the potential difference between point P and point T here?

VP minus VT, it's clear that that is 0, because there is no electric field here, and there is no electric field anywhere there, so the integral, obviously, is 0.

What is the capacitance of this plate?

The capacitance is the charge on one plate, divided by the potential difference, which I just use the word V now, it means it is this value VP minus VS.

So what is the charge on one of the plates?

It doesn't matter which one you take, that is sigma times A.

That's the definition of sigma, right?

It's charge per unit area.

So that's the charge on the plate, and the potential difference we just calculated, that is sigma D divided by epsilon 0, so the epsilon 0 comes upstairs, and so we find, now, that it is A times epsilon 0, divided by D.

Notice it's independent of sigma, of course.

Capacitance is geometry, it has nothing to do with how much charge you have on the capacitor.

I could ask you what is the electrostatic potential energy.

The electrostatic potential energy is the work that you would have to do to assemble the positive charges here and the negative charges there.

You could also look at it at the energy that it takes to create that electric field.

Same question.

So this work that has to be done to assemble it is the charge on one plate times the potential difference times one-half, or -- which is the same -- one-half C V squared.

So, what is one-half Q V?

Let's first take this one, there is one-half, Q is sigma times A, the potential difference V, we have, is here, sigma D divided by epsilon 0, so this is the answer that we will find from this one, and the answer that we find from the other one must, of course, be the same.

Let's check that.

This is C, it is A epsilon 0 divided by D, and now I must multiply by the potential difference squared, so I get a sigma squared, I get a D squared, and I get an epsilon 0 squared, and these two better be the same.

I have sigma sigma here, sigma squared, I have D squared divided by D, so I have only one D, I have an epsilon z- 0 squared here and here, an epsilon, so I have only 1 over epsilon 0, so they are, indeed, the same.

Now I could ask you where is the charge located?

Let's first go to the top plate, where is the charge located on the upper plate?

Some of you may say, "Oh, well, maybe the charge is in the plate, somewhere here.

That cannot be.

I make a Gaussian surface all around that charge.

Gauss's Law will tell me, then, that the surface, closed surface integral of E dot dA is not 0, because there is charge inside, and if there is charge inside, that closed surface integral is not 0 -- but we know that the electric field must be 0 everywhere, 0 in the conductor, everywhere it must be 0, so the closed surface integral must be 0.

So there cannot be any charge there.

Simple argument.

Some of you may say, "OK, maybe some of the charge here is at the top surface.

Not allowed either, in this configuration.

I make myself a small pillbox, which is my Gauss surface, these are flat ends.

Electric field is 0 here, electric field is 0 there, so the surface, closed surface integral must be 0 because the electric field is 0 everywhere, but there is charge inside the pillbox, and so Gauss's Law says that it cannot be 0.

Since it is 0, there's no charge inside.

And so there's only one solution, nature puts all the positive charge right here at the bottom of this plate, and the negative charge right there at the top of that plate.

That's the only solution in this case.

Charge cannot be anywhere else.

I can't believe it.

That thing is still running.

I want you to take a look at that.

You see that top is still happily running, so either this has to be a violation of the conservation of energy somehow, or it is black magic -- it can never be excluded in 26-100 -- or, perhaps, there is some simple physics behind it.

And whatever that simple physics is, I would like you to start thinking about.

All right.

Next subject.

Again, let me come with a -- with two plates, because I want to start talking about dielectrics, and I want to massage this idea of capacitance a little further.

I have here a parallel plate capacitor.

And I put on here, positive charge, plus sigma, I call it now, sigma free, there is no dielectric yet, but, later, there will be, so I call it sigma free, and here is minus sigma free, separation is D, surface area is A.

So in the beginning, it's going to be boring, we know that the electric field here is going in this direction, and that electric field is sigma free divided by epsilon 0.

The free goes with the sigma.

I charge it up using a power supply, and now -- and this is crucial -- I disconnect the power supply.

I take the leads off.

So I disconnect the power supply.

That means -- and this is crucial -- that whatever comes that this charge is trapped, can never change.

No matter what we're going to do.

Power supply has been disconnected, that sigma free is trapped.

I'm going to do various things now.

I'm going to change the distance between the plates, and then independently, I'm going to shove in a dielectric, we will do that separately, one at a time.

And so the equations that I can now trust, and that I will be looking at, are the following.

The -- the free charge that I have, Q free, is obviously sigma free times the area.

And that is the definition of surface charge density.

So I can trust that one.

The electric field between the plates is sigma free divided by epsilon 0, and now I get a kappa there, if we have a dielectric.

The potential difference between the plates is E D.

Provided I know the E, this is the E, it's always E D.

We just had that in our previous problem.

The capacitance itself, C, is the free charge on one plate divided by the potential difference between the plates, V is my potential difference, that is this V, and the electrostatic potential energy equals one-half Q free times V, but it is also one-half C V squared.

And so keep these in mind in what follows.

Take a look at them, the first one is correct, second one is correct, third one is correct, that one is correct, that one I can also live with.

Or you could write down, for the capacitance, if you wanted that, you can write down A times epsilon 0 divided by D, times kappa.

The first thing I'm going to do with the power supply disconnected, I'm going to increase the distance D between the plates.

And I'm going to increase them by a distance -- I'm going to double the distance.

So D goes up by a factor of 2.

But kappa remains 1.

Just air.

No dielectric yet.

What happened with the electric field?

E.

E can not change, because sigma free cannot change, kappa is 1 -- there's no kappa -- and if this cannot change, this cannot change.

So, as I move the plates apart, there is no change in the electric field.

Non-intuitive as that may be for you, the electric field remains a constant.

So what happens now with the potential difference between the plates?

That, now, must increase by a factor of 2, because if I increase D by a factor of 2, and if E is doing nothing, then V must go up by a factor of 2.

So V must go up by a factor of 2.

And I did a demonstration here, during one of my lectures, whereby I changed D from 1 millimeter to 10 millimeters, and I changed the potential difference from 1000 volts to 10000 volts, you've seen it in front of your own eyes, if you were here.

So, indeed, when you separate the plates with the power supply disconnected, the potential difference goes up.

What happens with the capacitance?

Well, the capacitance is Q free divided by V.

There's an R missing here.

This one doesn't change.

This one goes up by a factor of 2, so C must go down by a factor of 2.

What happens with the electrostatic potential energy?

Well, it is one-half Q free times V, Q free cannot change, V went up by a factor of 2, so U must go up by a factor of 2.

Remember that when I separated these plates and increased the potential difference, I told you I was doing work.

U is increasing.

If I move the plates apart, I have to do that work.

All right.

So this is the first part, whereby we change D.

Now, I go back to D, leave it as it was, and now I want to change kappa.

I'm going to move in a dielectric.

I'd like to stay working on the center board -- I have to change, have to -- can't see this any more.

So now D is as it was before, but now kappa becomes 3.

So I take dielectric and I shove it in, and sigma free is fixed, and so now, what happens with E?

Well, sigma free is fixed.

If kappa, all of a sudden, becomes 3, [krrk] E field goes down.

Is that surprising?

No, that is not surprising, because as you move in the dielectric, the -- this surface charge density is not going to change, but you are inducing now, on your dielectric, negative charge here and positive charge here as a result of that external electric field, and so that creates an induced electric field in this direction, and so, as a result of that, the net electric field goes down.

And that's what you see here, it goes down, in this case, by a factor of 3.

What happens with the potential difference over the plates?

Well, D wasn't changing, remember?

We kept D constant now.

So if E goes down by a factor of 3, V must go down by a factor of 3.

What happens with the capacitance, C?

Well, the capacitor is free charge divided by the potential difference.

The free charge is not changing, it's trapped.

The potential difference goes down by a factor of 3, capacitance goes up by a factor of 3.

What happens with the electrostatic potential energy?

Well, the electrostatic potential energy is one-half Q V.

But Q free could not change.

V went down by a factor of 3.

So U must go down by a factor of 3.

That means if the electrostatic potential energy goes down, that as I move in this dielectric, that I do negative work.

If I had to push it in, U would have gone up.

So, in a way, as I move the dielectric in, it's being sucked in.

There is a force that pulls it in.

Interesting all by itself.

I would like you, at home, to go to ex- through exactly the same questions, verbatim, with one difference.

And that is, you keep the power supply connected.

Now your answers are going to be very different.

For one thing, if the power supply is connected, and if you change D -- so your power supply is connected, and you go up in D by a factor of 2 -- the power supply is connected, there is one thing now that cannot change throughout, and that is V.

Potential difference cannot change, because the power supply is connected.

So now, if V cannot change, and you increase D by a factor of 2, E, now, must go down by a factor of 2.

And that's very different from what happened before, when E remained constant.

So it's very, very different physics.

Well, the physics is the same, but the results are very different.

And I want you do that, you have all the tools now, you can believe in those equations, and they should work for you.

All right, let's visit Ohm's Law and maybe look at Kirchhoff -- I prefer to stay on this center board, convenient for you, and it is also convenient for me.

A very simple network, keep in mind that, on an exam, all problems are extremely simple and very fundamental.

Nothing complicated.

You don't have the time for that.

I give here a problem in which I actually give numbers, on the exam you won't see any numbers, even, not in the sense of resistances, ohms, and so on, because there is no calculator necessary.

But here, you will see some numbers, this is a battery and this battery has an EMF, which is 10 volts.

That's a given.

Plus, minus.

And here, the current is going to split into three.

There is R1, which is 1 ohm, R2, which is 2 ohms, and then we have R3 -- put it a little lower -- R3 is 3 ohms, they come together here.

And here I have a resistor R4, which is 4 ohms, and I close the loop and go back to my battery.

Just to make it a little bit more interesting, I will introduce into this battery an internal resistance which is very small, which is 0.1 ohms.

You can't remove it, it's intrinsic into that battery.

And so the first question that I would ask you in this case is, what is the total current that is going to flow?

We're going to get a current I here.

Through here you get I1, through here you get I2, through here you get I3, I comes out here, I goes through here, through the fourth, come back, and I goes through the battery.

So what is I?

In a problem like this, there are many roads to success.

Not just one.

And it's a matter of taste which one you prefer.

If I call this point A and I call this point D, then what I would do, I would ask myself the question, if this is point A, and this is point D, what resistor -- which your book calls the equivalent resistor -- the equivalent resistance -- what resistor would I have to put here, instead of these three, for the current I to be exactly the same as what it is now?

So I'm going to replace these three by one resistor, which is this imaginary resistor.

As you have noticed in your book, where you undoubtedly read up on, 1 over R equivalent is 1 over R1 plus 1 over R2 plus 1 over R3.

You know all these numbers, and so you will find R equivalent is 0.55 ohms.

Check this at home, and I hope I didn't goof up on that one.

Notice that this resistor, this equivalent resistance, is smaller than the smallest one, which is 1 ohm.

That's obvious.

It has to be that way.

Think of these as water flows.

Water flow through this one, through this one, and through this one.

Remove these two.

Water is only flowing through this one.

Now you add these two pipes so more water can flow, so the equivalent resistance goes down.

Same with electricity.

So the equivalent resistance of parallel resistors is always lower than the smallest.

Now it's trivial to calculate the current I, I use Ohm's Law.

Ohm's Law says that potential difference that is available b- by the battery is E, it's 10 volts, is now the current, the total current times all resistances along the road.

I go once around, I have here R e- equivalent, then I have R4, because the full current goes through R4, and then I have this little stinky R of I.

Doesn't going to make much difference, but it's there.

And so you can find, now, what I is, because you know all other numbers, and you'll find that what I -- I is my goal, and I think I found 2.15 amperes, that is right.

2.15 amperes.

So we know what I is.

Is this the only way?

No.

But it is one way, it's very effective.

So now I want to know what I1, I2, and I3 is.

Well, if I know the potential difference between A and D, VA minus VD, that must be, according to Ohm's Law, I1 R1.

If I go this route.

But since we're dealing with conservative forces, I can also go through this path, the path doesn't matter, I must get the same potential difference.

So it's also I2 times R2, and so it must also be I3 times R3.

So if I only could find I2, then, of course, I would know the potential difference, out pops, immediately, I1 and out pops, immediately, I3.

And now I'm going to apply Kirchhoff's First Rule in order to find I2.

I could find I1, but I just decided on I2.

And Kirchhoff's First Rule says that the closed loop integral of E dot dL -- this, now, is a closed loop -- is 0.

In the future, you will see situations that it's not 0.

Here, 0.

I don't know why Kirchhoff got the credit for this, this was long known before him, but nevertheless, it's called Kirchhoff's First Rule.

So I go with closed path, and this is the closed path that I have decided to take.

And that closed loop integral E dot dL must be 0.

Any other path would also be 0.

I chose the one through R2, because my goal is to find I2.

Once I have I2, I1 and I3 follow immediately from this.

I have decided, very arbitrarily, that if I go down in potential, then I will call that -- I will give that a negative sign, and if I go down in potential, I will give that a plus sign.

You can reverse that.

That makes no difference, because the sum of them is going to be 0 anyhow.

So I will stick to my convention for now, that if I go down in potential, I will give it a minus sign, if I go up in potential, plus sign.

I go, first, from A to D, through R2.

The current is I2.

The resistance is R2.

And I go down in potential.

So I get my first term is minus I2 times R2.

I'm now at D.

I started at A, I'm now at D.

I went through 2.

I come out here, and I go through 4, R4.

I go down in potential.

The current through R4 is I.

So I get minus I R4.

I go up, and I see this battery in front of me, and I have to climb up in potential.

How much do I have to climb up?

That EMF, of the battery.

But that dinky toy little resistance R of I makes me go down a little bit, and so I get another minus I times R of I, and that, now, is 0.

And that's one equation with only one unknown, which is I2, because we already have I.

That's the 2.15 amperes.

And so you'll find, now, that I2 becomes 0.6 amperes.

And so you know now that VA minus VD, the potential difference, which was I2 R2, is now going to be 1.2 volts.

Because I2 is 0.6 amperes, but R2 -- there's a 2 here -- is 2 ohms.

So it's 1.2 volts.

And so the 1.2 volts is also I1 R1, and it's also I3 R3, so you get I1 and you get I3.

What is the power delivered by this battery?

That is the EMF times the total current I.

We know the EMF, 10 volts.

The total current is 2.15 amperes.

So this is 21.5 watts.

How does that show up, that energy?

Well, it comes out in the form of heat.

Heat in R4, heat in R1, R2, and R3, and a teeny, weeny little bit of heat inside that battery because of that 0.1 ohm internal resistance.

How much power comes out in resistance R2?

Well, that is, of course, the potential difference over R2, which was that VA minus VD times the current through R2.

That's power, power is potential difference times current.

This is the total power delivered by the battery.

That is the total potential difference available times the total current.

But of course, R2 only sees a potential difference which is 1.2 volts, and it has an I2 which is only 0.6 amperes, so this is only 0.72 watts.

So that is the number of joules per second, in terms of heat, that is produced in R2.

I think you'll believe me when I say that the top is still running.

And the clue I will give you is that the answer lies in 8.02.

Give it some thought, it's a very cute top.

All right, let's talk about kinetic energy increase due to charges that move over a potential difference.

I have two conductors, very funny in shape, but they are equipotentials, there's no current running inside the conductors.

And so conductor A is at a potential V of A, this is conductor A, potential is V of A, and this is conductor B, has a potential V of B, and let's assume that VA is larger than VB.

If you want to change that later, that's fine.

We put the whole thing in vacuum, because I'm going to release a charge here, plus Q, and the charge will now go to B.

Electric field configuration is a zoo.

I don't even want to think about what it is.

One way or another, if this is vacuum, than this charge will finds it way to B.

Let's say this is the routing that it takes.

It ends up on B, and the question now is, what is the speed at which it reaches B if I release it here at zero speed?

So the electric field is going to do work on this charge, and the work, in going from A to B, integral A to B, is the force dot dL.

That is the electric force on that charge.

Since it is a conservative field, it doesn't matter what your routing is, you will always get the same answer for this.

That electric force is also the force times the electric field, at any location along the line.

And so you see, you get here a Q times E dL.

But the integral of E dL is the potential difference between the two.

And so the net result, therefore, is that the work done by the electric fields, when this charge finally ends up here, that work is the charge Q times the potential difference VA minus VB, regardless of which path it chooses to go.

Let's take a practical case, we have a proton which has a mass 1.7 times 10 to the -27 kilograms.

The charge of the pro- proton is the same of that of the electron, but it is positive.

1.6 times 10 to the -19 Coulombs.

And let's suppose that the potential difference between A and B, this is now the potential difference, is a million volts.

Uh, let me put a delta here, because I don't want Vs on both sides.

But that's the difference VA minus VB.

So what is now the kinetic energy with which this proton reaches B?

So that kinetic energy must be Q times the potential difference, so that is 1.6 times 10 to the -19, times ten to the 6, so that is 1.6 times 10 to the -13 joules.

Almost no physicist would call this 1.6 times 10 to the -13 joules, but we would say, the kinetic energy of that proton is 1 MeV, 1 million electron volts.

And the reason why we do that is, an electron volt is the energy that an electron gains if it moves over a potential difference of 1 volt.

That's the definition of 1 electron volt.

Though the charge of the proton is that same of that of an electron, and it moves over a distance of 1 million volts, and so the energy is 1 million electron volts.

But this is not an SI unit, so be careful.

If you work SI, you've got to use this number.

But we would say that's a proton with a kinetic energy of 1 MeV, 1 million electron volts.

So what is the speed with which it finally, then, arrives?

Well, one-half M V squared is this number, this is the mass of the proton, this is the speed that the proton arrives at point B, and when you use that number, the mass, you will find that the velocity of that proton is about 1.4 times 10 to the 7 meters per second, which is about 5% of the speed of light.

In other words, we don't have to make relativistic corrections.

This answer is believable.

Several students have sent me e-mail, and they have asked me for practice exams.

I am equally surprised, as you are, that the previous lecturers of 8.02 did not list on the web, on their website, exams.

They didn't.

I was hoping they did, but they didn't.

Professor Belcher, however, mentioned one particular practice exam to me, and when you visit the 8.02 website, you can find that practice exam, but there are no solutions on that exam.

So you can discuss this exam with your instructors, with your tutors, I am available if you want to discuss it with me, I have no problems with that -- it's a little difficult for me to help 600 students, but I can help quite a few.

But my advice is that you take the study guide if you really want to do more practice, and go over some problems that have worked-out solutions.

And I wish you luck, and I'll see you next Friday.