Massachusetts Institute of Technology
Physics Department

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Solutions to Problem Set# 7

The solution is given for $g = 10.0m/s^2$. Of course you can use $9.8m/s^2$ and get the same credit.

Problem 1) Y&F 6-50, p189

a) Since the force $F = Mg$ with total body mass M, the work done in doing a chin-up is

$$W = F\Delta h = M\text{kg} \times 10\text{m/s}^2 \times 0.40\text{m},$$

and work per kilogram is

$$W/M = 4.0\text{J/kg}$$

b) Let the mass of the muscles involved in doing a chin-up be m. From (a) we find an equation for the total work done in doing a chin-up

$$W = M \times 4.0\text{J/kg} = m \times 70\text{J/kg} \Longrightarrow m/M = 4.0/70 = 5.7\%.$$

c) Now that the work per body mass is $W/M = 10\text{m/s}^2 \times \frac{1}{2} \times 0.40\text{m} = 2.0\text{J/kg}$, the ratio of mass of "used" muscle to total body mass is

$$m/M = 2.0/70 = 2.9\%.$$

d) Because children and adults have about the same percentage of muscle in their bodies, the problem depends on what percentage is needed in doing a chin-up. According to previous calculation a child needs a smaller percentage(2.9% in this problem) than an adult (5.7%). Thus children have advantage in doing chin-ups.

Problem 2) Y&F 6-54, p189

a) Following the usual procesure of decomposing the gravitational force into parallel ($mg \sin\theta$) and perpendicular($mg\cos\theta$) components relative to the direction of the inclined plane, the normal force between package and the ramp is balanced by, and therefore equal to in magnitude, $mg\cos\theta$. So friction $f = N\mu_k = mg\cos\theta\mu_k = 15.2\text{N}$. We find that the direction of frictional force is opposite to the direction of the package's velocity, which means the angle between them is 180^o. The work done by friction is

$$W_f = f \times 1.50\text{m}\cos 180^o = -15.2N \times 1.50m = -22.8J.$$

b) The package's height decreased by $1.50m \times \sin 12^o = 0.31m$. Work done by gravity is

$$W_g = 0.31m \times mg = 15.6N$$

c) Because it is perpendicular to the ramp at any instant, the normal force does no work on the package:

$$W_N = N \times 1.5m \times \cos 90^o = 0.$$

d) Total work $W_{tot} = W_f + W_g + W_n = -7.2N$. The work-kinetic energy theorem states that it is equal to change in the package's kinetic energy.

$$W_{tot} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \Longrightarrow v_f = 1.4\text{m/s}$$

Problem 3 Y&F 6-60, p190

It is convenient to calculate the potential energy function first. Without loss of generality the zero point of potential is at infinity $V(\infty) = 0$.

$$V(x) = -\int_{\infty}^{x} \frac{\alpha}{x^2} dx = \frac{\alpha}{x}.$$

In this problem total energy is conserved but the form of energy can be transformed between kinetic and potential energies.

a) By conservation of energy

$$E_{tot}(x = 5.00m) = E_{tot}(x = 8.00 \times 10^{-10}m) \Longrightarrow \frac{1}{2}mv_0^2 + \frac{\alpha}{5.00m} = \frac{1}{2}mv_1^2 + \frac{\alpha}{8.00 \times 10^{-10}m}$$

$$v_1 = \sqrt{v_0^2 + (\frac{\alpha}{5.00\text{m}} - \frac{\alpha}{8.00 \times 10^{-10}\text{m}})/(\frac{m}{2})} = 2.41 \times 10^5 m/s$$

b) At the closet position x_{min} the proton has all of its kinetic energy transformed into potential energy $(K = \frac{1}{2}m0^2 = 0$, $V(x)$ maximized.

$$E_{tot}(x = x_{min}) = \frac{\alpha}{x_{min}} = E_{tot}(x = 5.00m) \Longrightarrow x_{min} = 2.82 \times 10^{-10}\text{m}.$$

b) When the distance between proton and nucleus restored, there is no total work done on the proton so its speed is the same as itis moving toward the nucleus, 3.00×10^{-10}m/s

Problem 4 Y&F 6-71, p191

The only difference from problem 3 is that now the potential function is the well-known

$$V(x) = \frac{1}{2}kx^2.$$

a) At maximum compression the kinetic energy vanishes, so all energy goes into potential form.

$$E_{tot}(x_f) = \frac{1}{2}x_f^2 = \frac{1}{2}mv_0^2 \Longrightarrow x_f = 0.6\text{m}.$$

b) Again we need to solve the equation $E_{tot}(x_f) = \frac{1}{2}x_f^2 = \frac{1}{2}mv_0^2$, for v_0 this time, not for the given $x_f = 0.15$m. The solution for v_0 is

$$v_0 = 1.50\text{m/s}.$$