MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01X | Fall Term 2002

SOLUTIONS TO PROBLEM SET 12

1 Young & Friedman 11-24 Stress on a Mountaineer's Rope

A nylon rope used by mountaineers elongates 1.10 m under the weight of a 65.0-kg climber. If the rope is 45.0 m in length and 7.0 mm in diameter, what is Young's modulus for this material?

$$\begin{aligned} Y &= \frac{F_\perp}{A}\frac{L_0}{\Delta L} \\ &= \frac{(65.0\,\mathrm{kg})\left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)}{\pi\,(3.5\times10^{-3}\,\mathrm{m})^2}\frac{(45.0\,\mathrm{m})}{(1.10\,\mathrm{m})} \\ &= 6.8\times10^{8}\,\mathrm{Pa} \end{aligned}$$

2 Young & Friedman 14-17

An object of average density ρ floats at the surface of a fluid of density ρ_{fluid}.

(a) How must the two densities be related?

If $\rho < \rho_{\mathrm{fluid}}$ and the object is initially entirely submerged, then the buyant force will exceed the gravitational force on the object. It will rise to the surface and float there.

(b) In view of the answer to part (a), how can steel ships float in water?

If a piece of steel is hollow, its overall density will be less than that of solid steel. Since the *average* density of an object determines buoyancy, rather than total weight or the density of some specific part of it, a piece of steel can be hollowed out until its average density is less than that of water and it will float.

(c) In terms of ρ and ρ_{fluid}, what fraction of the object is submerged and what fraction is above the fluid? Check that your answers give the correct limiting behavior as $\rho \to \rho_{\mathrm{fluid}}$ and as $\rho \to 0$.

When floating at a constant level, the total force on the object is zero. This means that the buoyant force and the gravitational force have the same magnitude.

$$\begin{aligned} F_b &= W \\ \rho_{\mathrm{fluid}} V_{\mathrm{submerged}}\, g &= \rho V g \\ \frac{V_{\mathrm{submerged}}}{V} &= \frac{\rho}{\rho_{\mathrm{fluid}}} \end{aligned}$$

where V is the object's total volume. When the densities are the same, the object is not forced out of the fluid,

$$\lim_{\rho \to \rho_{\text{fluid}}} \frac{V_{\text{submerged}}}{V} = 1$$

and when the object is much less dense than the fluid, almost all of it will be floating,

$$\lim_{\rho \to 0} \frac{V_{\text{submerged}}}{V} = 0$$

(d) While aboard your yacht, your cousin Throckmorton cuts a rectangular piece (dimensions $5.0 \times 4.0 \times 3.0\,\text{cm}^3$) out of a life preserver and throws it into the ocean. The piece has a mass of 42 g. As it floats in the ocean, what percentage of its volume is above the surface?

The rectangular piece has a density of $\frac{42}{60}\,\frac{\text{g}}{\text{cm}^3}$, and water has by definition a density of $1\,\frac{\text{g}}{\text{cm}^3}$. Thus $\frac{42}{60} = 70\%$ of the piece is submerged, and 30% is above the surface.

3 Young & Friedman 14-34

A soft drink (mostly water) flows in a pipe at a beverage plant with a mass flow rate that would fill 220 0.355-L cans per minute. At point 2 in the pipe, the gauge pressure is 152 kPa and the cross-section area is $8.00\,\text{cm}^2$. At point 1, 1.35 m above point 2, the cross-section area is $2.00\,\text{cm}^2$.

(a) Find the mass flow rate.

Since we are dealing with constant flow, the derivative $\frac{dV}{dt}$ is the same as the discrete $\frac{\Delta V}{\Delta t}$.

$$\begin{aligned} \rho \frac{dV}{dt} &= \rho \frac{\Delta V}{\Delta t} \\ &= \left(1\,\frac{\text{kg}}{\text{L}}\right) \frac{(220)\,(0.355\,\text{L})}{(60\,\text{s})} \\ &= 1.30\,\frac{\text{kg}}{\text{s}} \end{aligned}$$

(b) Find the volume flow rate.

$$\begin{aligned} \frac{dV}{dt} &= \frac{(220)\,(0.355\,\text{L})}{(60\,\text{s})} \\ &= 1.30\,\frac{\text{L}}{\text{s}} \end{aligned}$$

(c) Find the flow speeds at points 1 and 2.

$$
\begin{aligned}
\frac{dV}{dt} &= Av \\
v &= \frac{\frac{dV}{dt}}{A} \\
v_1 &= \frac{\left(1.30\times10^{-3}\,\frac{\mathrm{m}^3}{\mathrm{s}}\right)}{(2.00\times10^{-4}\,\mathrm{m}^2)} = 6.51\,\frac{\mathrm{m}}{\mathrm{s}} \\
v_2 &= \frac{\left(1.30\times10^{-3}\,\frac{\mathrm{m}^3}{\mathrm{s}}\right)}{(8.00\times10^{-4}\,\mathrm{m}^2)} = 1.63\,\frac{\mathrm{m}}{\mathrm{s}}
\end{aligned}
$$

(d) Find the gauge pressure at point 1.

Since we have an incompressible beverage, Bernoulli's equation can be used if we also ignore the small viscosity.

$$
\begin{aligned}
p_1+\rho g y_1+\frac{1}{2}\rho v_1^2 &= p_2+\rho g y_2+\frac{1}{2}\rho v_2^2 \\
p_1 &= p_2+\rho g\,(y_2-y_1)+\frac{1}{2}\rho\left(v_2^2-v_1^2\right) \\
&= p_2+\rho\left(g\Delta y+\frac{1}{2}\left(\frac{dV}{dt}\right)^2\left(\frac{1}{A_2^2}-\frac{1}{A_1^2}\right)\right) \\
&= (152\,\mathrm{kPa})+\left(10^3\,\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(\left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)(-1.35\,\mathrm{m})+\frac{1}{2}\left(1.30\times10^{-3}\,\frac{\mathrm{m}^3}{\mathrm{s}}\right)^2\left(-\frac{15}{64}\times10^8\,\frac{1}{\mathrm{m}^4}\right)\right) \\
&= 152\,\mathrm{kPa}-13.2\,\mathrm{kPa}-19.9\,\mathrm{kPa} \\
&= 119\,\mathrm{kPa}
\end{aligned}
$$

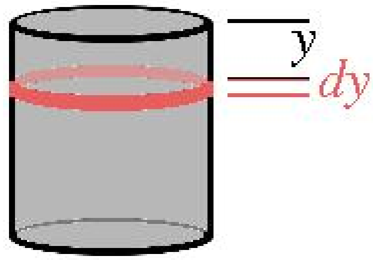
Note that (y_2-y_1) is negative because point 2 is *lower* than point 1, and $\left(v_2^2-v_1^2\right)$ is negative because the pipe is *wider* at point 2 than at point 1.

4 Young & Friedman 14-56 The Great Molasses Flood

On the afternoon of January 15, 1919, an unusually warm day in Boston, a 27.4-m high (90 ft), 27.4-m-diameter cylindrical metal tank used for storing molasses ruptured. Molasses flooded into the streets in a 9-m-deep stream, killing pedestrians and horses, and knocking down buildings. The molasses had a density of $1600\,\frac{\mathrm{kg}}{\mathrm{m}^3}$.

If the tank was full before the accident, what was the force the molasses exerted on its sides?

(*Hint*: Consider the outward force on a circular ring of the tank wall of width dy at a depth y below the surface. Integrate to find the total outward force. Assume that before the tank ruptured, the pressure at the surface of the molasses was equal to the air pressure outside the tank.)



At a depth y the molasses pressure was $p(y) = p_0 + \rho g y$, where p_0 is atmospheric pressure. If the tank had a diameter d, the area of the tank wall at y (a ring of height dy) was $dA = (\pi d)\,dy$. Force being pressure times area,

$$
\begin{aligned}
dF &= (p_0 + \rho g y)(\pi d)\,dy \\
F &= \int_0^h dy\,(p_0 + \rho g y)(\pi d) \\
&= \pi d \left(p_0 h + \frac{\rho g}{2} h^2\right) \\
&= \pi (27.4\,\mathrm{m}) \left(\left(1.01 \times 10^5\,\frac{\mathrm{N}}{\mathrm{m}^2}\right)(27.4\,\mathrm{m}) + \frac{1}{2}\left(1600\,\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)(27.4\,\mathrm{m})^2\right) \\
&= 2.38 \times 10^8\,\mathrm{N} + 5.07 \times 10^8\,\mathrm{N} \\
&= 7.45 \times 10^8\,\mathrm{N}
\end{aligned}
$$

This is the total force exerted by the molasses on the tank wall. Note that there was also an *inward* force of $\pi d\,p_0 h = 2.38 \times 10^8\,\mathrm{N}$ on the tank wall due to the air outside the tank. The net force on the wall was thus $\pi d \frac{\rho g}{2} h^2 = 5.07 \times 10^8\,\mathrm{N}$ outward.