Final Review #4

Experiments

Statistical Mechanics, Kinetic Theory Ideal Gas Thermodynamics Heat Engines Relativity

8.01t

December 8, 2004

Review of 8.01T Experiments

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Experiment 01: Introduction to DataStudio

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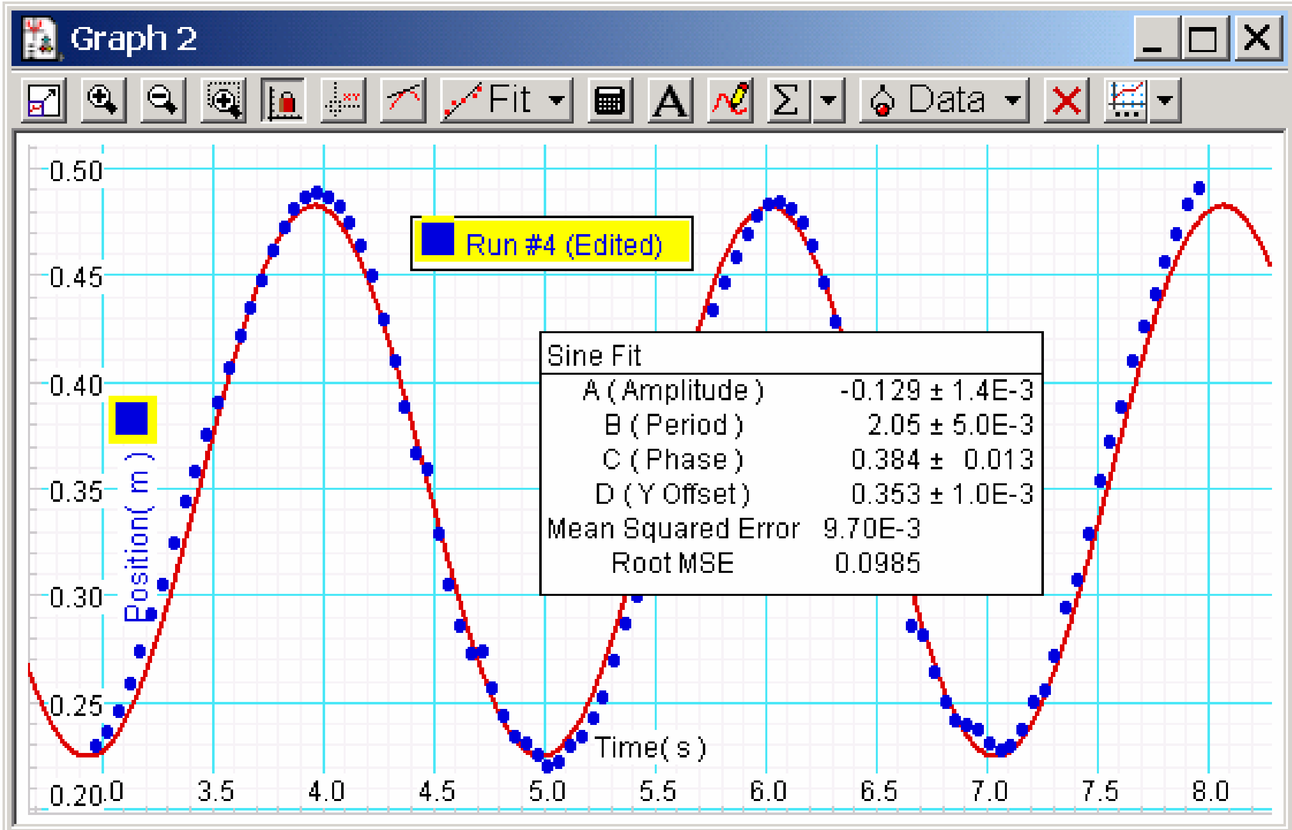
Position of hand motion



Where is the velocity zero? Where does the magnitude of the velocity reach its maximum?

Experiment 02: Projectile Motion

Reminder on projectile motion

- Horizontal motion (x) has no acceleration.
- Vertical motion (y) has acceleration –*g*.
- Horizontal and vertical motion may be treated separately and the results combined to find, for example, the trajectory or path.
- Use the kinematic equations for x and y motion:

$$x(t) = x_0 + v_0 t \cos\theta$$

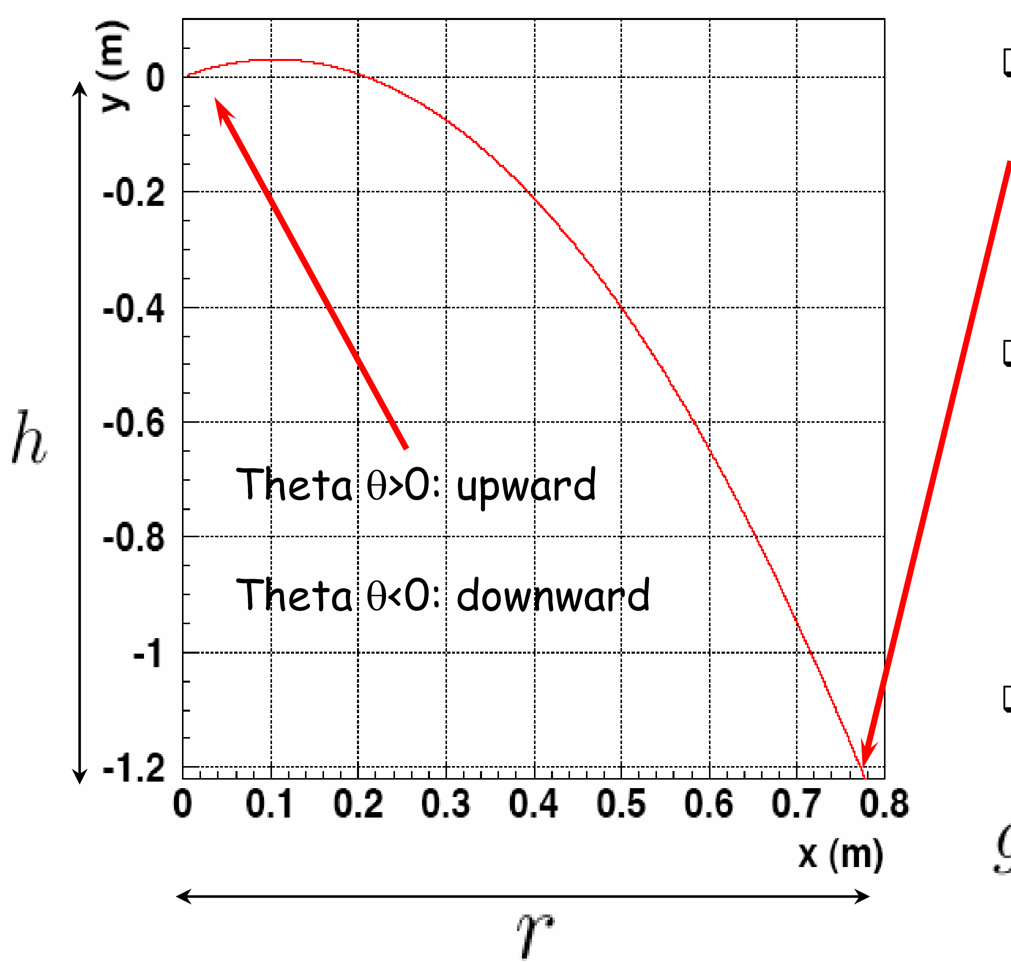
$$y(t) = y_0 + v_0 t \sin\theta - \frac{1}{2} g t^2$$

Experimental setup

- Coordinate system

$$x_0 = 0 \qquad y_0 = 0$$

$$y(x) = x\tan\theta - \frac{g}{2v_0^2\cos^2\theta}x^2$$



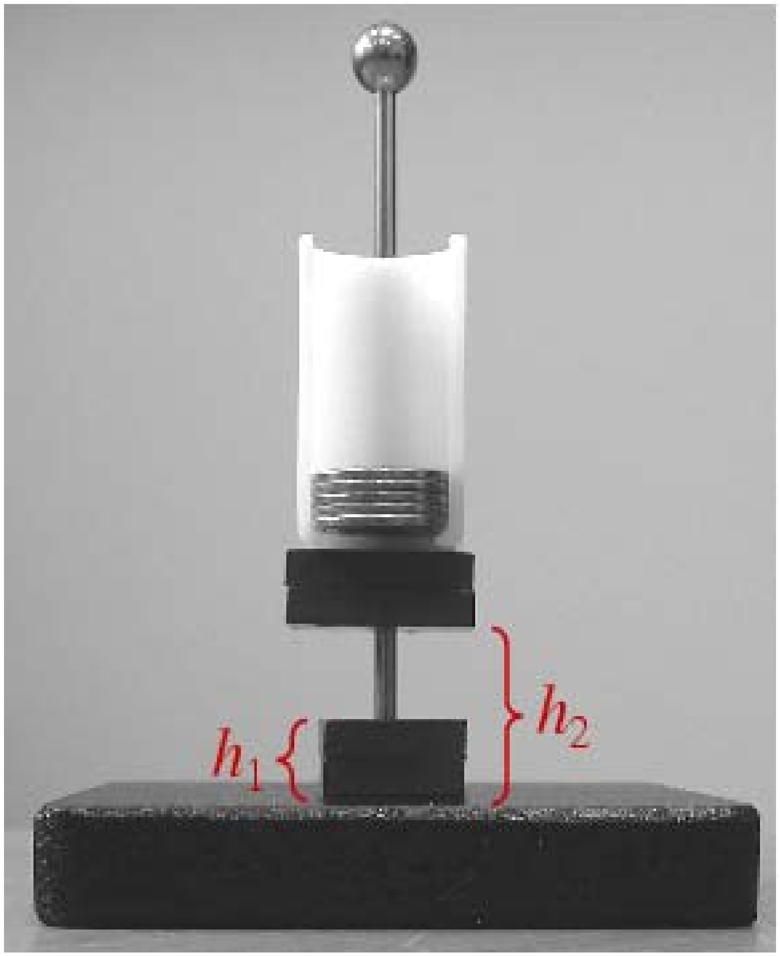
- Impact point:
  - Height: h
  - Horizontal displacement: r
- With chosen coordinate system:
  - Height: y=-h
  - Horizontal displacement: x=r
- Solve above equation for *g*:

$$g = \frac{2v_0^2\cos^2\theta}{r^2}\left[r\tan\theta + h\right]$$

Experiment 03: Modeling Forces

Experimental setup

- Measuring the magnet gap h_2-h_1



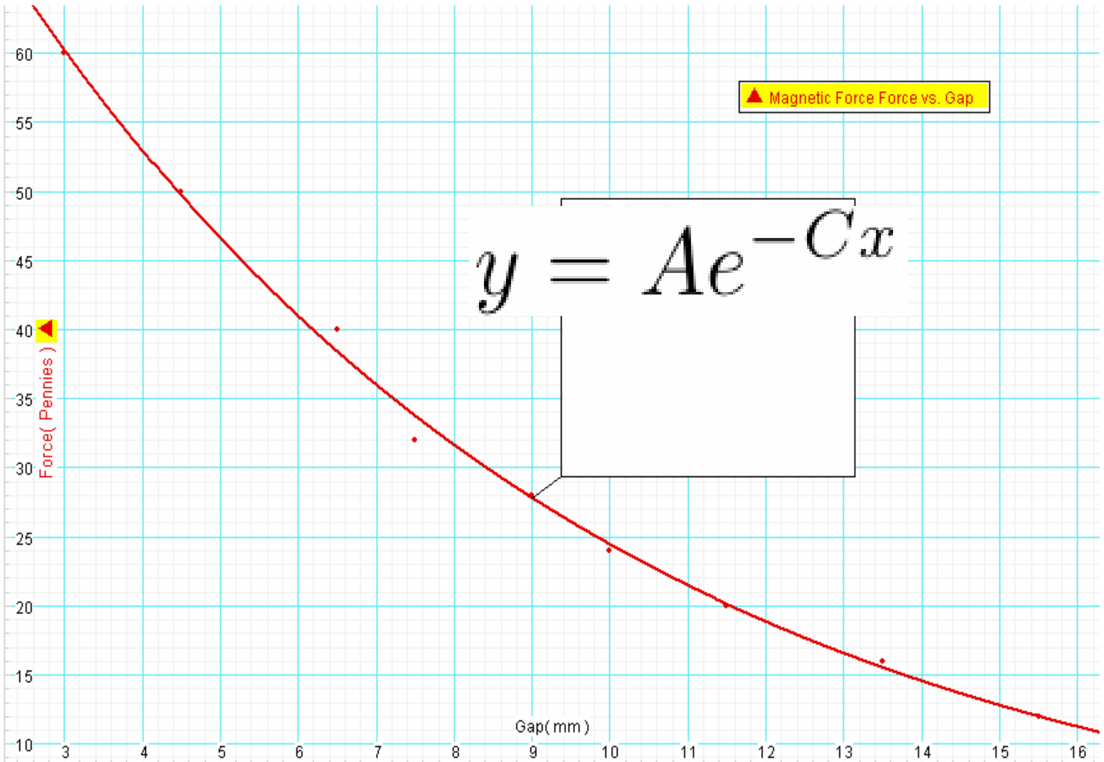
- Measure heights h_1 and h_2 with your ruler, and subtract them. (h_1 will be constant.)

- The two magnets stuck together weigh 6.0 pennies. The plastic coin holder weighs 4.0 pennies.

- Enter the gap (in mm) and the <u>total</u> weight (in pennies) into a table in DataStudio.

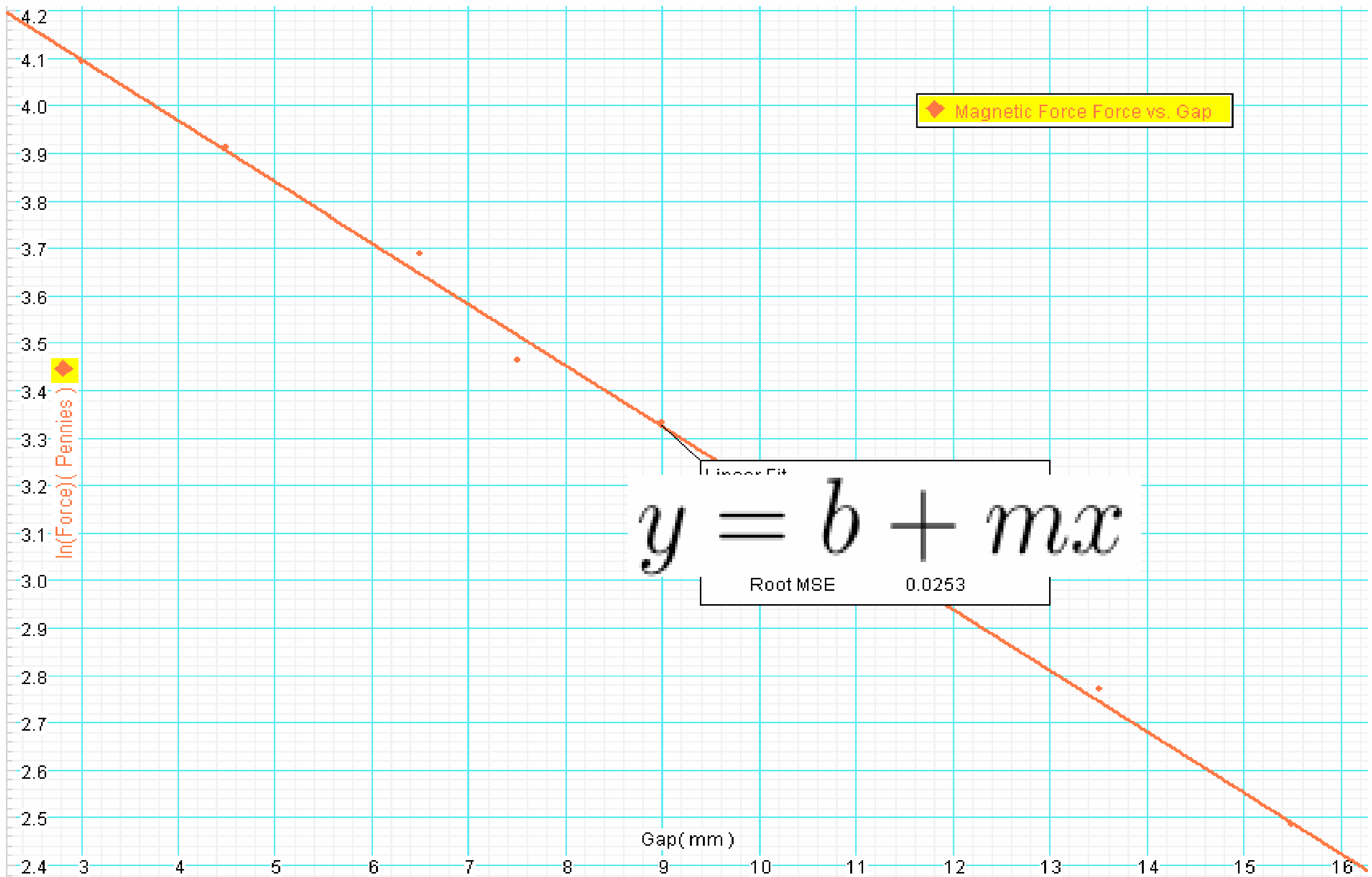
- The gap goes in the X (left) column of the table.

Exponential fit



- Carry out a user-defined fit of: $y = Ae^{-Cx}$
- Record *A* and *C* for part (a) and answer question about the characteristic length l over which the force drops by a factor 1/e

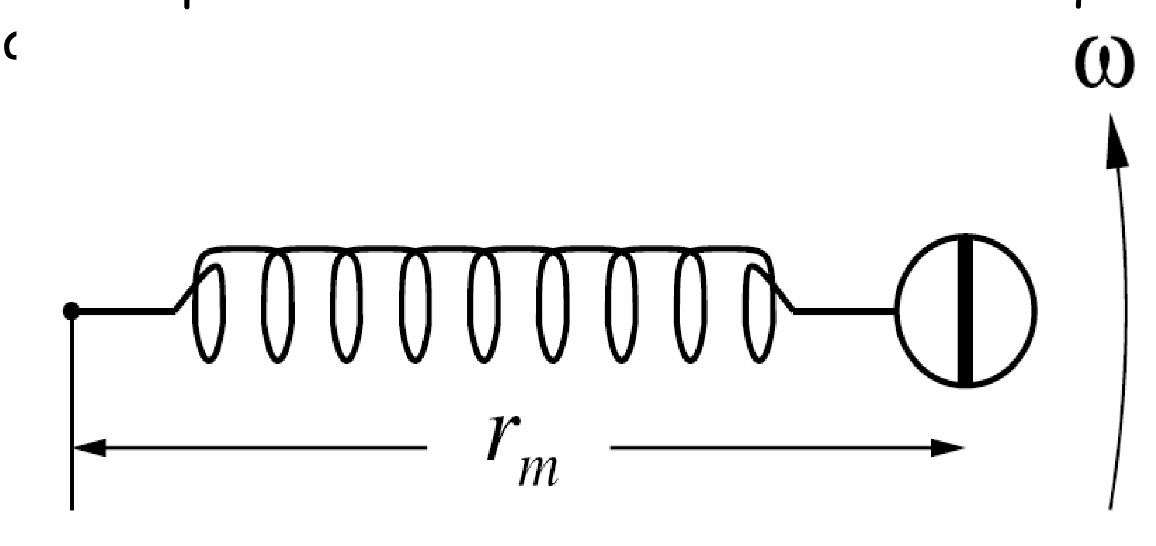
Semi-log plot and linear fit II



Experiment 04: Uniform Circular Motion

Goal

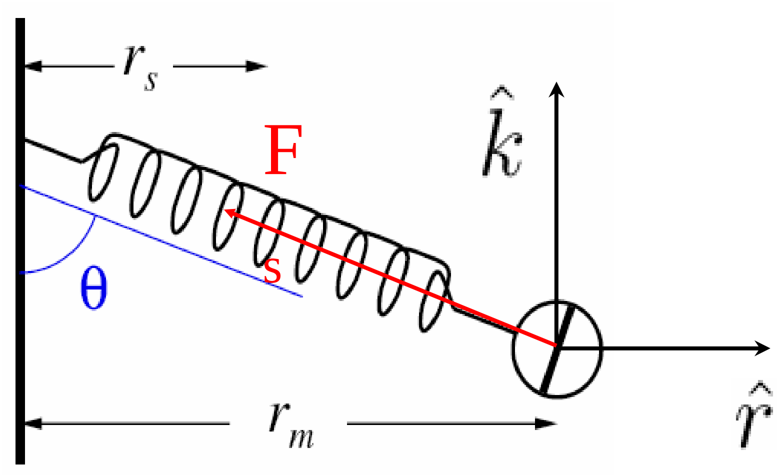
- Study a conical pendulum and measure the force required to produce c



- Extract from measurement of angular frequency and r_m the spring constant k and pre-tension F_0

- Understand how an instability in this system occurs at a critical frequency ω_0 and how to extract ω_0 from your measurements

Analysis of conical pendulum



$$\sum_i F_i = ma_i$$

$\hat{r}$ $$-F_s \cdot \sin\theta = -mr_m\omega^2$$

$\hat{k}$ $$-mg + F_s\cos\theta = 0$$

From my measurements of r_m, ω: $\theta \approx 88°$

$$\tan\theta = \frac{r_m\omega^2}{g}$$

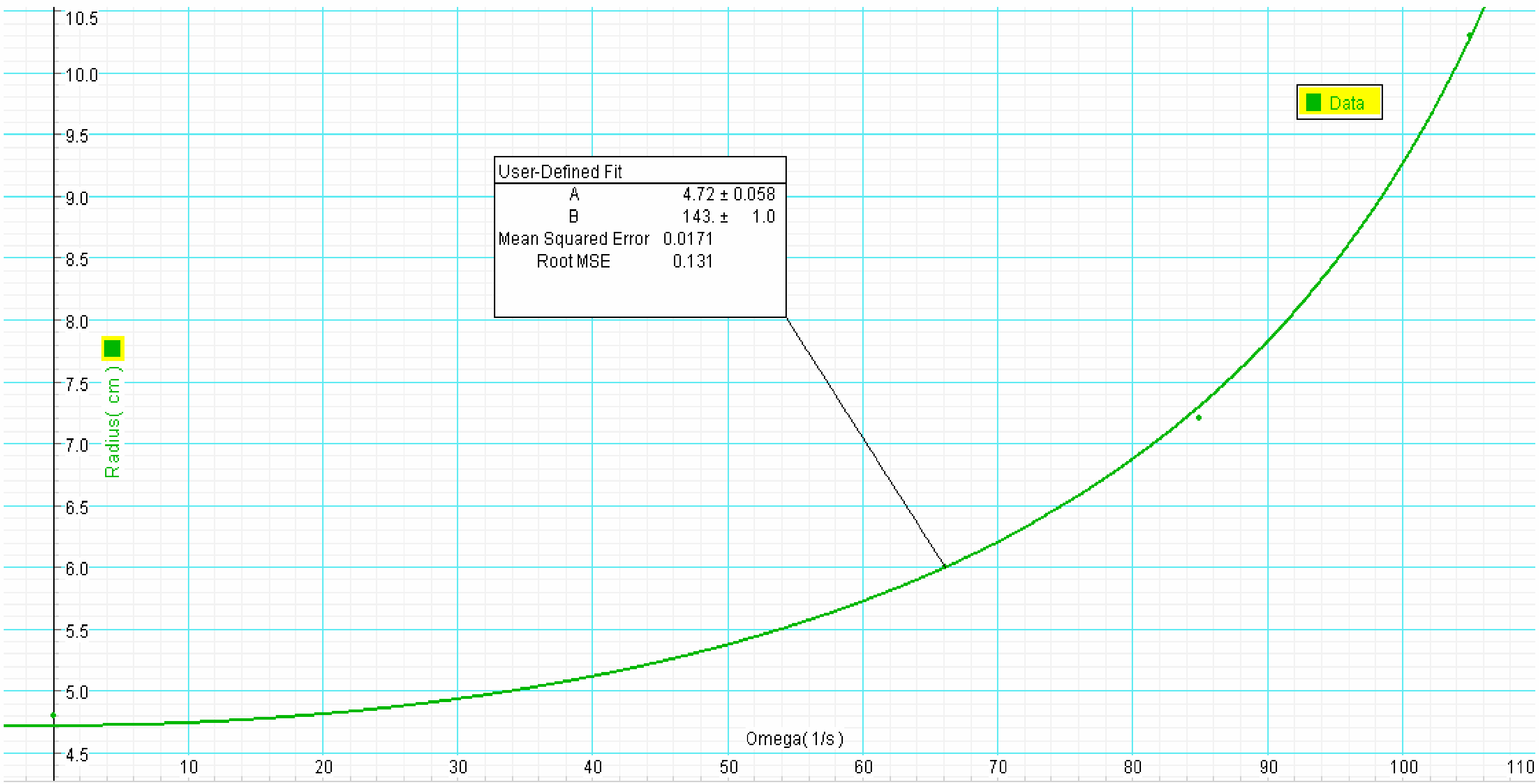
Therefore: Ignore effect of gravitation!

$$F_s = \Delta r \cdot k = (r_m - r_0)k = mr_m\omega^2$$

$$r_m = \frac{r_0}{1 - \frac{m\omega^2}{k}} = \frac{r_0}{1 - \frac{\omega^2}{\omega_c^2}}$$

Fitting

Perform a User-Defined fit to: `A/(1-x*x/(B*B))`

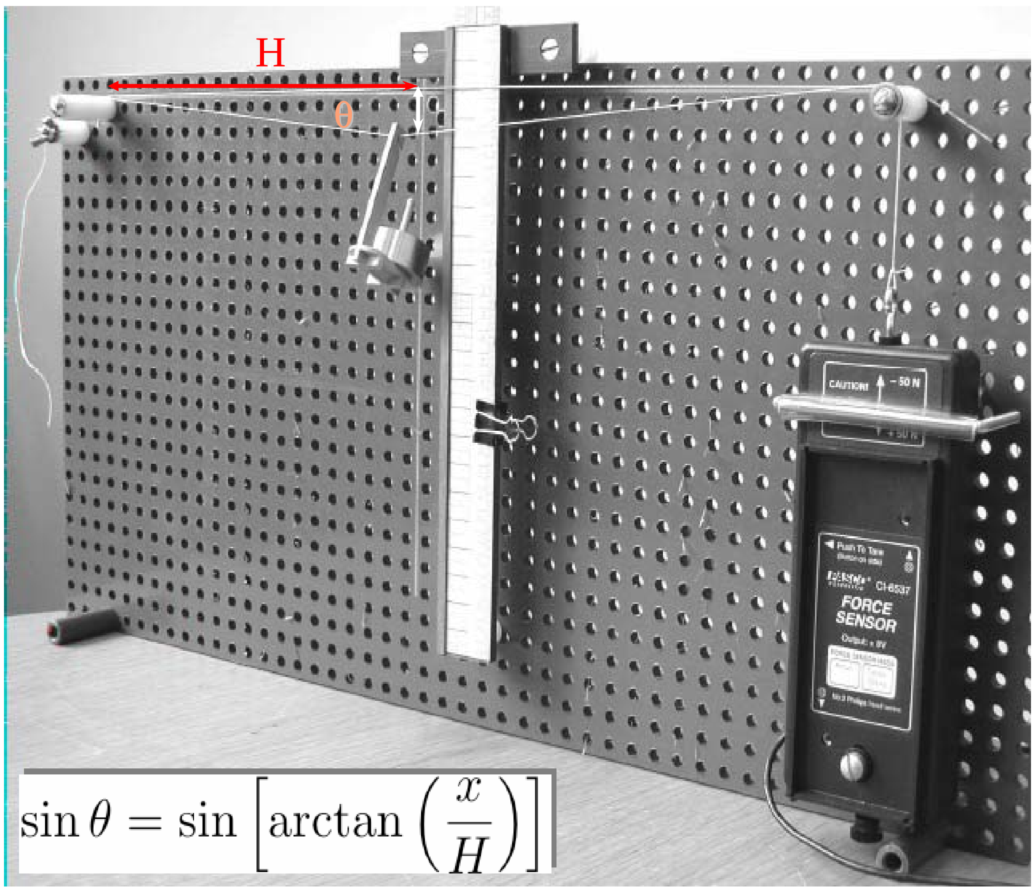


Experiment 05A: Static equilibrium

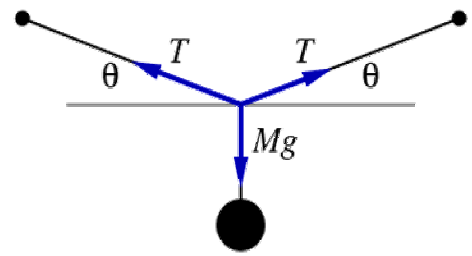
Goal

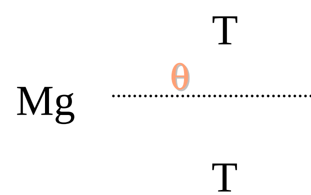
When a weight is suspended by two strings in the center as shown in the photograph below, the tension is given as follows:

$$T = \frac{Mg}{2\sin\theta}$$



$$\sin\theta = \sin\left[\arctan\left(\frac{x}{H}\right)\right]$$

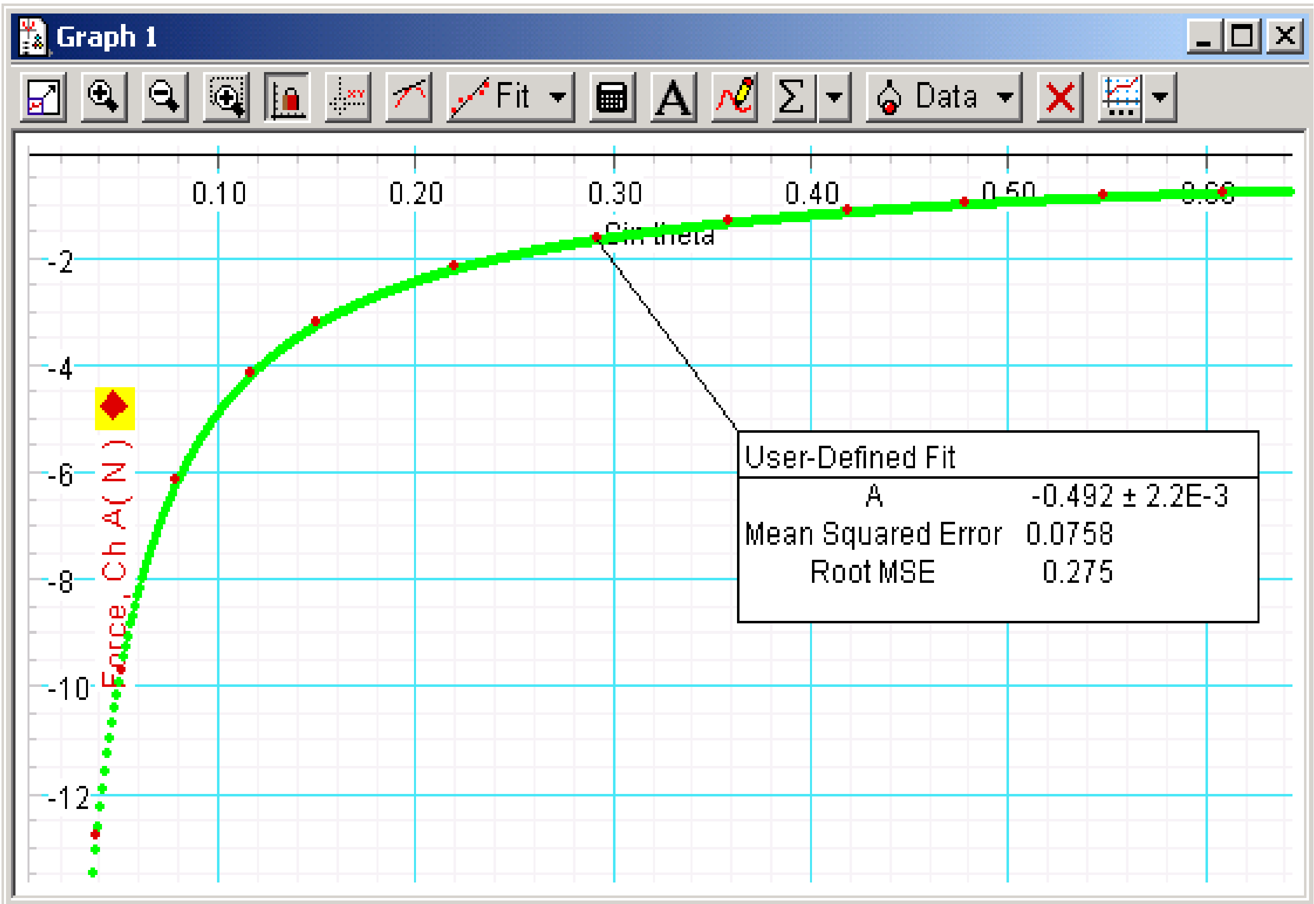




Goal: Measure T for several values of θ using measurements of , H (fixed), to verify the equation above!

Analyzing data

- Calculate sinθ from your vertical drop measurements (see write up).
- Plot force on y axis, sinθ on x axis.
- Fit y = A/x (User-defined fit) to your data.



Experiment 06: Work, Energy and the Harmonic Oscillator

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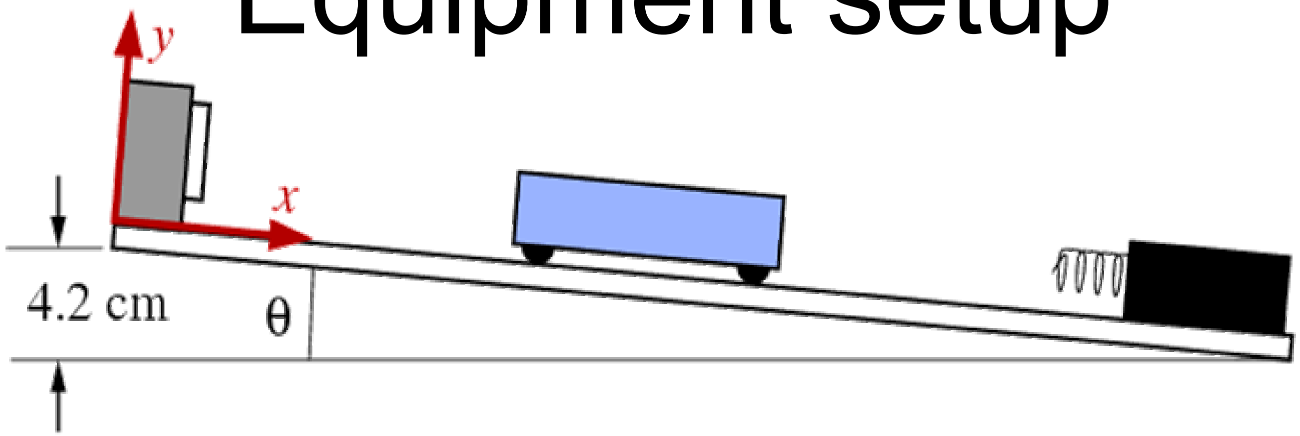
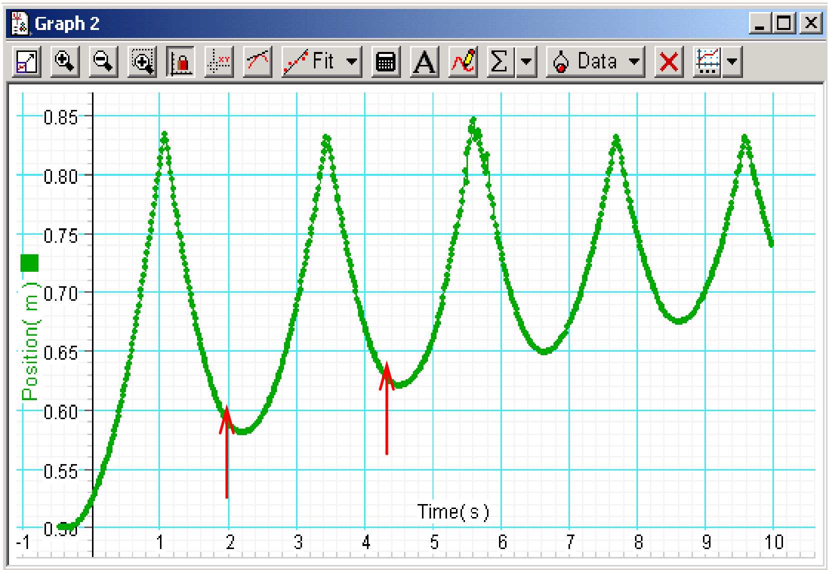
Equipment setup

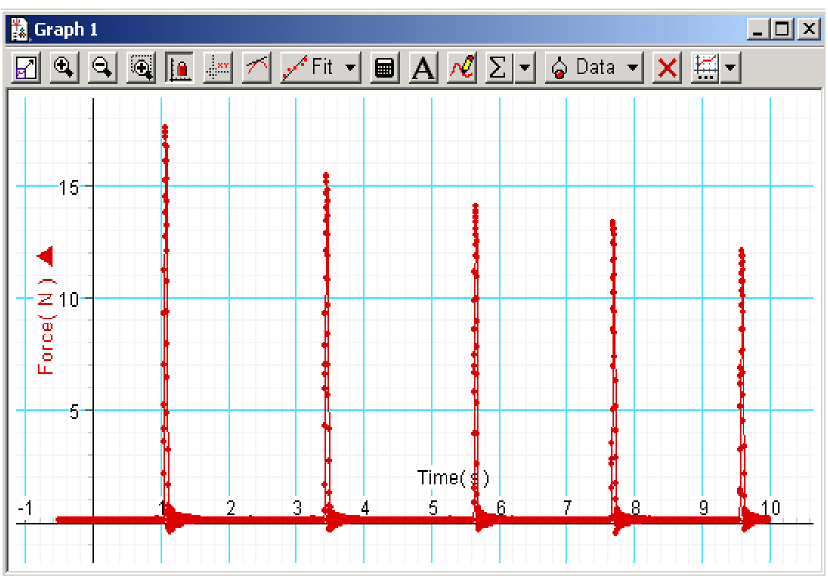


- Use the heavy spring on the force sensor.
- Put two 250g weights in the cart.
- Clip motion sensor to other end of track, and support it on a piece of 2x4.

Measurement Results

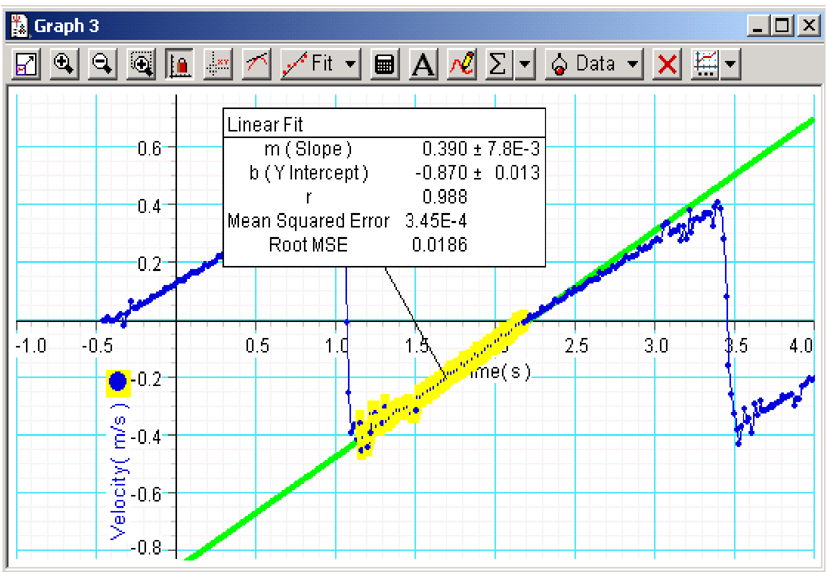


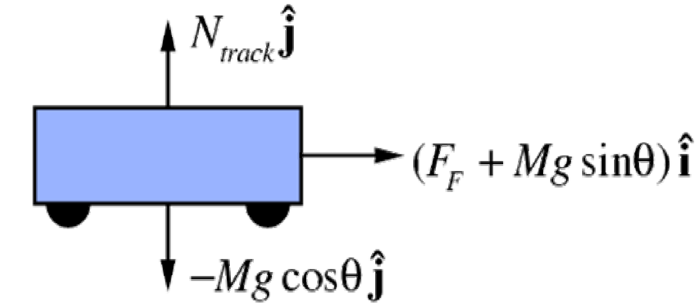
- Position vs. Time: Measure maximum heights either side of 2nd bounce, calculate loss of potential energy, and friction force. Enter in table!



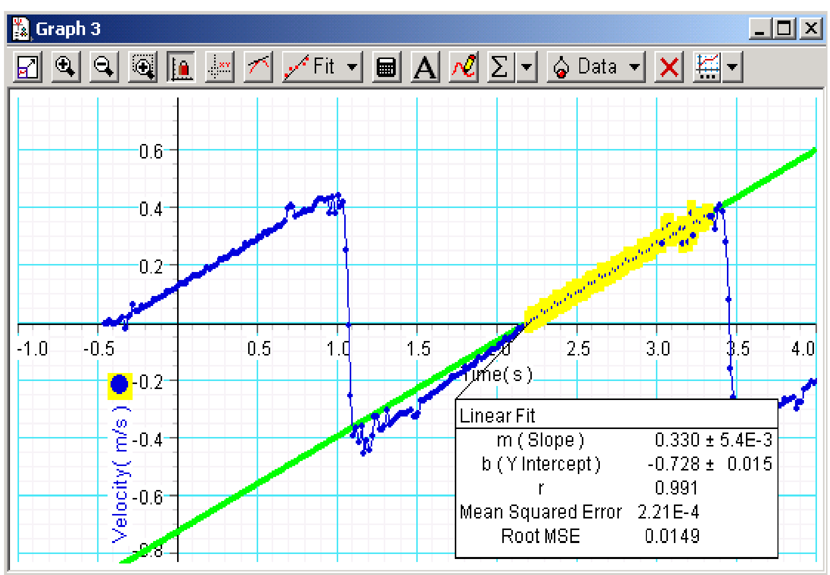
- Force vs. Time: Expand force peak around 2nd bounce.

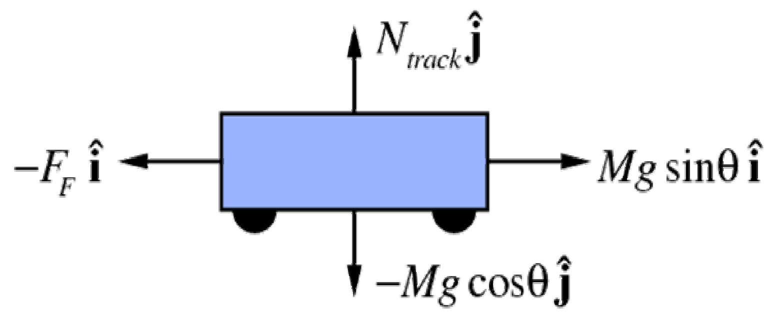
Finding Acceleration Up & Down





Linear fit to find a_{up}

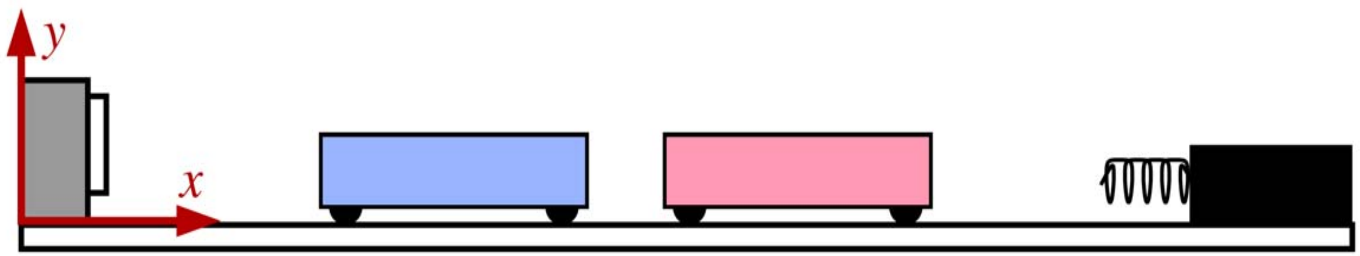




Linear fit to find a_{down}

Experiment 07: Momentum and Collisions

Equipment setup



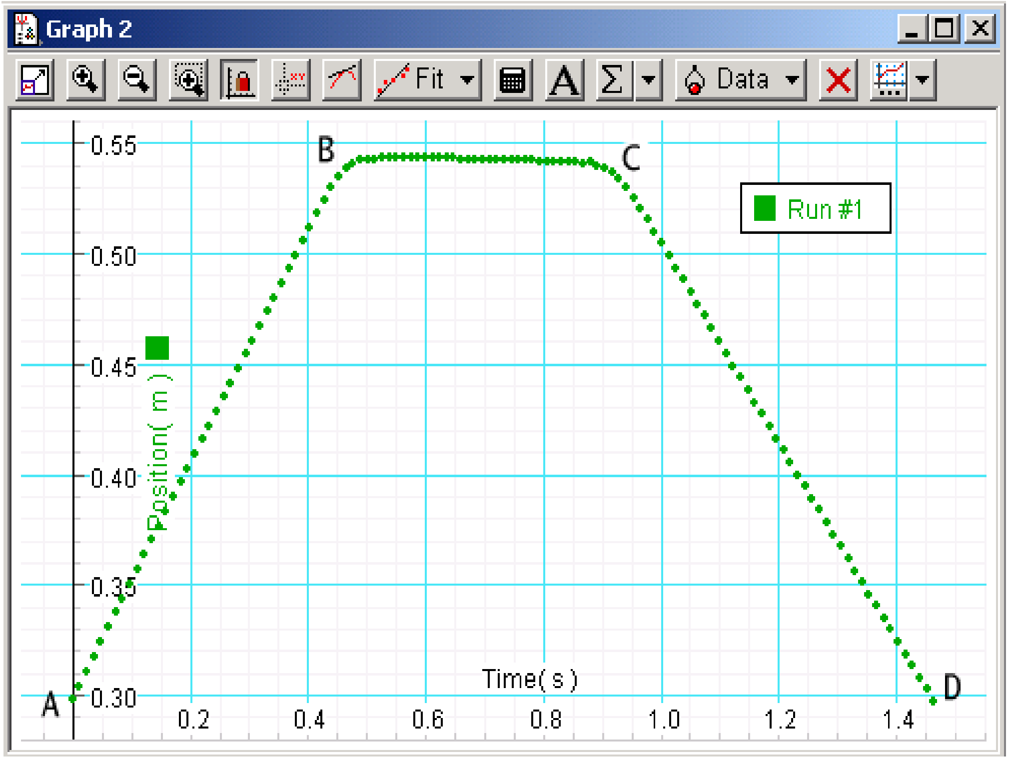
- Use the lighter spring on the force sensor.
- Clip the motion sensor to the end of the track.
- Level the track.

- Place Target cart at rest about 10cm from the spring.
- Place Incident cart about 16-20cm from motion sensor.
- Note: Velcro facing = inelastic and magnets facing = elastic.
- Roll incident cart just hard enough to come back to its starting point. Practice this first before you take your data!
- Make measurements with different weights of incident and target cart.

Graph 1

Two equal mass carts A and B collide.
This is X_A vs. time.

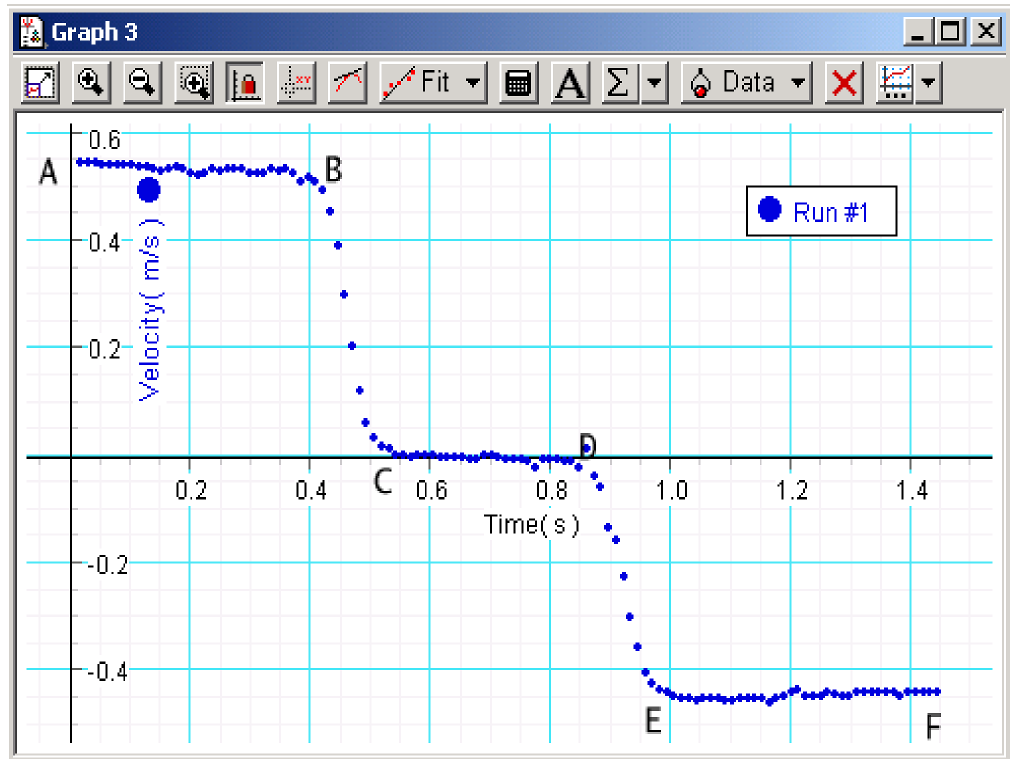
1. Along line AB?
2. At point B?
3. Along line BC?
4. At point C?
5. Along line CD?



At approximately what time does cart B hit the spring?

Graph 2

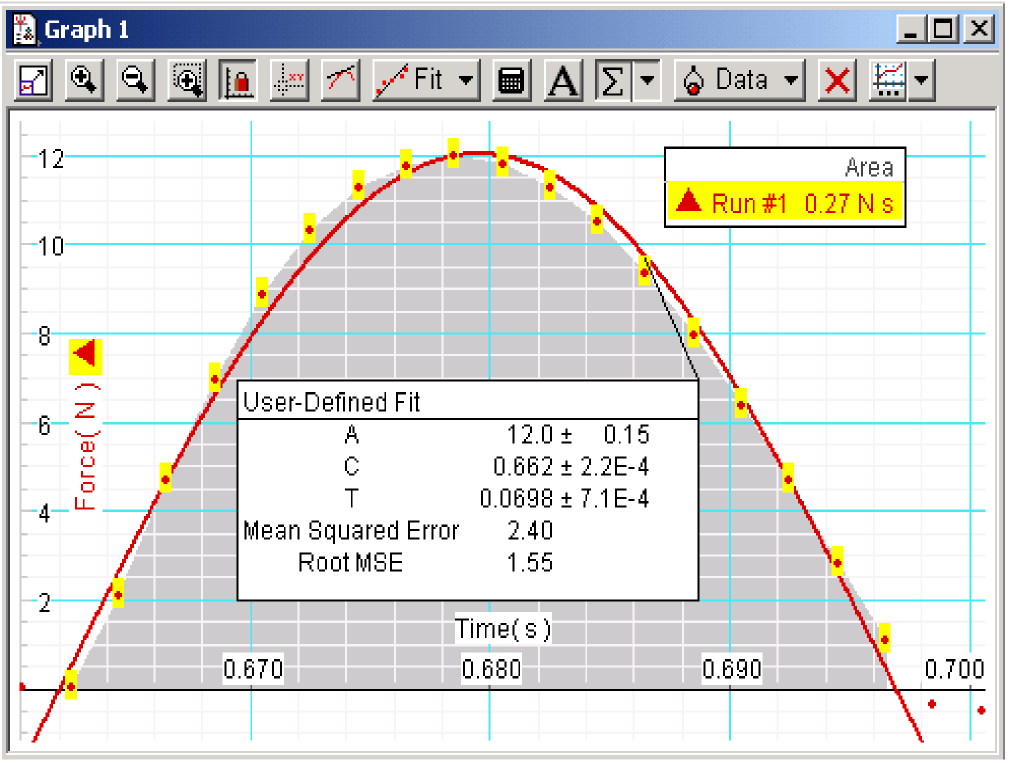
Two equal mass carts A and B collide.
This is V_A vs. time.



1. Along line AB?
2. Along line BC?
3. Along line CD?
4. Along line DE?
5. Along line EF?

Graph 3

A cart of mass 0.25kg collides with a spring on the force sensor. Here is the force during the collision. The fit is to: $A*\sin(2\pi(x-C)/T)$



What does the area under the curve tell you?

What can you learn from the parameter T?

Experiment 08: Physical Pendulum

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Goals

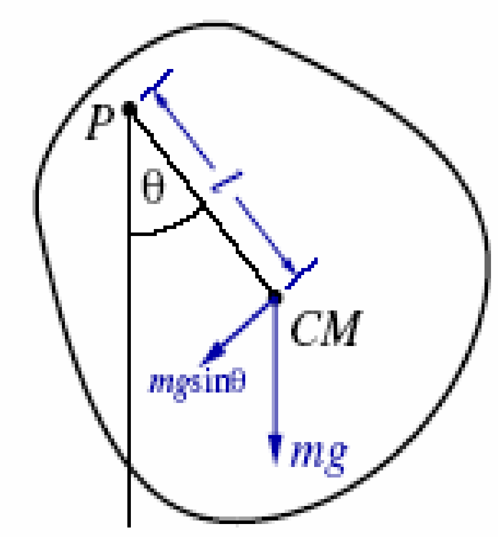
- Investigate the oscillation of a real (physical) pendulum and compare to an ideal (point mass) pendulum.

- Angular frequency calculation:

$$\vec{\tau} = I\vec{\alpha} \qquad -lmg\sin\theta = I\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{lmg}{I}\sin\theta = 0$$

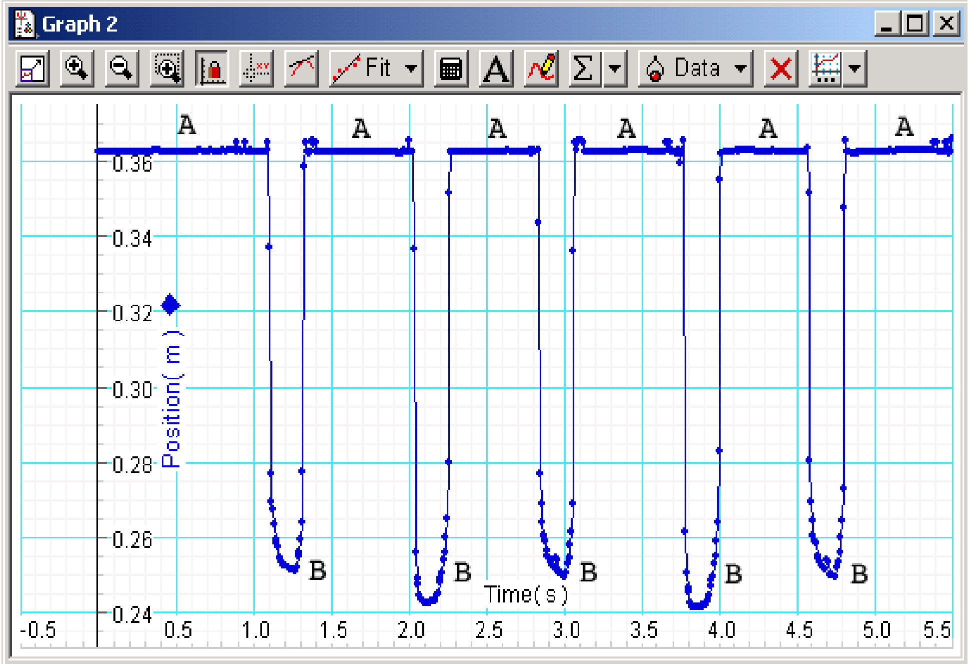
$$\text{With} \quad \sin\theta \simeq \theta \ : \ \omega = \sqrt{\frac{lmg}{I}} \qquad \frac{d^2\theta}{dt^2} + \frac{lmg}{I}\theta = 0$$



- Practice calculating moments of inertia, using them, and solving the $\tau = I\alpha$ equation of motion.

Understanding the graphs

Position vs. time data from the motion sensor.



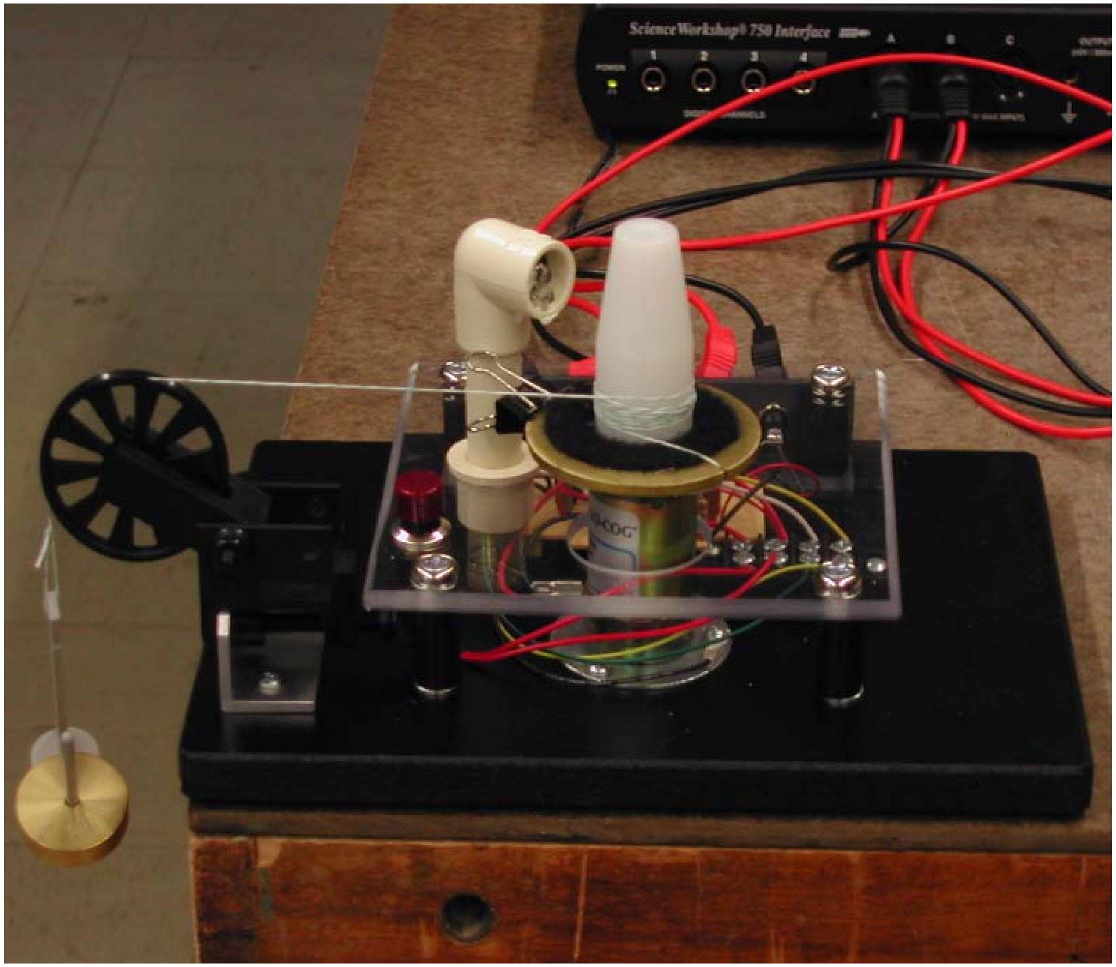
What is happening:

1. Along the top plateaus marked by A?
2. At the downward peaks marked by B?

How do you use this graph to find the period of oscillation of the pendulum?

Experiment 09: Angular momentum

Measure rotor I_R



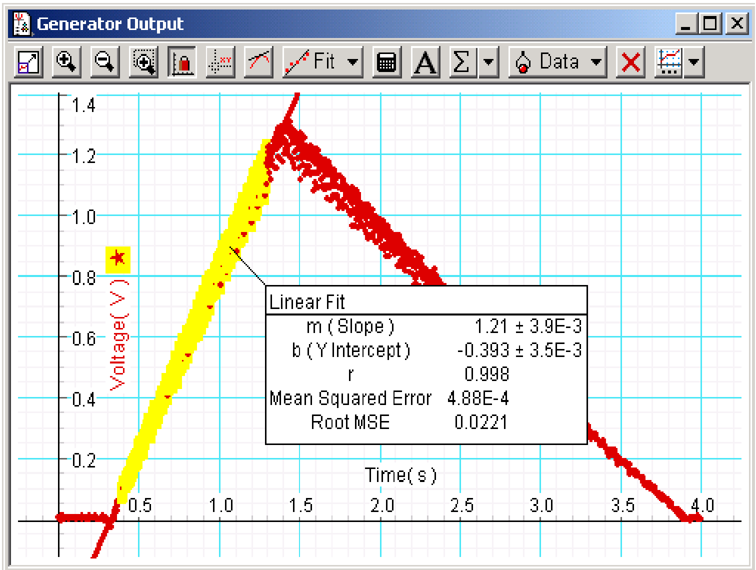
- Plot only the generator voltage for rest of experiment.
- Use a 55 gm weight to accelerate the rotor.
- Settings:
  - Sensitivity: Low
  - Sample rate 500 Hz.
  - Delayed start: None
  - Auto Stop: 4 seconds
- Start DataStudio and let the weight drop.

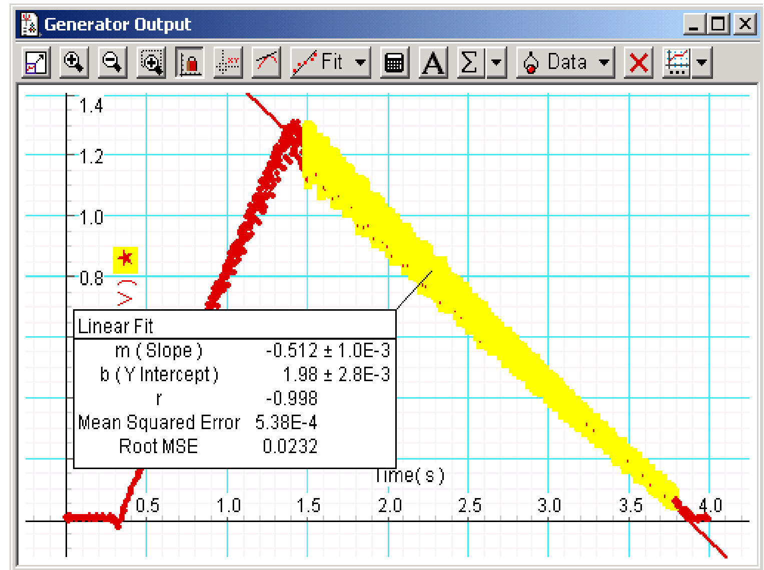
Understand graph output to measure I_R

- Generator voltage while measuring I_R. What is happening:
  1. Along line A-B ?
  2. At point B ?
  3. Along line B-C ?

- How do you use this graph to find I_R ?

Measure I_R results





- Measure and record α_{up} and α_{down}.
- For your report, calculate I_R:

$$I_R|\alpha_{down}| = |\tau_f| \qquad I_R = \frac{mr(g - r\alpha_{up})}{\alpha_{up} + |\alpha_{down}|}$$

Experiment 10: Energy transformation

Equipment setup

- Mechanical equivalent to heat: A motor applies a known friction torque τ_f at a known ω to a plastic jar with a known mass of H_2O.

$$c = \frac{\tau_f \omega}{m\left(dT/dt\right)}$$

- Electrical equivalent to heat: Apply voltage (2.5V) across a resistor (2.5Ohms) and use resulting electrical heat: Resistor is connected to right pair of posts.

$$c = \frac{\Delta VI}{m\left(dT/dt\right)}$$

- How can one double the temperature increase in both setups for a fixed amount of time?

$$\Delta V = R \cdot I$$

REVIEW #4

Statistical Mechanics,
Kinetic Theory
Ideal Gas
Thermodynamics
Heat Engines
Relativity

Statistical Mechanics and Thermodynamics

- Thermodynamics Old & Fundamental
  - Understanding of Heat (I.e. Steam) Engines
  - Part of Physics Einstein held inviolate
  - Relevant to Energy Crisis of Today

- Statistical Mechanics is Modern Justification
  - Based on mechanics: Energy, Work, Momentum
  - Ideal Gas model gives observed thermodynamics

- Bridging Ideas

  Temperature (at Equilibrium) is Average Energy

  Equipartition - as simple/democratic as possible

Equation of State

- A condition that the system must obey
  - Relationship among state variables

- Example: Perfect Gas Law
  - Found in 18th Century Experimentally
  - $pV = NkT = nRT$
  - K is Boltzmann's Constant 1.38×10^{-23} J/K
  - R is *gas constant* 8.315 J/mole/K

$PV = n R T = N k T$
chemists vs physicists

- Mole View (more Chemical) = nRT
  - R is *gas constant* 8.315 J/mole/K
- Molecular View (physicists) = NkT
  - N is number of molecules in system
  - K is Boltzmann's Constant 1.38×10^{-23} J/K

Ideal Gas Law Derivation: Assumptions

- Gas molecules are hard spheres without internal structure
- Molecules move randomly
- All collisions are elastic
- Collisions with the wall are elastic and instantaneous

Gas Properties

- N number of atoms in volume, m mass of molecule
- n_m moles in volume
- M_{molar} is atomic mass ($^{12}C = 12$)
- mass density $\rho = \frac{m_T}{V} = \frac{n_m M_{molar}}{V} = \frac{n_m N_A m}{V}$
- Avogadro's Number $N_A = 6.02 \times 10^{23} molecules \cdot mole^{-1}$

Pressure of a Gas: Microscopic to Macroscopic

Pressure

$$P_{pressure} = \frac{\left|\vec{\mathbf{F}}\right|_{gas,wall}}{A} = \frac{1}{3}\rho\left\langle v^2 \right\rangle$$

Root Mean Square Velocity

$$v_{rms} = \sqrt{\frac{3P}{\rho}}$$

Equipartition of Energy Theorem

$$\frac{1}{2}m(v^2)_{ave} = \frac{3}{2}kT$$

Ideal Gas Eq. Of State

$$P = \frac{n_m N_A kT}{V} = \frac{n_m RT}{V}$$

Degrees of Freedom in Motion

- Three types of degrees of freedom for molecule

1. Translational
2. Rotational
3. Vibrational

- Ideal gas Assumption: only 3 translational degrees of freedom are present for molecule with no internal structure

Equipartition of Energy: Kinetic Energy and Temperature

- Equipartition of Energy Theorem

$$\frac{1}{2}m(v^2)_{ave} = \frac{(\#\text{degrees of freedom})}{2}kT = \frac{3}{2}kT$$

- Boltzmann Constant $k = 1.38 \times 10^{-23}\, J \cdot K^{-1}$

- Average kinetic of gas molecule defines kinetic temperature

Heat

- If two bodies are in contact but initially have different temperatures, heat will transfer or flow between them if they are brought into contact.

- heat is the energy transferred, given the symbol Q.

Temperature and Equilibrium

- Temperature is Energy per Degree of Freedom

- Heat flows form hotter to colder object

  Until temperatures are equal

  Faster if better thermal contact

  Even flows at negligible Δt (for reversible process)

- The Unit of Temperature is the Kelvin

  Absolute zero (no energy) is at 0.0 K

  Ice melts at 273.15 Kelvin (0.0 C)

  Fahrenheit scale is arbitrary

State Variables of System

- **State Variables - Definition**

  Measurable Static Properties

  Fully Characterize System (if constituents known)

  e.g. Determine Internal Energy, compressibility

  Related by Equation of State

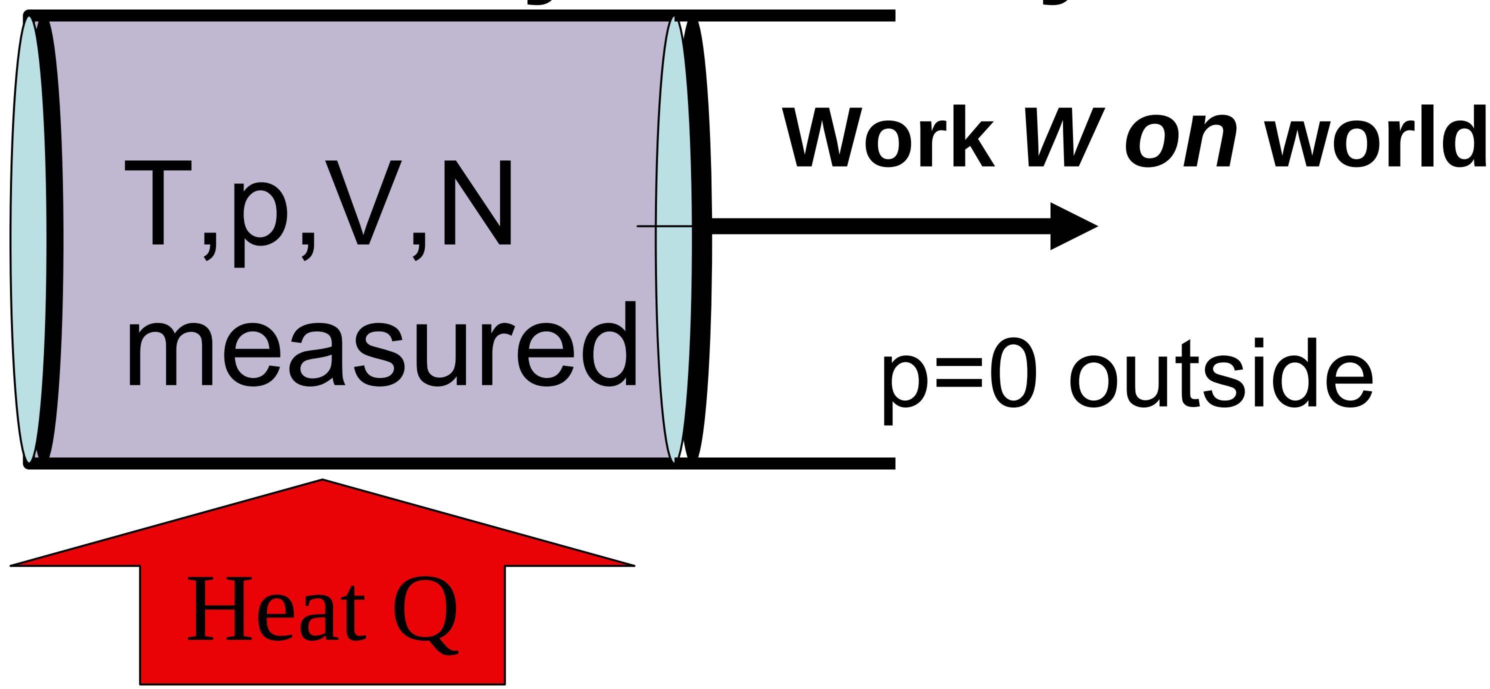
- **State Variables: Measurable Static Properties**
  - Temperature - measure with thermometer
  - Volume (size of container or of liquid in it)
  - Pressure (use pressure gauge)
  - Quantity: Mass or Moles or Number of Molecules
    - Of each constituent or phase (e.g. water and ice)

Heat and Work are Processes

- Processes accompany/cause state changes
  - Work **along particular path** to state B from A
  - Heat added along path to B from A

- Processes are **not state variables**
  - Processes change the state!
  - But Eq. Of State generally obeyed

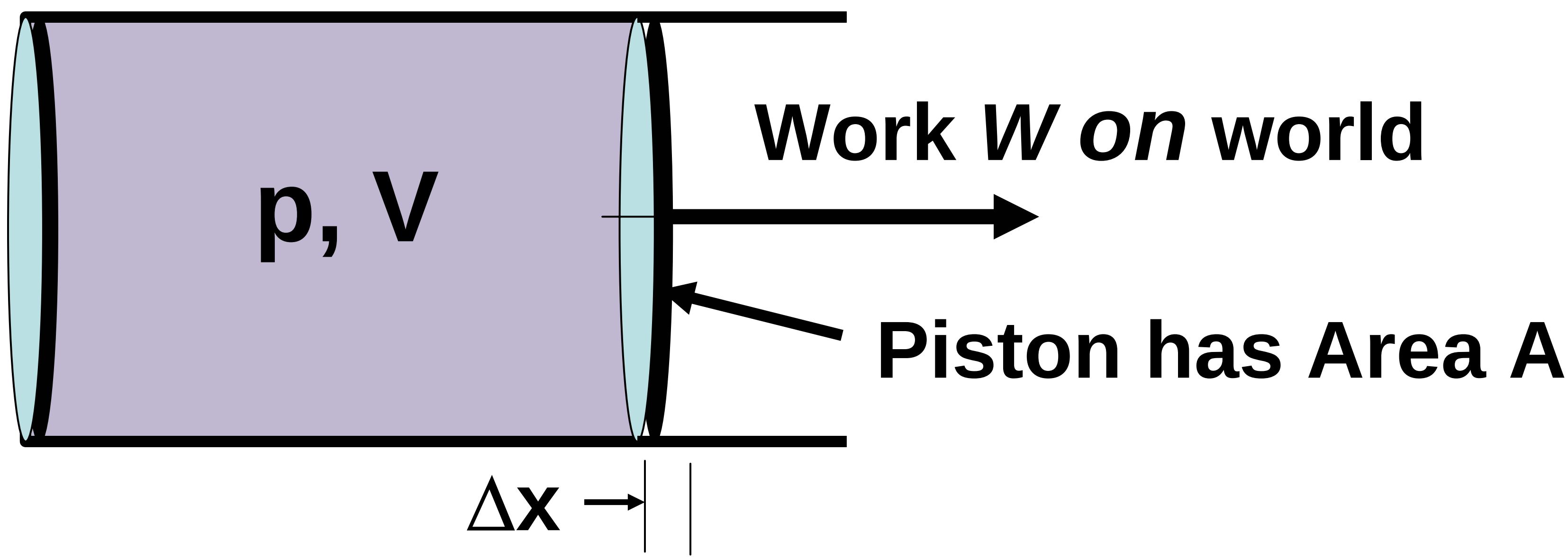
Thermodynamic Systems



The state variables are changed only in response to Q and W

No other work or heat enters

Work



Find Work if piston moves Δ**x**:

$$W = F\,\Delta x = pA\,\Delta x = p\Delta V$$

$$\text{In General:} \quad W_{fi} = \int_i^f p(V,T)\; dV$$

Variables in First Law

- Q is the **Heat Added**
  - Could find from Temperature Gradient
    - But need Heat Conductivity and Area
  - Generally determine from First Law

- W is the **Work done by** system
  - Equal to pDV

- U is the **Internal Energy of** system
  - It is determined by state variables
  - From equipartition, proportional to T

Internal Energy

Based on Equipartition:

-each coordinate of each particle

$$1/2\ m \langle v_x^2 \rangle = 1/2\ m \langle v_y^2 \rangle = 1/2\ k_B T$$

$$1/2\ \mu \langle v_{rel}^2 \rangle = 1/2\ k_B T \text{ ..molecule}$$

For an ideal monatomic gas:

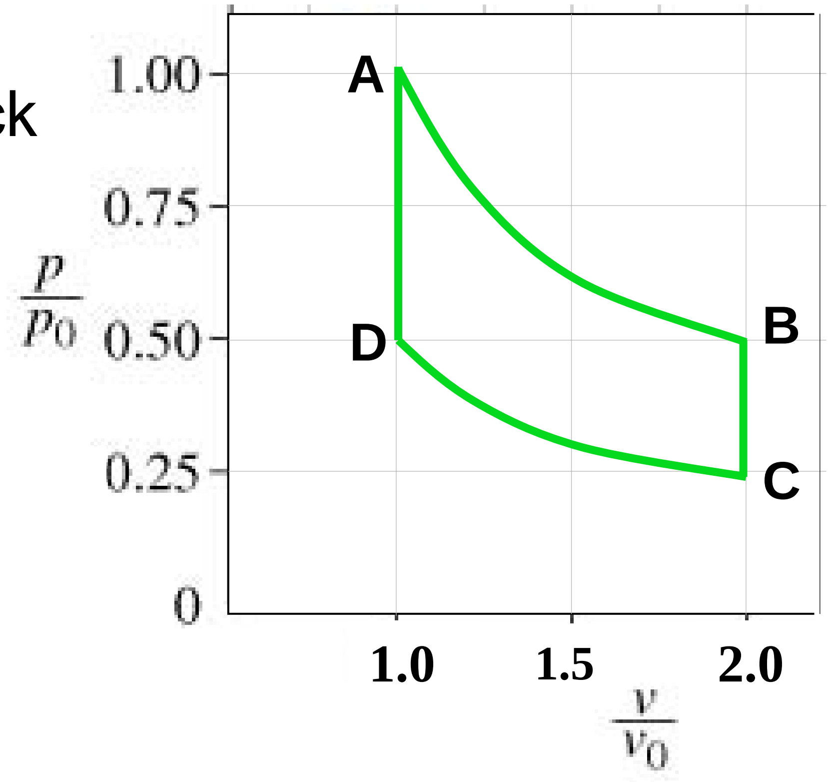
$$U(T) = 3/2\ N\ k\ T$$

For an ideal diatomic molecular gas:

$$U(T) = 5/2\ N\ k\ T \quad \text{(no vibration)}$$

Thermodynamic cycles

A thermodynamic cycle is any process that brings a system back to its original state.

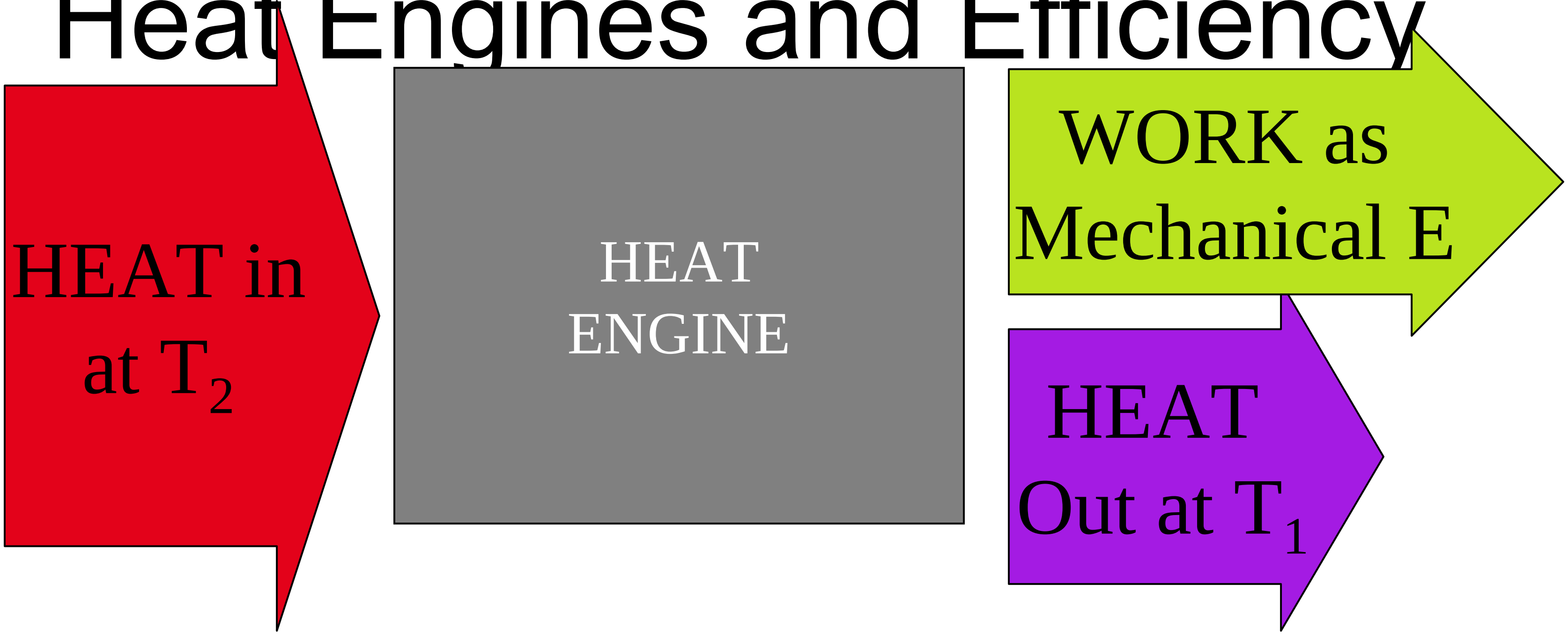


The cycle involves a path in state space over which various processes may act.

Addition/Removal of heat and work are typical processes.

Often the objective is to get work from heat or vice versa, as in a *heat engine* or *heat pump.*

Heat Engines and Efficiency



Energy Conserved $Q(\text{in at } T_2) = W_{cycle} + Q(\text{out at } T_1)$

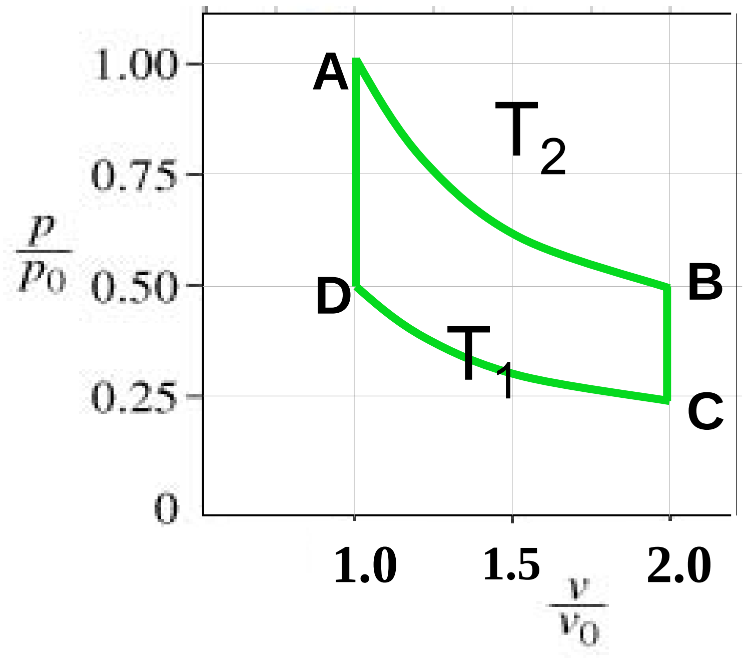
Heat Engine Efficiency: $$\varepsilon = \frac{W_{cycle}}{Q(\text{in at } T_2)}$$

Reversibility of Cycle

We showed that any leg of this cycle is reversible.

Therefore, the entire cycle (heat engine) could operate in reverse

In this case the total Work and the Heat flow will be reversed.

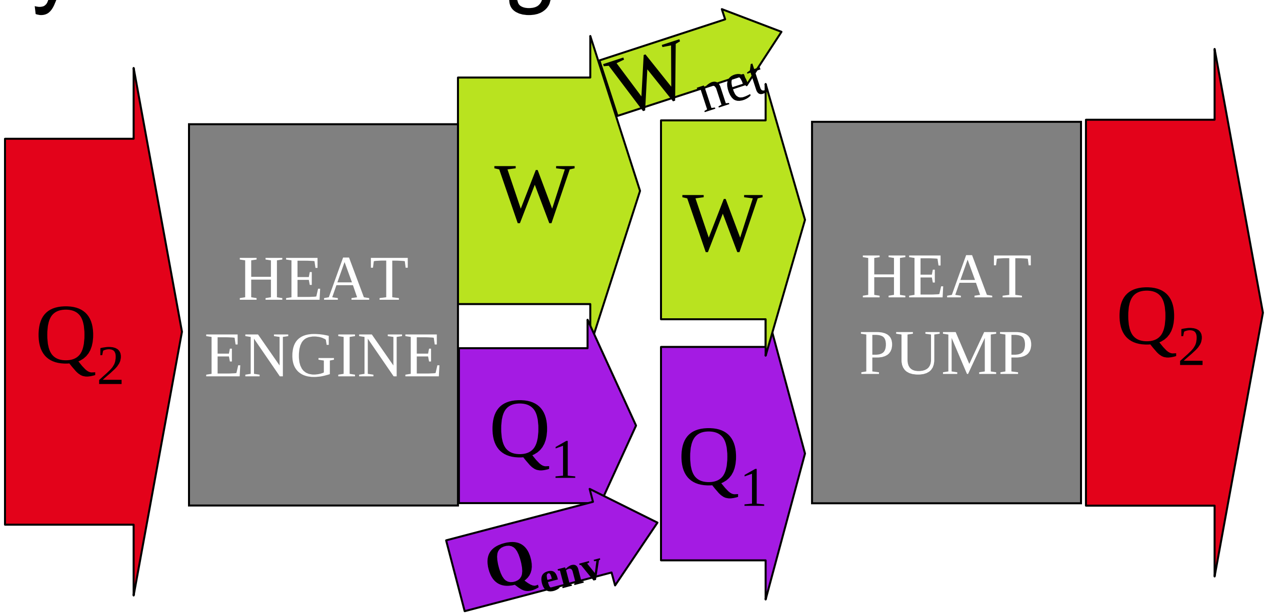


Operated in reverse it is a refrigerator that removes heat at the lower temperature

Or a heat pump delivering heat at the temperature T_2

Why Carnot is Maximum

Say a heat engine exceeded Carnot Limit



Hook it to a perfect gas heat pump with the same T_1

Net Effect: Heat Q_{env} becomes work W_{net}

Second law of Thermodynamics

No process shall have the only result that Heat is turned into Work

or

No process shall have the only result that Heat is transferred from cooler to hotter.

The second law is the formal statement of the irreversibility of Nature on a classical scale: friction can irreversibly convert mechanical energy into heat; nothing can reverse this process.

Postulates of Relativity

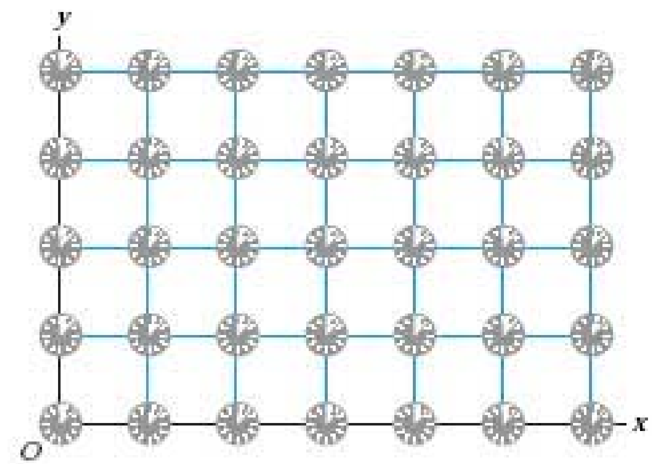
Relativity Postulate: All inertial frames are equivalent with respect to all the laws of physics.

Speed of Light Postulate: The speed of light in empty space always has the same value c.

This is really special case of Relativity Postulate since Maxwell's Eqn's predict speed of light.

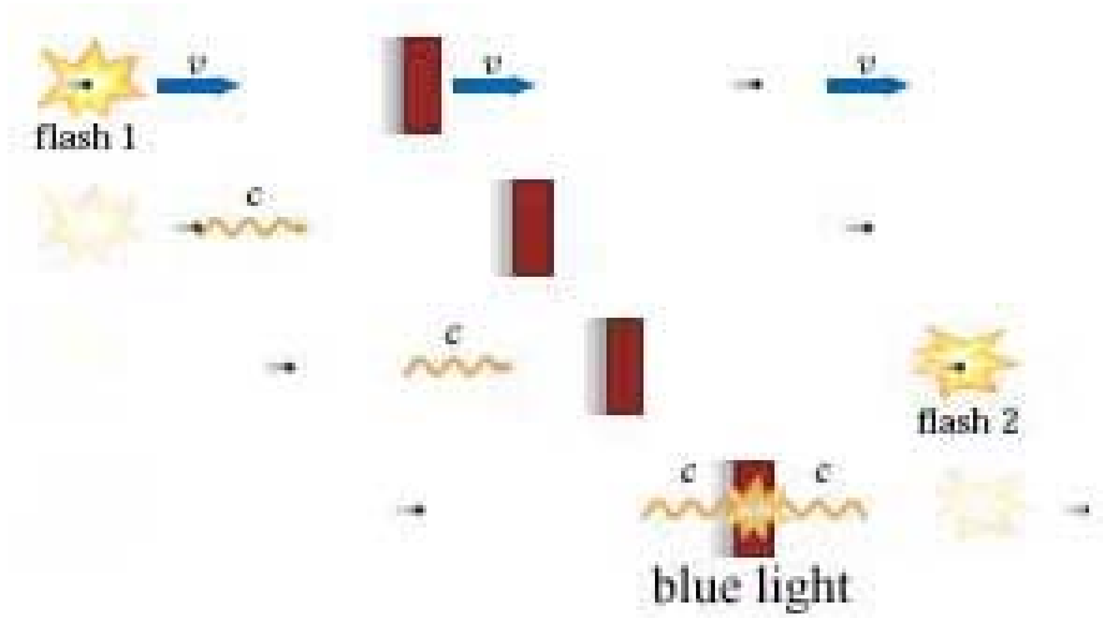
Alternative Synchronization

- two identical clocks that are at rest in a reference frame at different spatial points will run at identical rates.
- a lamp at the origin emits a pulse of light at time .
- Every point in space has a clock that will begin running when the light pulse reaches it.
- Each clock has been pre-set to the time $t = d/c$ where d is the distance from the clock to the origin and is the speed of light.
- When the light pulse reaches the clock, it begins to run at the same identical rate as all the other clocks.
- All the clocks are now synchronized (also according to Einstein's definition)
- Only good in that coordinate frame



The Relativity of Simultaneity

Events that a synchronous in one reference frame are not synchronous in another reference frame



Simultaneous flashes on the train aren't simultaneous
To an observer at rest on ground

Causality

- If event A causes event B, it must be observed to occur before B in any and all reference frames. The time ordering cannot be reversed!
- If a light pulse from A reaches B before event B occurs, then all observers will observe that event A happened first.
- Such events have *timelike separation*
- Events that are not timelike are *spacelike*

Time Dilation

Moving clocks run slow.

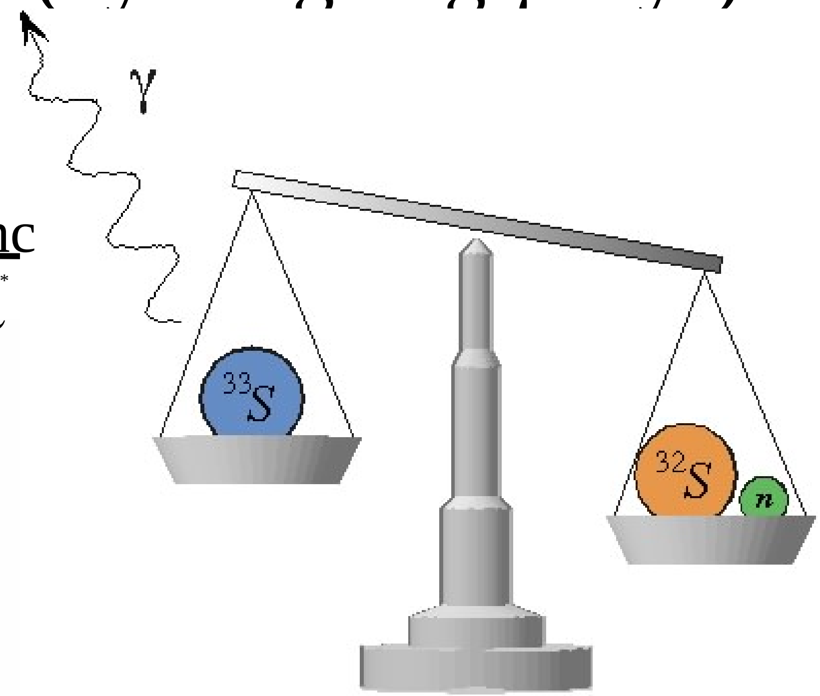
Time interval Δt´ between two space-time events measured by muon clock in frame in which muon clock is moving dilates compared to same time as measured in frame in which muon clock is at rest

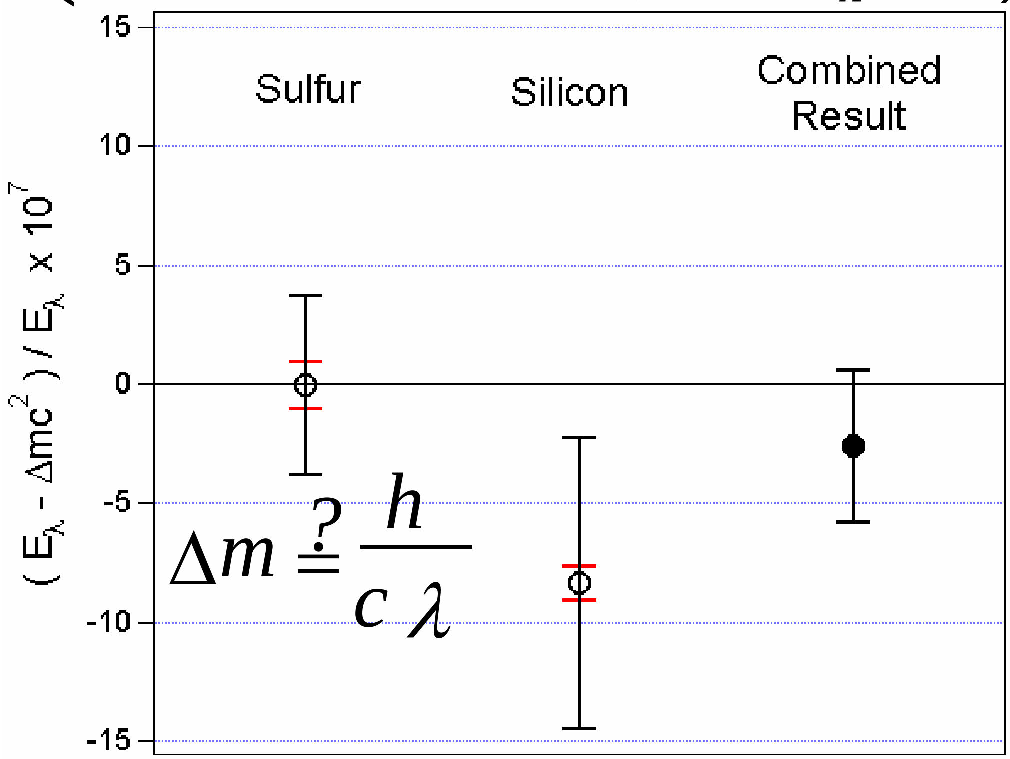
$$\Delta t' = \gamma \Delta t \qquad \gamma = 1 / \sqrt{(1 - v^2 / c^2)}$$

Testing $E=mc^2$ (by weighing γ-rays)

$$n + {}^{32}S \rightarrow {}^{33}S + \gamma$$

$$\left(M({}^{32}S) + M(n) - M({}^{33}S)\right) c^2 \left(\frac{10^3}{N_A}\right) = \frac{hc}{\lambda^*}$$





$$\Delta m \overset{?}{=} \frac{h}{c\,\lambda}$$

$$\left(E/mc^2 - 1\right) = -2.5 \pm 3.3 \text{ X } 10^{-7}$$

We measured ^{29}Si and ^{33}S

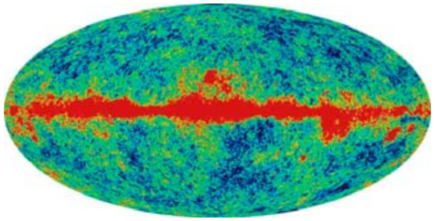
NIST measured λ's

Combined error X40 improvement of direct tests

If $E \neq mc^2$

Special Relativity results from the relativity postulate and observation of a limiting velocity, c. If $E \neq mc^2$ possibly:

Yes, there is a preferred frame



Michelson-Morley, Hughes-Drever look for anisotropy and place limits assuming CMB is preferred frame.

(see M.P. Haugan and C.M. Will, *Physics Today*, May 1987)

c doesn't equal c_{light}

Limited since high energy protons not radiating Cherenkov radiation

Einstein's logic faulty

Unlikely; it's been widely checked

Our experiments are wrong

We were very careful to check systematics BEFORE comparing with NIST

New physics in our experiments

There is no evidence from our experiment

Newtonian Mechanics

Explain and predict phenomena ranging from scale of 100’s atoms to Intergalactic distances

Theory built on Two Foundations

1. Principles of Dynamics embodied in the concept of Force Laws (based on experiments) and Newton’s Three Laws of Motion

2. Conservation Laws: Momentum, Energy, and Angular Momentum

Constants and Fundamental Scales

1. Speed of Light

$$c = 2.99792458 \times 10^{8}\,\mathrm{m \cdot s^{-1}}$$

2. Universal Gravitational Constant

$$G = 6.6726 \times 10^{-11}\,\mathrm{N \cdot m^{2} \cdot kg^{-2}}$$

3. Boltzmann's Constant

$$k = 1.3806503 \times 10^{-23}\,\mathrm{kg \cdot m^{2} \cdot s^{-2} \cdot K^{-1}}$$

4. Planck's Constant

$$h = 6.626068 \times 10^{-34}\,\mathrm{kg \cdot m^{2} \cdot s^{-1}}$$

Fundamental Scales: Planck Mass

Mass $$\dim\left[\left(\frac{hc}{G}\right)^{1/2}\right] = \left(\frac{(mass)(length)^3}{(time)^2} \bigg/ \frac{(length)^3}{(mass)(time)^2}\right)^{1/2} = (mass)$$

$$\left(\frac{hc}{G}\right)^{1/2} = \left(\frac{\left(6.626068\times10^{-34}\,\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-1}\right)\left(2.998\times10^{8}\,\mathrm{m}\cdot\mathrm{s}^{-1}\right)}{\left(6.6726\times10^{-11}\,\mathrm{N}\cdot\mathrm{m}^2\cdot\mathrm{kg}^{-2}\right)}\right)^{1/2} = 5.456\times10^{-8}\,\mathrm{kg}$$

Fundamental Scales: Planck Time

Time

$$\dim\left[\left(\frac{hc}{G}\right)^{1/2}\right]=\left(\frac{(mass)(length)^3}{(time)^2}\Big/\frac{(length)^3}{(mass)(time)^2}\right)^{1/2}=(mass)$$

$$\dim\left[\frac{G}{c^3}\right]=\frac{(time)}{(mass)} \qquad \dim\left[\frac{G}{c^3}\left(\frac{hc}{G}\right)^{1/2}\right]=\dim\left[\left(\frac{hG}{c^5}\right)^{1/2}\right]=(time)$$

$$\left(\frac{hG}{c^5}\right)^{1/2}=\left(\frac{\left(6.626068\times10^{-34}\,\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-1}\right)\left(6.6726\times10^{-11}\,\mathrm{N}\cdot\mathrm{m}^2\cdot\mathrm{kg}^{-2}\right)}{\left(2.998\times10^{8}\,\mathrm{m}\cdot\mathrm{s}^{-1}\right)^5}\right)^{1/2}=1.35\times10^{-43}\,\mathrm{s}$$

Fundamental Scales: Planck Length

Length

$$\dim\left[\left(\frac{hc}{G}\right)^{1/2}\right] = \left(\frac{(mass)(length)^3}{(time)^2} / \frac{(length)^3}{(mass)(time)^2}\right)^{1/2} = (mass)$$

$$\dim\left[\frac{G}{c^2}\right] = \frac{(length)}{(mass)} \qquad \dim\left[\left(\frac{hG}{c^3}\right)^{1/2}\right] = length$$

$$\left(\frac{hG}{c^3}\right)^{1/2} = \left(\frac{\left(6.626068\times10^{-34}\,\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-1}\right)\left(6.6726\times10^{-11}\,\mathrm{N}\cdot\mathrm{m}^2\cdot\mathrm{kg}^{-2}\right)}{\left(2.998\times10^{8}\,\mathrm{m}\cdot\mathrm{s}^{-1}\right)^3}\right)^{1/2} = 4.071\times10^{-35}\,\mathrm{m}$$

Fundamental Scales: Planck Temperature

Temperature $$\dim\left[\frac{G}{c^3}\left(\frac{hc}{G}\right)^{1/2}\right] = \dim\left[\left(\frac{hG}{c^5}\right)^{1/2}\right] = (time)$$

$$\dim[h/k] = (energy)(time)/\,(energy)\cdot(temp^{-1}) = (time)(temp)$$

$$\dim[\frac{h}{k}/\frac{G}{c^3}\left(\frac{hc}{G}\right)^{1/2}] = (temp)$$

$$\frac{h}{k}/\frac{G}{c^3}\left(\frac{hc}{G}\right)^{1/2} = 3.55499\times 10^{32}\ \mathrm{K}$$

Limits of Newtonian Mechanics

- Special Relativity: Rapidly moving objects when $v/c \rightarrow 1$
- Quantum Mechanics: Atomic and Molecular Scales
- General Relativity: Non-Euclidian Structure of Universe
- Statistical Mechanics: Large Number of Particles
- Classical Chaos: Initial Conditions don't predict future
- Quantized Gravity: String Theory, Planck Length: