Collision Theory

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Momentum and Impulse

- momentum $$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$
- change in momentum $$\Delta\vec{\mathbf{p}} = m\Delta\vec{\mathbf{v}}$$
- *average impulse* $$\vec{\mathbf{I}}_{ave} = \vec{\mathbf{F}}_{ave}\Delta t = \Delta\vec{\mathbf{p}}$$

Conservation of Momentum

- *The total change in momentum of a system and its surroundings between the final state and the initial state is zero,*

$$\Delta\vec{\mathbf{p}}_{system} + \Delta\vec{\mathbf{p}}_{surroundings} = 0$$

Conservation of Momentum

- completely isolate system from the surroundings

$$\vec{\mathbf{F}}_{ext}^{total} = \vec{\mathbf{0}}$$

- change in momentum of the system is also zero

$$\Delta\vec{\mathbf{p}}_{system} = \vec{\mathbf{0}}$$

Conservation of Momentum: Isolated System

When the total external force on a system is zero, then the total initial momentum of the system equals the total final momentum of the system,

$$\vec{\mathbf{p}}_0^{total} = \vec{\mathbf{p}}_f^{total}$$

Problem Solving Strategies: Momentum Diagram

- Identify the objects that compose the system
- Identify your initial and final states of the system
- Choose symbols to identify each mass and velocity in the system.
- Identify a set of positive directions and unit vectors for each state.
- Decide whether you are using components or magnitudes for your velocity symbols.

Momentum Diagram

Since momentum is a vector quantity, identify the initial and final vector components of the total momentum

Initial State

$$\vec{\mathbf{p}}_0^{total} = m_1\vec{\mathbf{v}}_{1,0} + m_2\vec{\mathbf{v}}_{2,0} + \ldots$$

x-comp:

$$p_{x,0}^{total} = m_1\left(v_x\right)_{1,0} + m_2\left(v_x\right)_{2,0} + \ldots$$

y-comp:

$$p_{y,0}^{total} = m_1\left(v_y\right)_{1,0} + m_2\left(v_y\right)_{2,0} + \ldots$$

Final State

$$\vec{\mathbf{p}}_f^{total} = m_1\vec{\mathbf{v}}_{1,f} + m_2\vec{\mathbf{v}}_{2,f} + \ldots$$

x-comp:

$$p_{x,f}^{total} = m_1\left(v_x\right)_{1,f} + m_2\left(v_x\right)_{2,f} + \ldots$$

y-comp:

$$p_{y,f}^{total} = m_1\left(v_y\right)_{1,f} + m_2\left(v_y\right)_{2,f} + \ldots$$

Strategies: Conservation of Momentum

- If system is isolated, write down the condition that momentum is constant in each direction

$$p_{x,0}^{total} = p_{x,f}^{total}$$

$$m_1 \left(v_x\right)_{1,0} + m_2 \left(v_x\right)_{2,0} + \ldots = m_1 \left(v_x\right)_{1,f} + m_2 \left(v_x\right)_{2,f} + \ldots$$

$$p_{y,0}^{total} = p_{y,f}^{total}$$

$$m_1 \left(v_y\right)_{1,0} + m_2 \left(v_y\right)_{2,0} + \ldots = m_1 \left(v_y\right)_{1,f} + m_2 \left(v_y\right)_{2,f} + \ldots$$

Planar Collision Theory: Energy

Types of Collisions in Two Dimensions:

- Elastic: $K_f = K_i$
- Inelastic: $K_f < K_i$
- Completely Inelastic: Only one body emerges
- Superelastic: $K_f > K_i$

Elastic Collisions

- Kinetic Energy does not change.

$$K_0^{total} = K_f^{total}$$

$$\frac{1}{2}m_1 v_{1,0}^{\ 2} + \frac{1}{2}m_2 v_{2,0}^{\ 2} + \ldots = \frac{1}{2}m_1 v_{1,f}^{\ 2} + \frac{1}{2}m_2 v_{2,f}^{\ 2} + \ldots$$

PRS Question

Suppose a golf ball is hurled at a heavy bowling ball initially at rest and bounces elastically from the bowling ball. After the collision,

1. The golf ball has the greater momentum and the greater kinetic energy.
2. The bowling ball has the greater momentum and the greater kinetic energy.
3. The golf ball has the greater momentum but has the smaller kinetic energy.
4. The bowling ball has the greater momentum but has the smaller kinetic energy.

PRS Question

Consider the exothermic reaction (final kinetic energy is greater than the initial kinetic energy).

$$H + H \rightarrow H_2 + 5ev$$

Two hydrogen atoms collide and produce a diatomic hydrogen molecule. Using only the principles of classical mechanics, this reaction

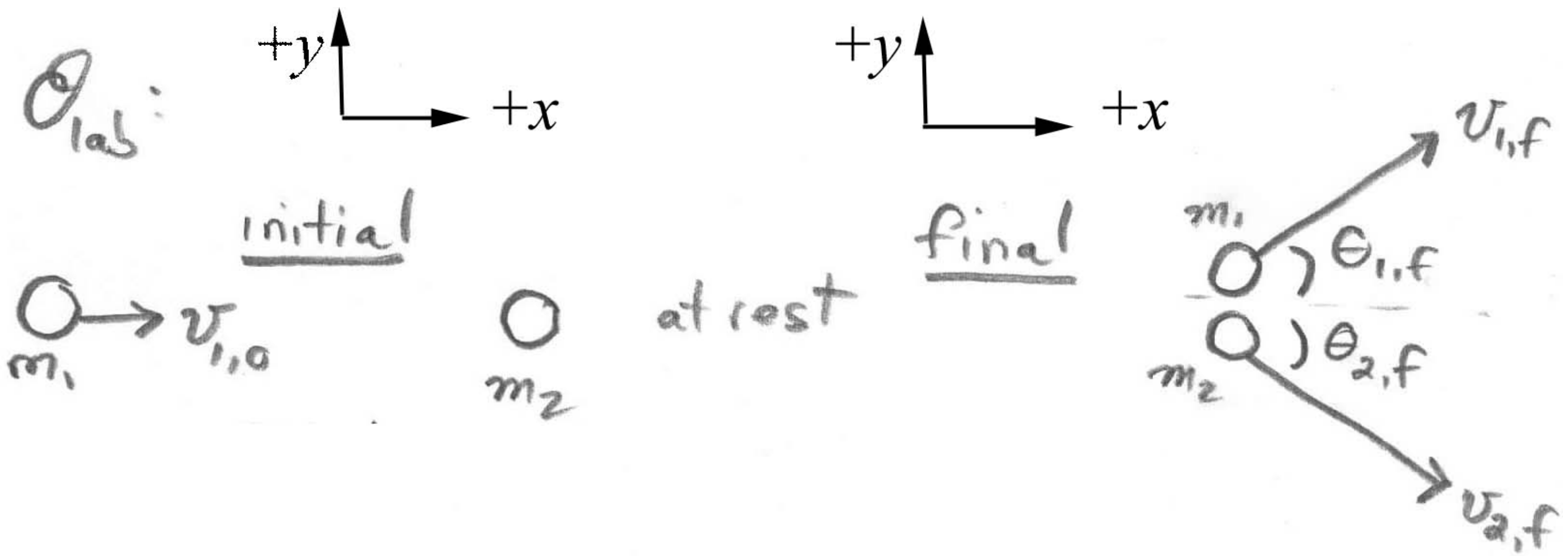
1. Is possible.
2. either violates conservation of energy or conservation of momentum but not both
3. satisfies conservation of energy and momentum but is not possible for other reasons.

Worked Example: Two Dimensional Elastic Collision

Consider an *elastic* collision between two particles. In the laboratory reference frame, the first particle with mass m_1, the incident particle, is moving with an initial given velocity v_{10}. The second 'target' particle is of mass $m_2 = m_1$ and at rest. After the collision, the first particle moves off at a measured angle $\theta_{1f} = 45^o$ with respect to the initial direction of motion of the incident particle with an unknown final velocity v_{1f}. Particle two moves off at unknown angle θ_{2f} with an unknown final velocity v_{2f}. Find v_{1f} , v_{2f} , and θ_{2f}.

Momentum Diagram

- Momentum is a vector!



Analysis

Momentum is constant:

x-direction

$$\hat{\mathbf{i}}: m_1 v_{1,0} = m_1 v_{2,f} \cos\theta_{2,f} + m_1 v_{1,f} \cos\theta_{1,f}$$

y-direction

$$\hat{\mathbf{j}}: 0 = m_1 v_{2,f} \sin\theta_{2,f} - m_1 v_{1,f} \sin\theta_{1,f}$$

Elastic collision:

$$\frac{1}{2} m_1 v_{1,0}{}^2 = \frac{1}{2} m_1 v_{2,f}{}^2 + \frac{1}{2} m_1 v_{1,f}{}^2$$

Strategy

Three unknowns: θ_{2f} , v_{1f}, and v_{2f}

1. Eliminate θ_{2f} by squaring momentum equations and adding equations and solve for v_{2f} in terms of v_{1f}

2. Substitute expression for v_{2f} kinetic energy equation and solve possible quadratic equation for v_{1f}

3. Use result to find expression for v_{2f}

4. Divide momentum equations to obtain expression for θ_{2f}

Carry out Plan: Step 1

- x-momentum: $v_{2,f}\cos\theta_{2,f} = v_{1,0} - v_{1,f}\cos\theta_{1,f}$

$$v_{2,f}{}^2\cos^2\theta_{2,f} = v_{1,f}{}^2\cos^2\theta_{1,f} - 2v_{1,f}\cos\theta_{1,f}v_{1,0} + v_{1,0}{}^2$$

- y-momentum: $v_{2,f}\sin\theta_{2,f} = v_{1,f}\sin\theta_{1,f}$

$$v_{2,f}{}^2\sin^2\theta_{2,f} = v_{1,f}{}^2\sin^2\theta_{1,f}$$

- Add using $\sin^2\theta + \cos^2\theta = 1$

$$v_{2,f}{}^2 = v_{1,f}{}^2 - 2v_{1,f}\cos\theta_{1,f}v_{1,0} + v_{1,0}{}^2$$

Carry out Plan: Step 2

- Energy: $$\frac{1}{2}m_1 v_{1,0}{}^2 = \frac{1}{2}m_1 v_{2,f}{}^2 + \frac{1}{2}m_1 v_{1,f}{}^2$$

- Substitute result from momentum:

$$v_{1,0}{}^2 = v_{2,f}{}^2 + v_{1,f}{}^2 = 2v_{1,f}{}^2 - 2v_{1,f}\cos\theta_{1,f}v_{1,0} + v_{1,0}{}^2$$

$$0 = 2v_{1,f}{}^2 - 2v_{1,f}\cos\theta_{1,f}v_{1,0}$$

- Easy to solve (no quadratic) for v_{1f}:

$$v_{1,f} = v_{1,0}\cos\theta_{1,f}$$

Carry out Plan: Step 3

- Use results from step 1 to solve for v_{2f}:

$$v_{2,f} = \left(v_{1,f}{}^{2} - 2v_{1,f}\cos\theta_{1,f}v_{1,0} + v_{1,0}{}^{2}\right)^{1/2}$$

$$v_{2,f} = \left(\left(v_{1,0}\cos\theta_{1,f}\right)^{2} - 2\left(v_{1,0}\cos\theta_{1,f}\right)\cos\theta_{1,f}v_{1,0} + v_{1,0}{}^{2}\right)^{1/2}$$

$$v_{2,f} = v_{1,0}\left(1-\cos^{2}\theta_{1,f}\right)^{1/2} = v_{1,0}\sin\theta_{1,f}$$

Carry out Plan: Step 4

- Divide momentum equations to solve for θ_{2f}:

$$v_{2,f} \sin\theta_{2,f} = v_{1,f} \sin\theta_{1,f}$$

$$v_{2,f} \cos\theta_{2,f} = v_{1,0} - v_{1,f} \cos\theta_{1,f}$$

$$\text{cotan}\,\theta_{2,f} = \frac{v_{1,0} - v_{1,f} \cos\theta_{1,f}}{v_{1,f} \sin\theta_{1,f}}$$

- Substitute result for v_{1f}: $v_{1,f} = \cos\theta_{1,f} v_{1,0}$

$$\theta_{2,f} = \text{cotan}^{-1}\left(\frac{v_{1,0} - v_{1,f} \cos^2\theta_{1,f}}{v_{1,0} \cos\theta_{1,f} \sin\theta_{1,f}} \right)$$