Problem Solving Strategies: Simple Harmonic Oscillator and Mechanical Energy

8.01t

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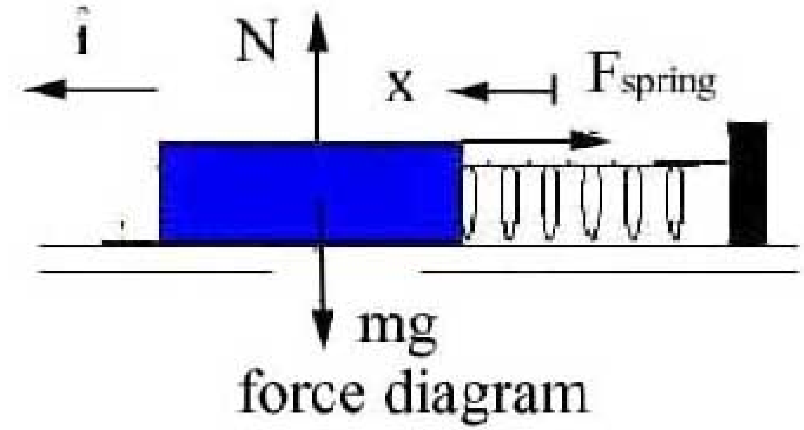
Modeling the Motion: Newton 's Second Law

- Equation of Motion: $\hat{\mathbf{i}}: -kx = m\dfrac{d^2x}{dt^2}$

- Possible Solution: Period of oscillation is T



- Position $x = A\cos(\dfrac{2\pi}{T}t)$

- Initial Position t = 0: $A = x_0$

- Velocity: $v = \dfrac{dx}{dt} = -\dfrac{2\pi}{T}A\sin(\dfrac{2\pi}{T}t)$

- Velocity at t = T/4 $v_{eq} = -\dfrac{2\pi}{T}A = -\dfrac{2\pi}{T}x_0$

Mechanical Energy is Constant

$$E_{eq} - E_0 = \frac{1}{2} m v_{eq}{}^2 - \frac{1}{2} k x_0{}^2 = 0$$

Solve for velocity at equilibrium position $v_{eq} = -\sqrt{\frac{k}{m}} x_0$

Period T: Condition from Newton’s Second Law

$$-\frac{2\pi}{T} x_0 = v_{eq}$$

Solve for Period

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}} \qquad T = 2\pi \sqrt{\frac{m}{k}}$$

The Gravitational Field of a Spherical Shell of Matter

- The gravitational force on a mass placed outside a spherical shell of matter of uniform surface mass density is the same force that would arise if all the mass of the shell were placed at the center of the sphere.
- The gravitational force on a mass placed inside a spherical shell of matter is zero.

The Gravitational Field of a Spherical Shell of Matter

$$\vec{\mathbf{F}}_{m,s}(r) = \begin{cases} -G\dfrac{mm_s}{r^2}\hat{\mathbf{r}}, & r > R \\ \vec{\mathbf{0}}, & r < R \end{cases}$$

where $\hat{\mathbf{r}}$ is the unit vector located at the position of mass and pointing radially away from the center of the shell.

Gravitational Force Inside Uniform Sphere

- Place mass m at distance r < R_E from center

- Force only due mass of earth inside sphere of radius r

$$m_r = \rho(4/3)\pi r^3$$

- Density

$$\rho = m_e /(4/3)\pi R_e^{\,3}$$

- Force law

$$\vec{\mathbf{F}}_{grav} = -\frac{Gmm_r}{r^2}\hat{\mathbf{r}} =$$