Problem Solving Strategies: Mechanical Energy

8.01t

Oct 20, 2004

Class Problem: Block-Spring System

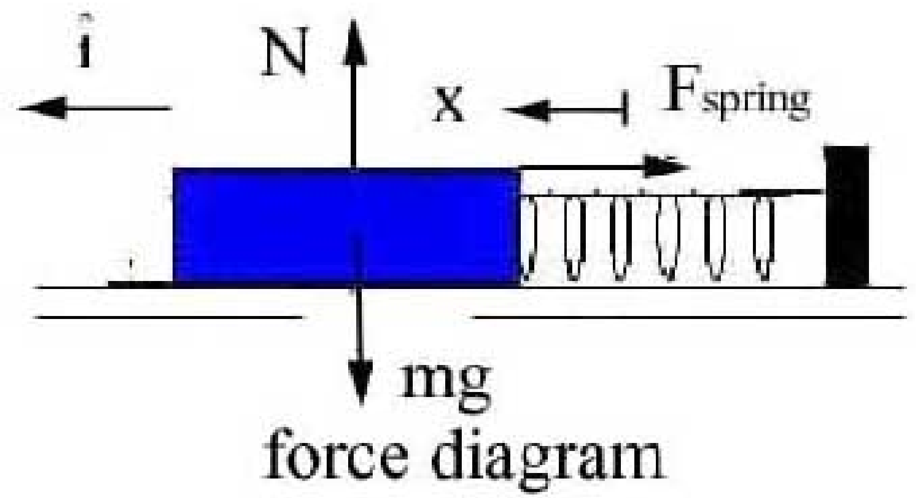
Example 1: A block of mass m is attached to a spring and is free to slide along a horizontal frictionless surface. At t = 0 the block-spring system is stretched an amount $x_0 > 0$ from the equilibrium position and is released from rest.

1. Is the mechanical energy of the block-spring constant?
2. What is the velocity of the block when it first comes back to the equilibrium?
3. What is the period of oscillation of the block?

Modeling the Motion: Newton ‘s Second Law

- Define system, choose coordinate system

- Draw force diagram



$$\vec{\mathbf{F}}_{spring} = -kx\hat{\mathbf{i}}$$

- Newton’ Second Law

$$\hat{\mathbf{i}} : -kx = m\frac{d^2x}{dt^2}$$

PRS Question

Which of the following functions x(t) of the variable t have a second derivative which is proportional to the negative of the function

$$d^2x/dt^2 \sim -x$$

1. $x(t) = (1/2)at^2$
2. $x(t) = Ae^{t/T}$
3. $x(t) = Ae^{-t/T}$
4. $x(t) = A\cos((2\pi/T)t)$

PRS Question

The first derivative $v = dx/dt$ of the sinusoidal function

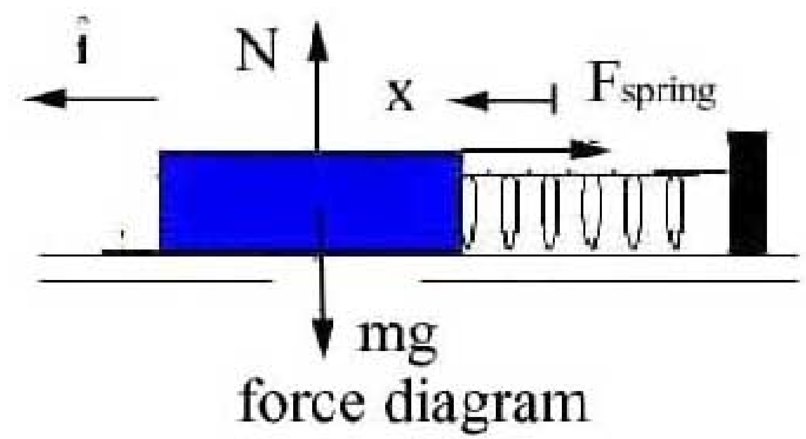
$$x(t) = A\cos((2\pi/T)t)$$

is:

1. $v(t) = A\cos((2\pi/T)t)$
2. $v(t) = -A\sin((2\pi/T)t)$
3. $v(t) = -(2\pi/T)\,A\sin((2\pi/T)t)$
4. $v(t) = (2\pi/T)\,A\cos((2\pi/T)t)$

Modeling the Motion: Newton 's Second Law

- Equation of Motion: $\hat{\mathbf{i}}: -kx = m\dfrac{d^2x}{dt^2}$
- Possible Solution: Period of oscillation is T



- Position $x = A\cos(\dfrac{2\pi}{T}t)$
- Initial Position t = 0: $A = x_0$
- Velocity: $v = \dfrac{dx}{dt} = -\dfrac{2\pi}{T}A\sin(\dfrac{2\pi}{T}t)$
- Velocity at t = T/4 $v_{eq} = -\dfrac{2\pi}{T}A = -\dfrac{2\pi}{T}x_0$

Modeling the Motion: Energy

- Choose initial and final states
- Determine external work

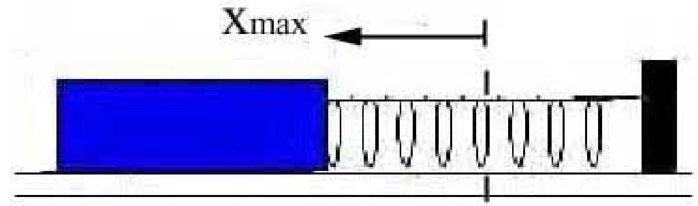
$$W_{nc} = 0$$

- Choose zero point for potential energy

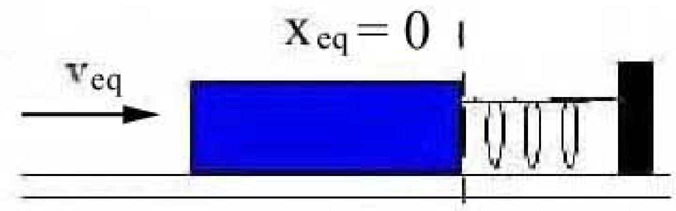
$$U(x = 0) = 0$$

- Mechanical energy is constant

$$\Delta K + \Delta U = 0$$



initial state: maximum stretch



final state: unstretched

Energy Diagram

Initial state: at rest with maximum stretch x_0 >0
and initial velocity v_0 = 0

- Kinetic energy $K_0 = 0$
- Potential energy $U_0 = \frac{1}{2}kx_0^2$
- Mechanical energy $E_0 = \frac{1}{2}kx_0^2$

Final state: equilibrium position x_{eq} = 0 and velocity v_{eq} < 0

- Kinetic energy $K_{eq} = \frac{1}{2}mv_{eq}^2$
- Potential energy $U_{eq} = 0$
- Mechanical energy $E_{eq} = \frac{1}{2}mv_{eq}^2$

PRS Question

A block of mass m is attached to a spring and is free to slide along a horizontal frictionless surface. At t = 0 the block-spring system is stretched an amount x_0 from the equilibrium position and is released from rest. What is the velocity of the block when it first comes back to the equilibrium?

1. $V_{eq} = -x_0 T/4$
2. $V_{eq} = x_0 T/4$
3. $V_{eq} = -(k/m)^{1/2} x_0$
4. $V_{eq} = (k/m)^{1/2} x_0$

Mechanical Energy is Constant

$$E_{eq} - E_0 = \frac{1}{2} m v_{eq}{}^2 - \frac{1}{2} k x_0{}^2 = 0$$

Solve for velocity at equilibrium position $v_{eq} = -\sqrt{\frac{k}{m}} x_0$

Period T: Condition from Newton's Second Law

$$-\frac{2\pi}{T} x_0 = v_{eq}$$

Solve for Period

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Class Problem: Block-Spring System with Friction

Example 2: A block of mass m slides along a horizontal surface with speed v_0. At t = 0 it hits a spring with spring constant k and begins to experience a friction force. The coefficient of friction is variable and is given by $\mu_k = bx$ where b is a constant. Find the loss in mechanical energy when the block has first come momentarily to rest.

Experiment 06: Work, Energy and the Harmonic Oscillator

TEAL
MIT Studio Physics
8.01T

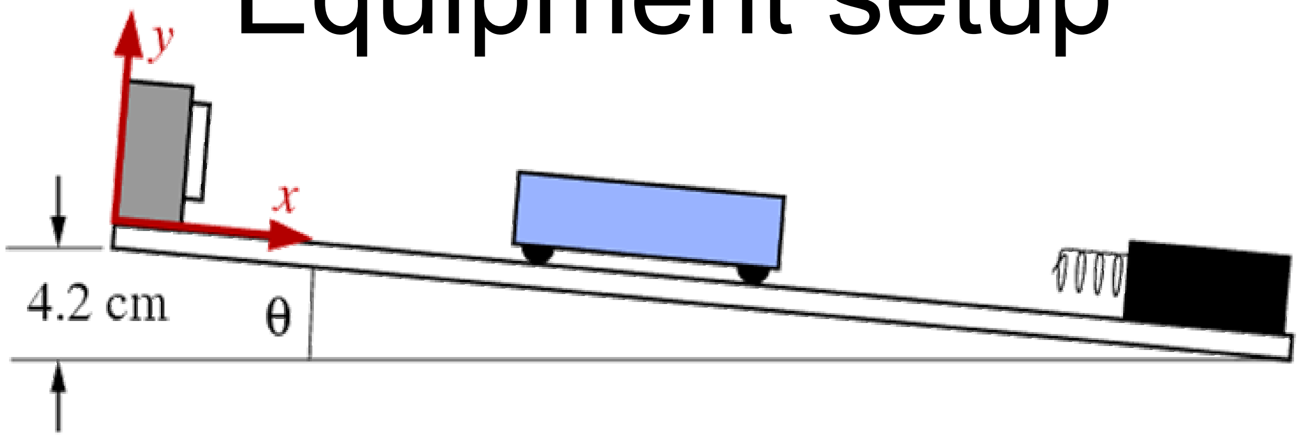
Goals

- Investigate the work-mechanical energy theorem.

- Observe how forms of mechanical energy are converted from one to another and lost by non-conservative work.

- Study the behavior of a simple harmonic motion with a high quality low-loss spring.

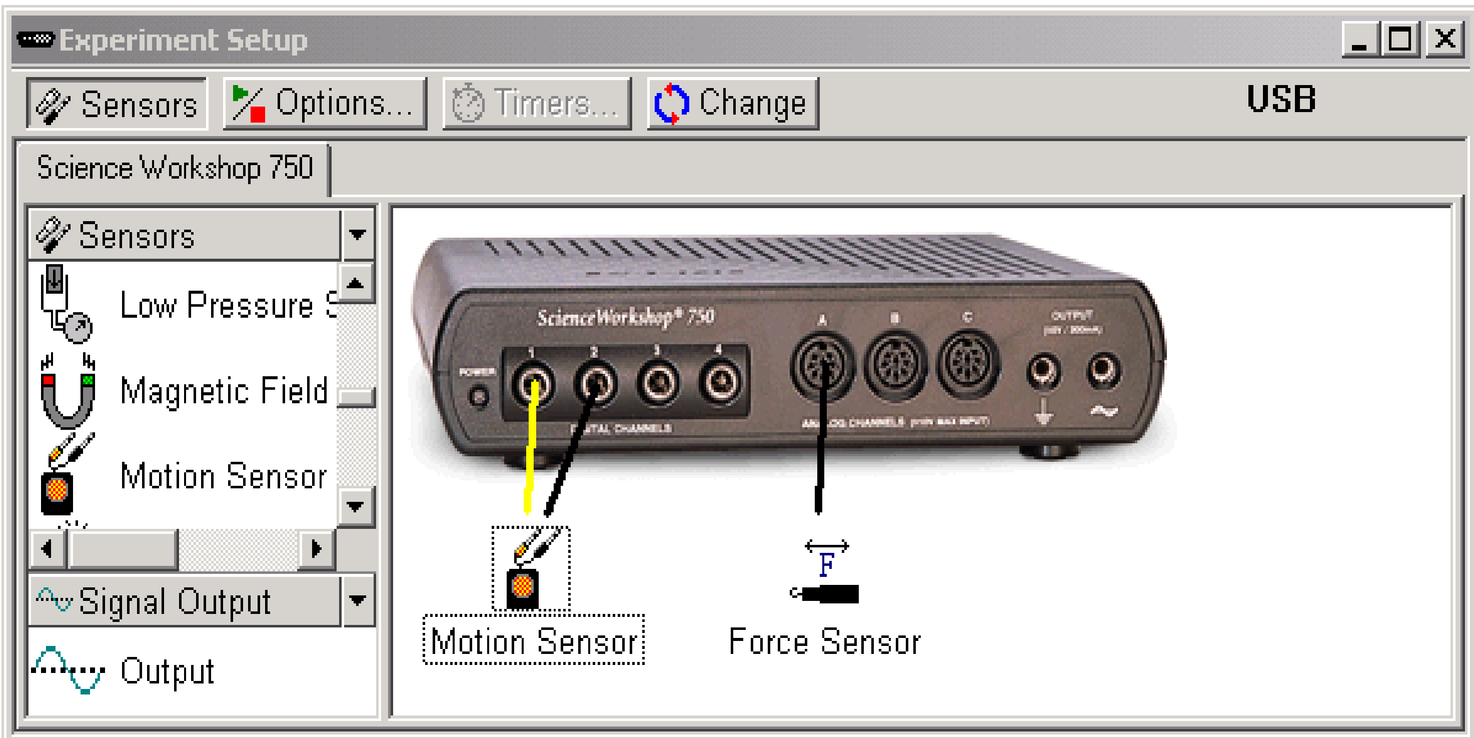
Equipment setup



- Use the heavy spring on the force sensor.
- Put two 250g weights in the cart.
- Clip motion sensor to other end of track, and support it on a piece of 2x4.

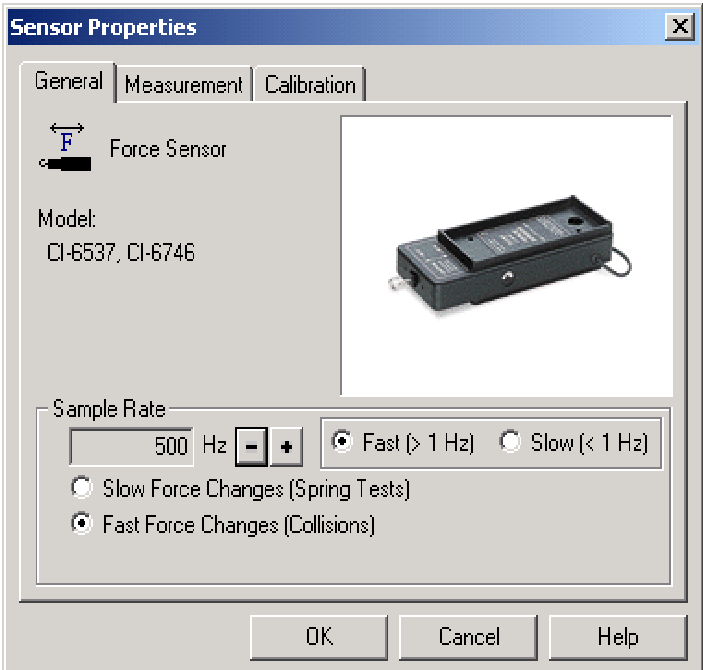
Starting DataStudio

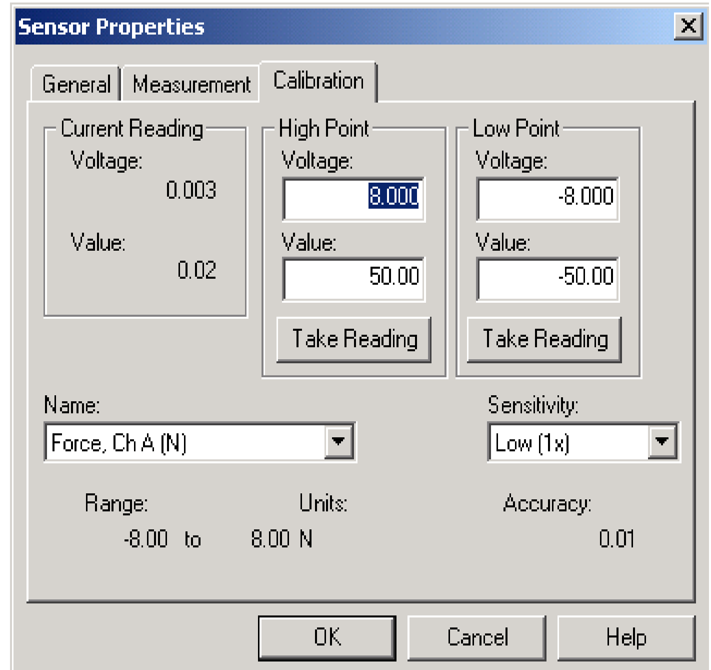
- ❑ Create a new experiment.
- ❑ Plug force and motion sensors into the 750 and
- ❑ drag their icons to inputs in the Setup window.



- ❑ Double-click the Force Sensor icon.

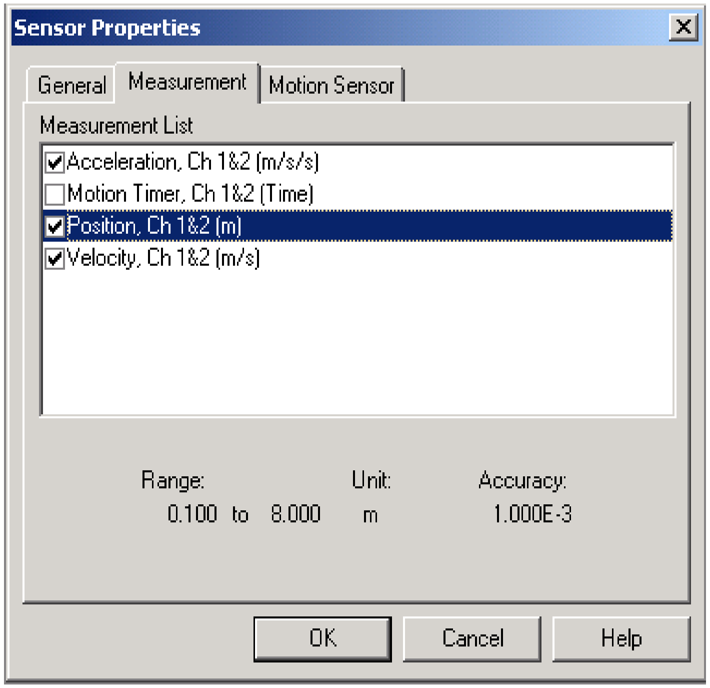
Force Sensor

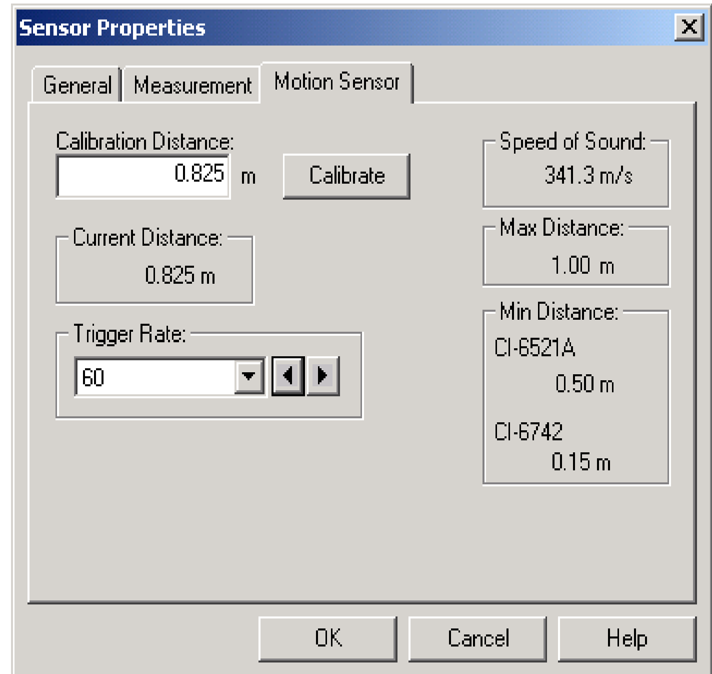




- ❑ Set Sample Rate to 500Hz and Sensitivity to Low.
- ❑ Double-click the Motion Sensor Icon.

Motion Sensor

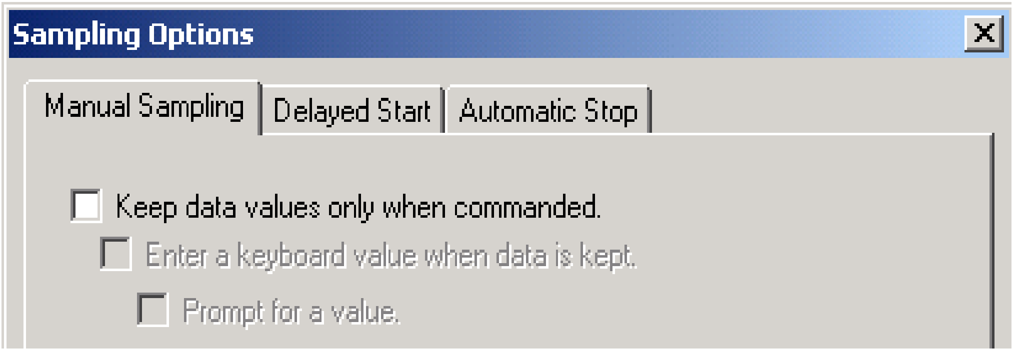




- Ensure to have Acceleration, Position and Velocity checked
- Set Trigger Rate to 60Hz and
- calibrate distance to cart when it is resting against the spring.

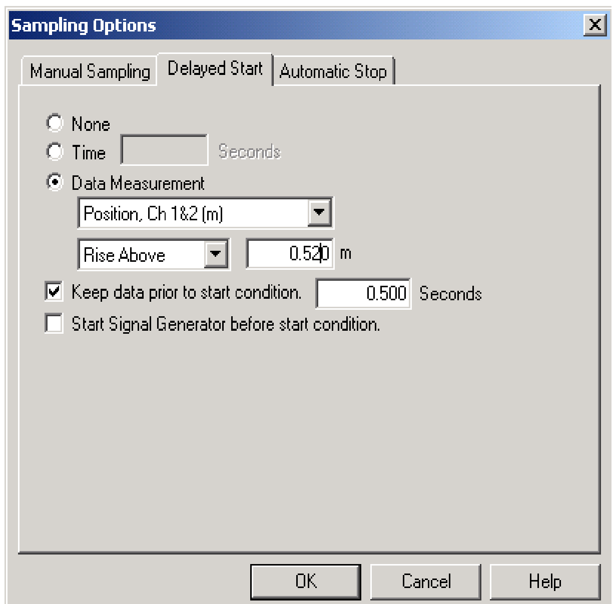
Click 

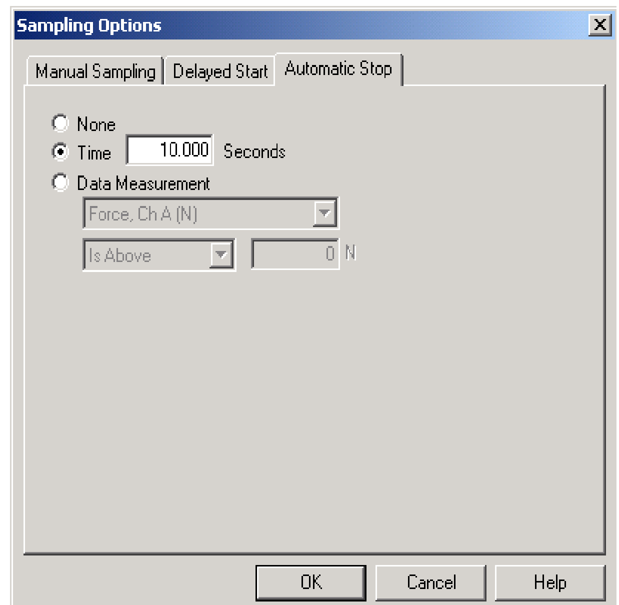
Sampling Options



No boxes checked!

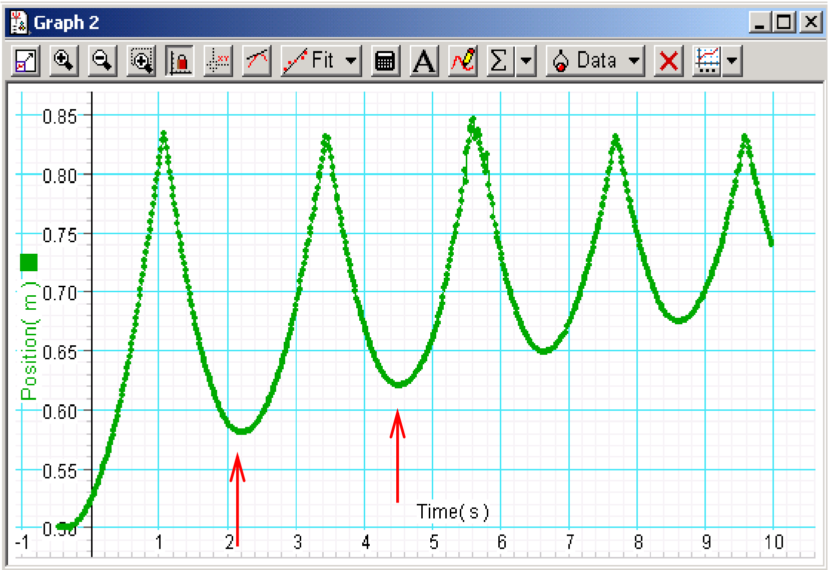
Delayed start on position measurement!



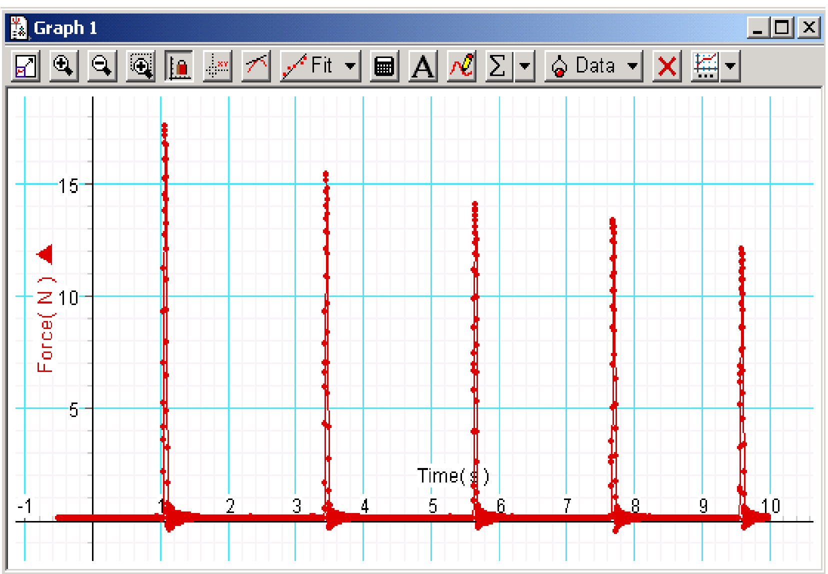


Stop after 10s!

Measurement Results

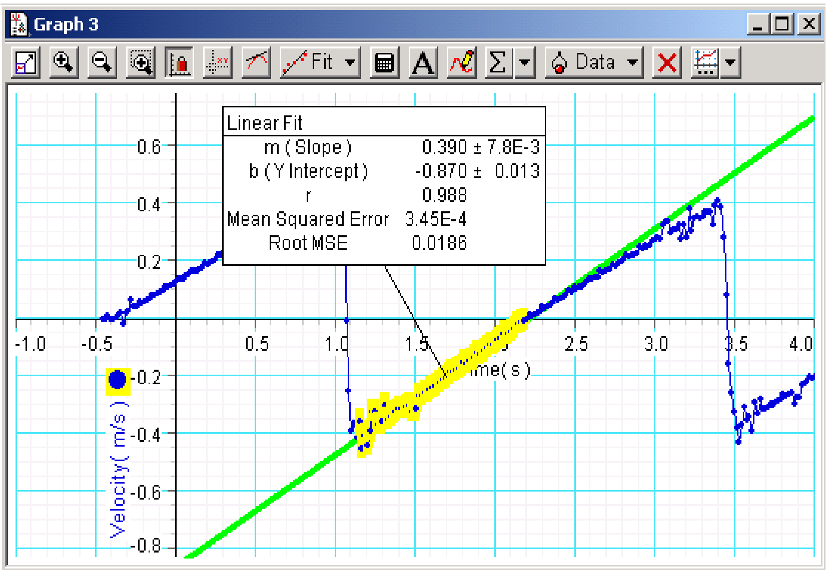


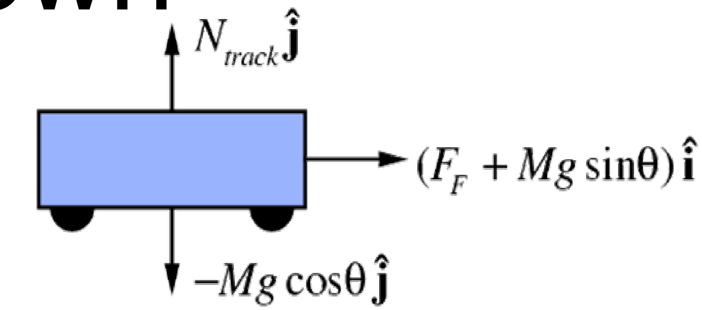
- Position vs. Time: Measure maximum heights either side of 2nd bounce, calculate loss of potential energy, and friction force. Enter in table!



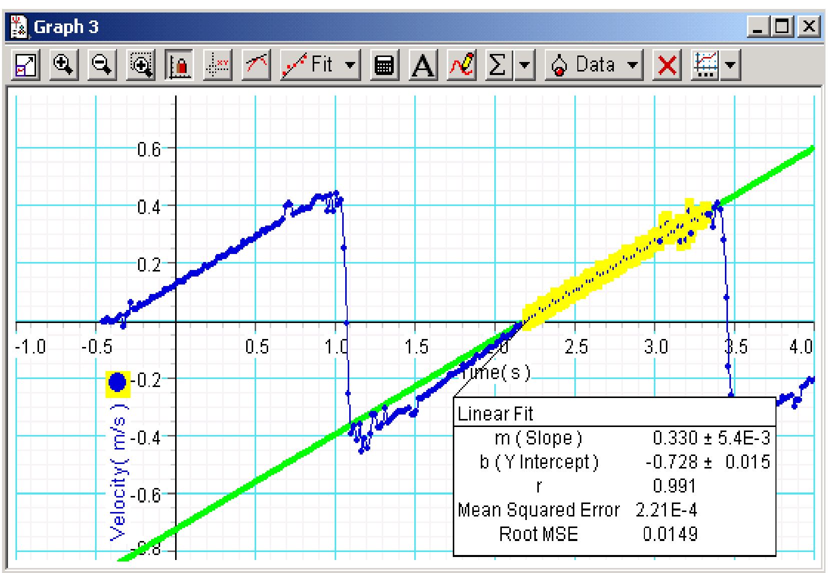
- Force vs. Time: Expand force peak around 2nd bounce.

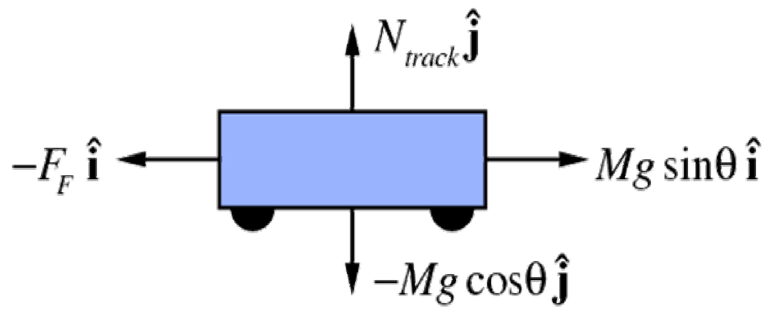
Finding Acceleration Up & Down





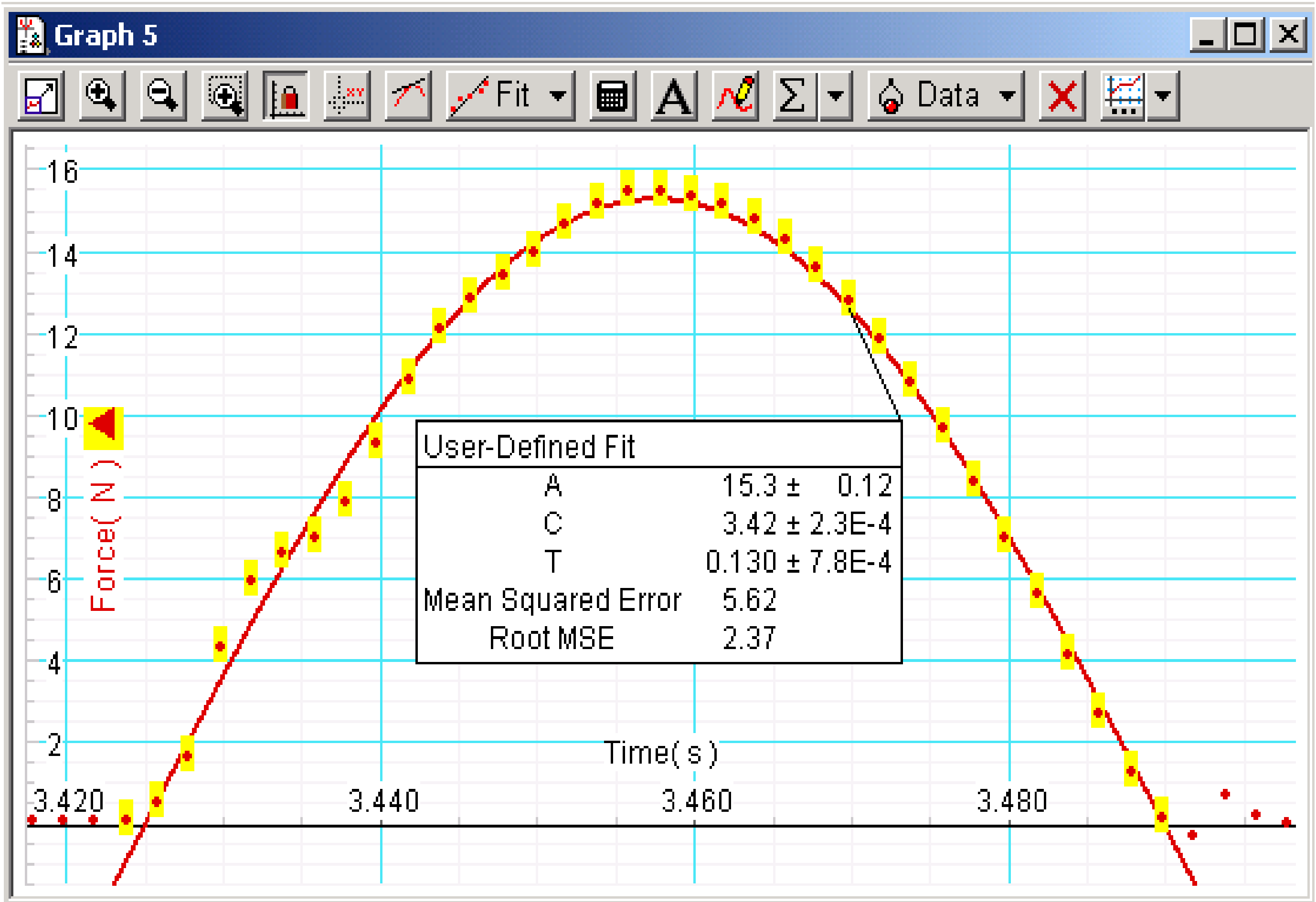
Linear fit to find a_{up}





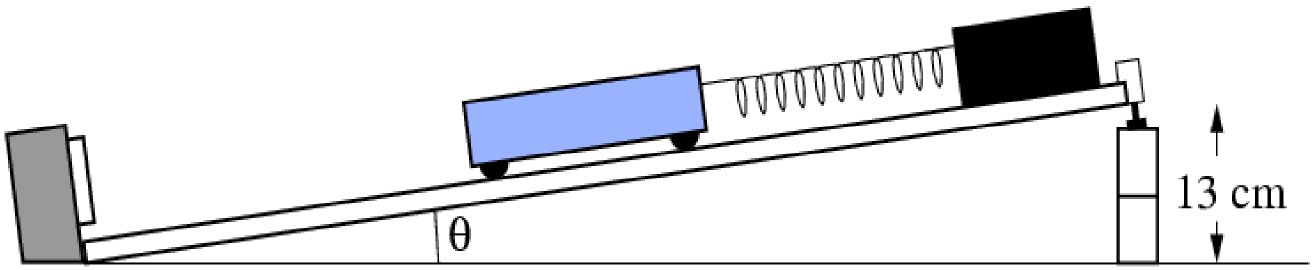
Linear fit to find a_{down}

Analysis Force Peak



User-Defined Fit to A*sin(2*pi(x-C)/T)

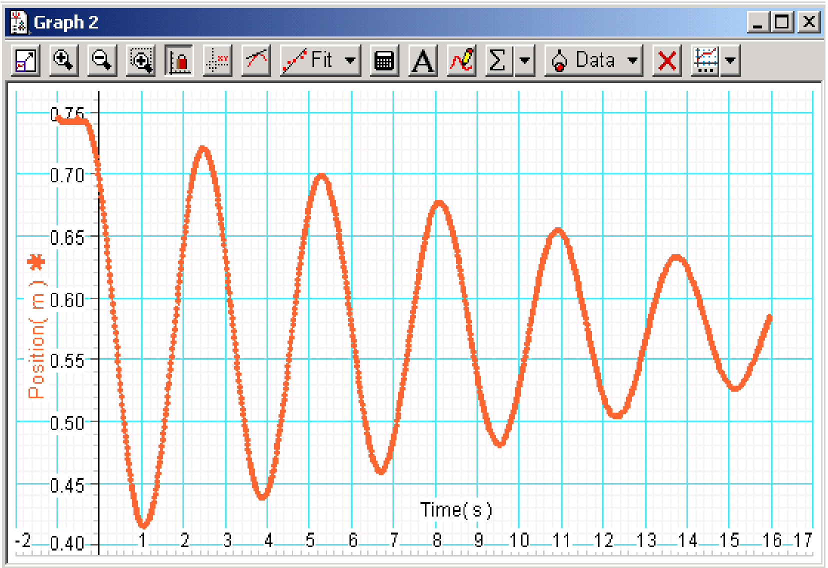
Harmonic Oscillator



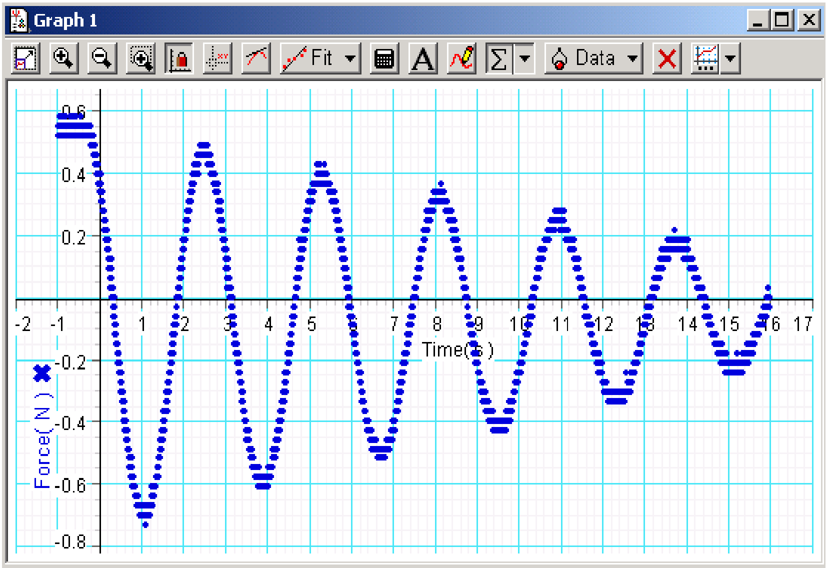
Unclip motion sensor, raise the force sensor end of track

Attach spring to plunger on cart with a binder clip and to the hook on force sensor.
Add two 250g weights in the cart.
Place motion sensor on table touching other end of track.
Set Delayed Start and Auto Stop.

Harmonic Oscillator Results



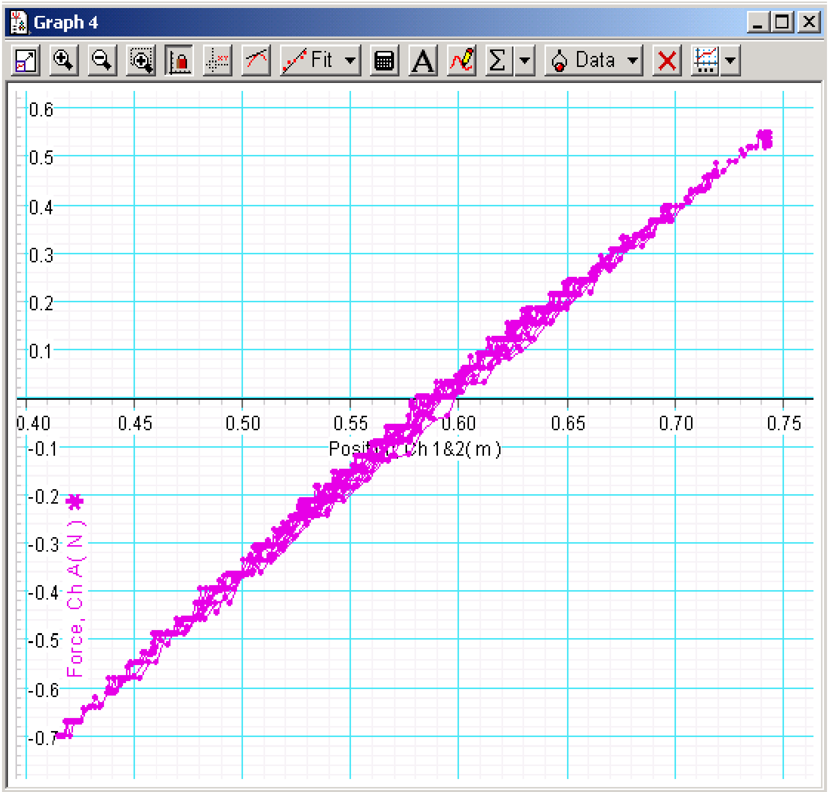
Position vs. Time:
Measure the period, and calculate spring constant k from M = 0.75 kg.



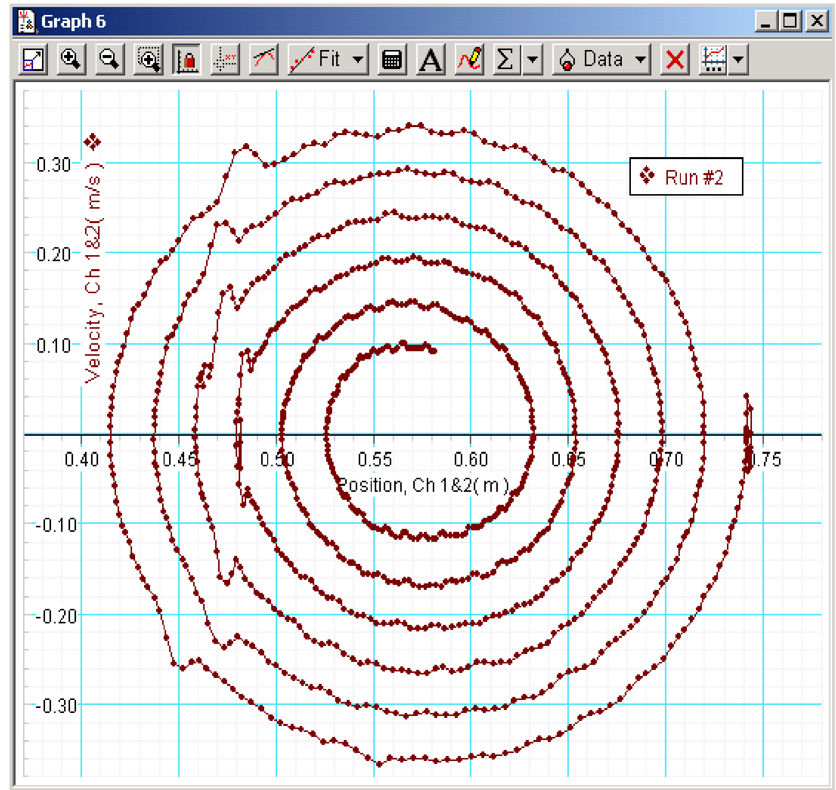
Force vs. Time.

Make a plot of force vs. position.

Lissajous Patterns

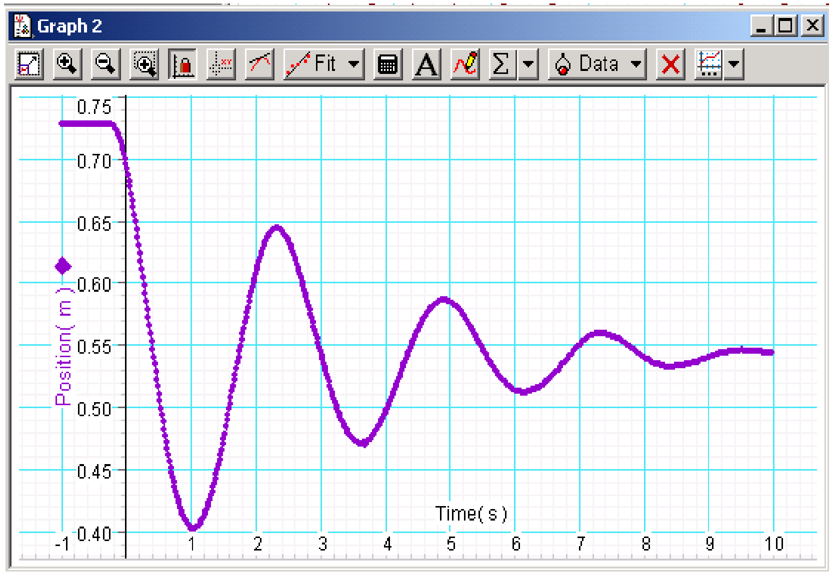


Force vs. Position: Find k from a Linear Fit.



Velocity vs. Position.

Rubber Band Spring - Optional



Position vs. Time:
Note increased damping.

Force vs. Position.
Not linear.