

Circular Motion and Universal Law of Gravitation

8.01t

Oct 4, 2004

Summary: Circular Motion

- arc length $s = R\theta$
- tangential velocity $v = \frac{ds}{dt} = R \frac{d\theta}{dt} = R\omega$
- tangential acceleration $a_\theta = \frac{dv_\theta}{dt} = R \frac{d^2\theta}{dt^2} = R\alpha$
- centripetal acceleration $|a_r| = v\omega = \frac{v^2}{R} = R\omega^2$

Circular Motion Vector Description

- Use plane polar coordinates

- Position $\vec{\mathbf{r}}(t) = R\hat{\mathbf{r}}(t)$

- Velocity $\vec{\mathbf{v}}(t) = R \frac{d\theta}{dt} \hat{\boldsymbol{\theta}}(t) = R\omega\hat{\boldsymbol{\theta}}(t)$

$$\vec{\mathbf{a}} \equiv a_r \hat{\mathbf{r}} + a_\theta \hat{\boldsymbol{\theta}}$$

- Acceleration

$$a_\theta = r\alpha$$

$$a_r = -r\omega^2 = -(v^2 / r)$$

Modeling Problems: Circular Motion

- Always has a component of acceleration pointing radially inward
- May or may not have tangential component of acceleration
- Free Body Diagram for all forces
- mv^2/r is not a force but mass times acceleration and does not appear on force diagram
- Choose one unit vectors to point in the radial direction

Equations of Motion

	$\vec{\mathbf{F}} \doteq m\vec{\mathbf{a}}$	

	physics	mathematics
$\hat{\mathbf{r}}$:	force decom-	= $-mv^2 / r$
$\hat{\theta}$:	position from	= $mr d^2\theta / dt^2$
$\hat{\mathbf{k}}$:	force diagram	= 0

PRS Question 1

You are a passenger in a racecar approaching a turn after a straight-away. As the car turns left on the circular arc at constant speed, you are pressed against the car door. Which of the following is true during the turn (assume the car doesn't slip on the roadway)?

1. A centripetal force pushes you against the door.
2. There is no force that pushes you against the door.
3. The frictional force of the ground pushes you against the door.
4. There is no centripetal force acting on you.
5. You cannot analyze this situation in terms of the forces on you since you are accelerating.
6. Two of the above.
7. None of the above.

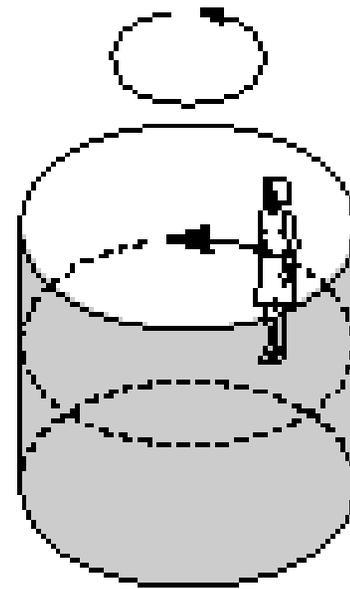
PRS Question 2

Which of the following types of forces can produce a centripetal acceleration ?

1. Normal force
2. Frictional force
3. Tension force
4. Gravitational force
5. All of the above

PRS Question 3

The "barrel of fun" must spin at a certain minimum angular speed in order for a rider of mass m_1 to stick to the wall. Does this minimum angular speed change for a rider of mass $m_2 > m_1$?

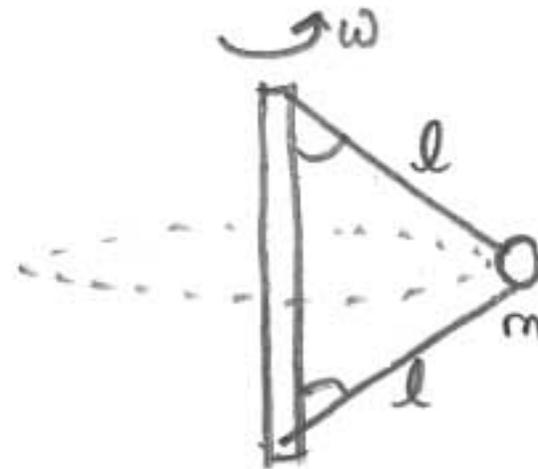


1. yes
2. no
3. Not enough information to determine.

PRS Question 4

A ball is suspended from a vertical rod by two strings of equal strength and equal length. The strings are very light and do not stretch. The rod is spun with a constant angular acceleration. Which string breaks first?

1. the upper string
2. the lower string
3. they break simultaneously
4. cannot tell without more information



Analysis of Acceleration

- Circular motion always has radial inward component

$$mv^2/r$$

- Uniform Circular Motion means

$$d^2\theta/dt^2 = 0$$

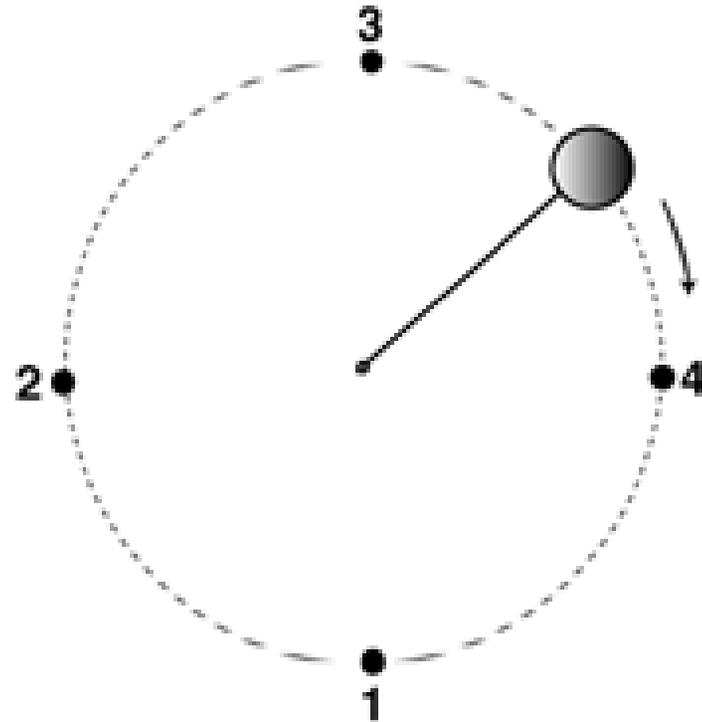
- Vertical direction

$$d^2z/dt^2 = 0$$

PRS Question 5

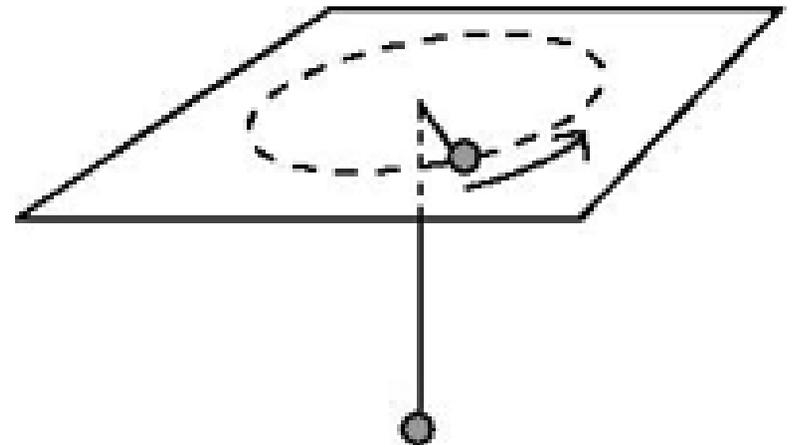
A stone attached to a string is whirled in a vertical plane. Let T_1 , T_2 , T_3 , and T_4 be the tensions at locations 1, 2, 3, and 4 required for the stone to have a given speed v_0 at these four locations.

1. $T_3 > T_2 > T_1 = T_4$
2. $T_1 = T_2 = T_3 = T_4$
3. $T_1 > T_2 = T_4 > T_3$
4. none of the above



PRS Question 6

A puck of inertia M is moving in a circle at uniform speed on a frictionless table as shown above. It is held by a string which holds a suspended bob, also of inertia M , at rest below the table. Half of the length of the string is above the tabletop and half below. What is the centripetal acceleration of the moving puck?



1. less than g
2. g
3. greater than g
4. zero
5. insufficient information

Universal Law of Gravitation

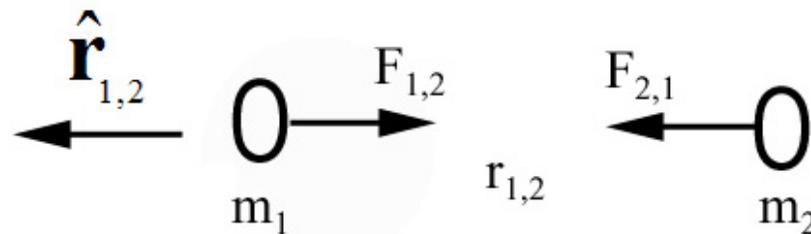
- The force on body 1 due to the gravitational interaction between two bodies of masses m_1 and m_2 is

$$\vec{\mathbf{F}}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{\mathbf{r}}_{1,2}$$

where

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}$$

and $\hat{\mathbf{r}}_{1,2}$ is the unit vector located at the position of body 1 and pointing from body 2 towards body 1.

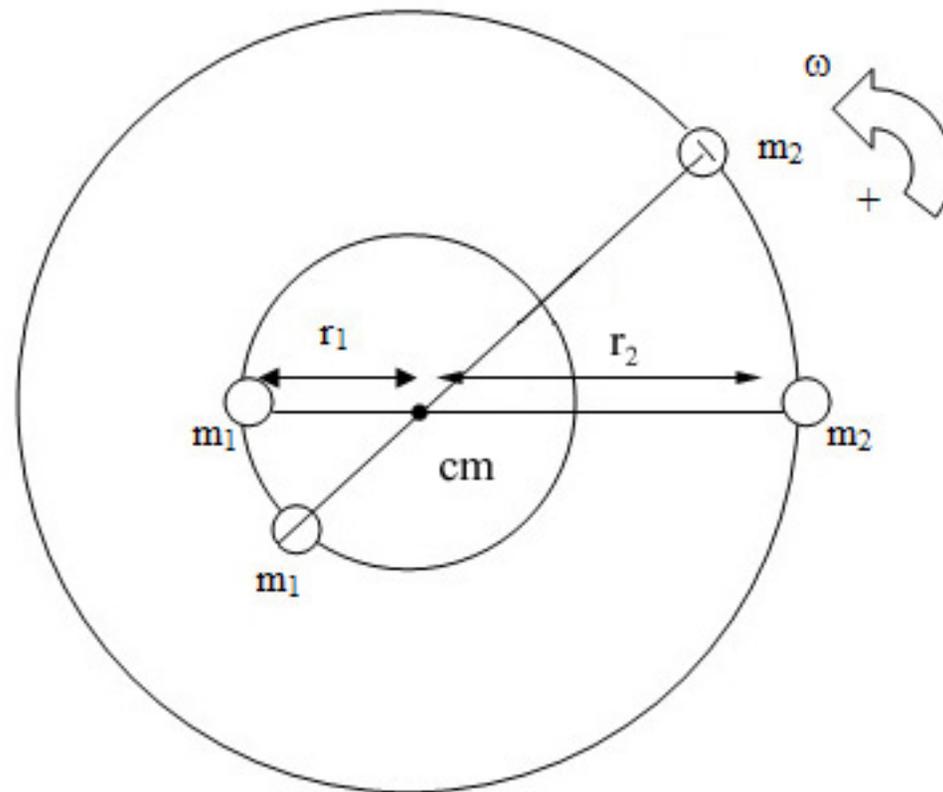


Double Star System

- **Example 1: Double Star System**

Consider a double star system under the influence of gravitational force between the stars. Star 1 has mass m_1 . Star 2 has mass m_2 . Assume that each star undergoes uniform circular motion about the center of mass of the system. If the stars are always a fixed distance r apart, what is the period of the orbit?

Class Problem 1 (con't)



The Gravitational Field of a Spherical Shell of Matter

- The gravitational force on a mass placed outside a spherical shell of matter of uniform surface mass density is the same force that would arise if all the mass of the shell were placed at the center of the sphere.
- The gravitational force on a mass placed inside a spherical shell of matter is zero.

The Gravitational Field of a Spherical Shell of Matter

$$\vec{\mathbf{F}}_{m,s}^{\square}(r) = \begin{cases} -G \frac{mm_s}{r^2} \hat{\mathbf{r}}, & r > R \\ \vec{\mathbf{0}}, & r < R \end{cases}$$

where $\hat{\mathbf{r}}$ is the unit vector located at the position of mass and pointing radially away from the center of the shell.