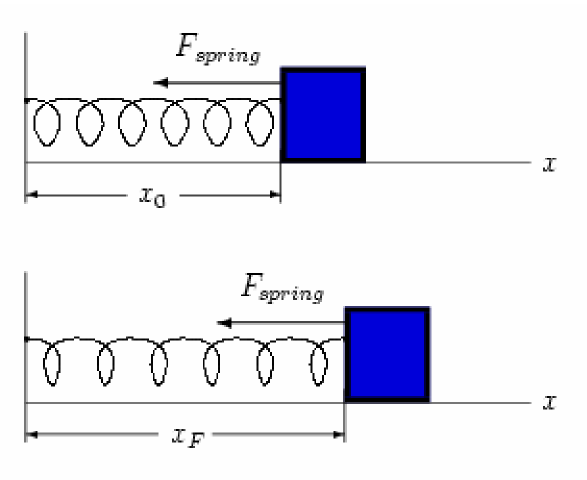
Hooke's Law and Circular Motion

8.01T

Sept 29, 2004

Stretching a Spring



Suppose we have a spring whose relaxed length is x_0, and stretch it to a length x_F.

According to Hooke's law, the restoring force will be proportional to the amount of stretch,

$$F_{spring} = -k(x_F - x_0) = -k\Delta x$$

where k is the spring *force constant*.

This is an example of *elastic* behavior; it is an experimental "law" that is valid only for a limited range of Δx.

We shall meet Hooke's law again and again in many different contexts. Hooke was a contemporary of Newton.

Experiment 04: Uniform Circular Motion

MIT Studio Physics
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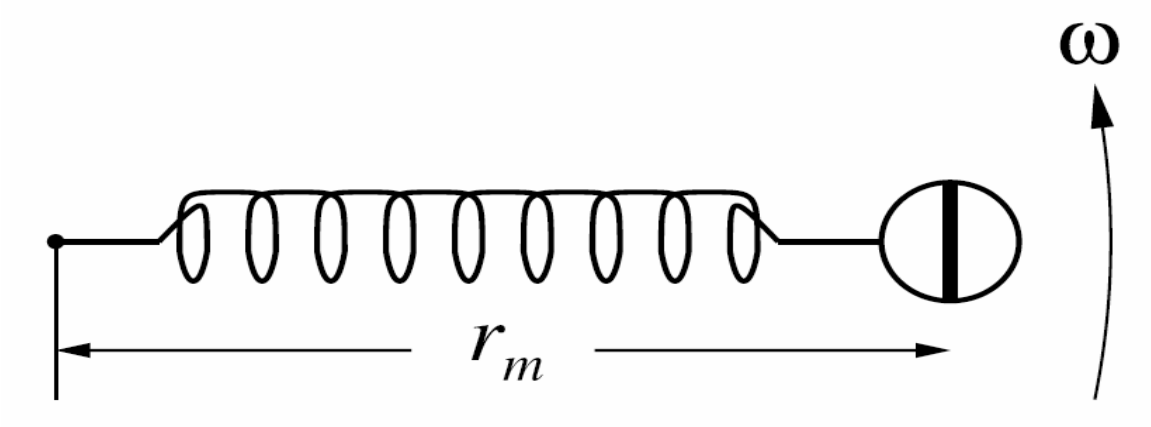
Goal

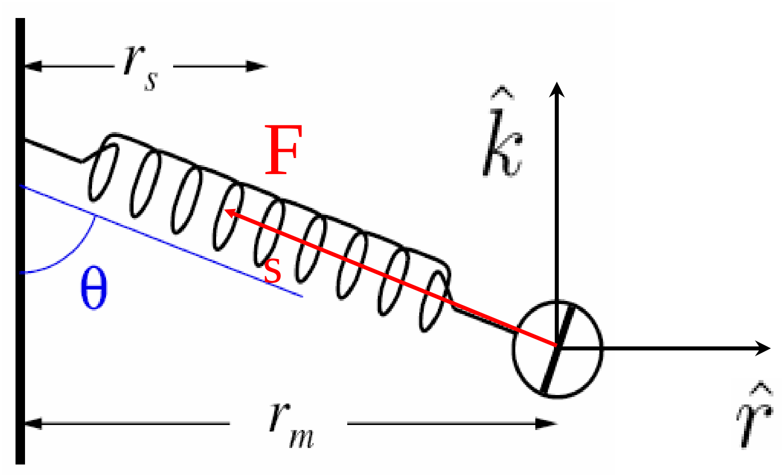
- Study a conical pendulum and measure the force required to produce a centripetal acceleration



- Extract from measurement of angular frequency and r_m the spring constant k and pre-tension F_0

- Understand how an instability in this system occurs at a critical frequency ω_0 and how to extract ω_0 from your measurements

Analysis of conical pendulum



$$\sum_i F_i = ma_i$$

$\hat{r}$ $$-F_s \cdot \sin\theta = -mr_m\omega^2$$

$\hat{k}$ $$-mg + F_s\cos\theta = 0$$

From my measurements of r_m, ω: $\theta \approx 88°$

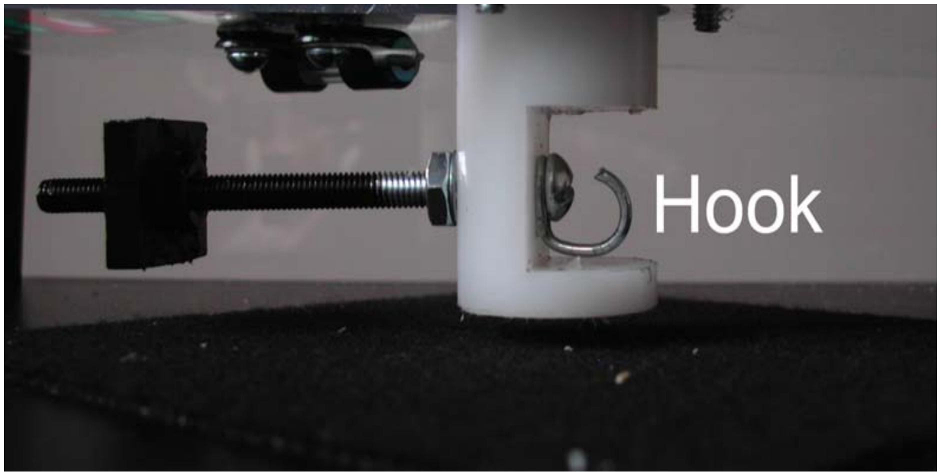
$$\tan\theta = \frac{r_m\omega^2}{g}$$

Therefore: Ignore effect of gravitation!

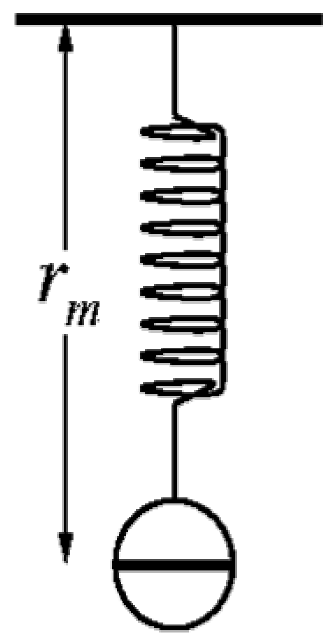
$$F_s = \Delta r \cdot k = (r_m - r_0)k = mr_m\omega^2$$

$$r_m = \frac{r_0}{1 - \frac{m\omega^2}{k}} = \frac{r_0}{1 - \frac{\omega^2}{\omega_c^2}}$$

Attaching the mass

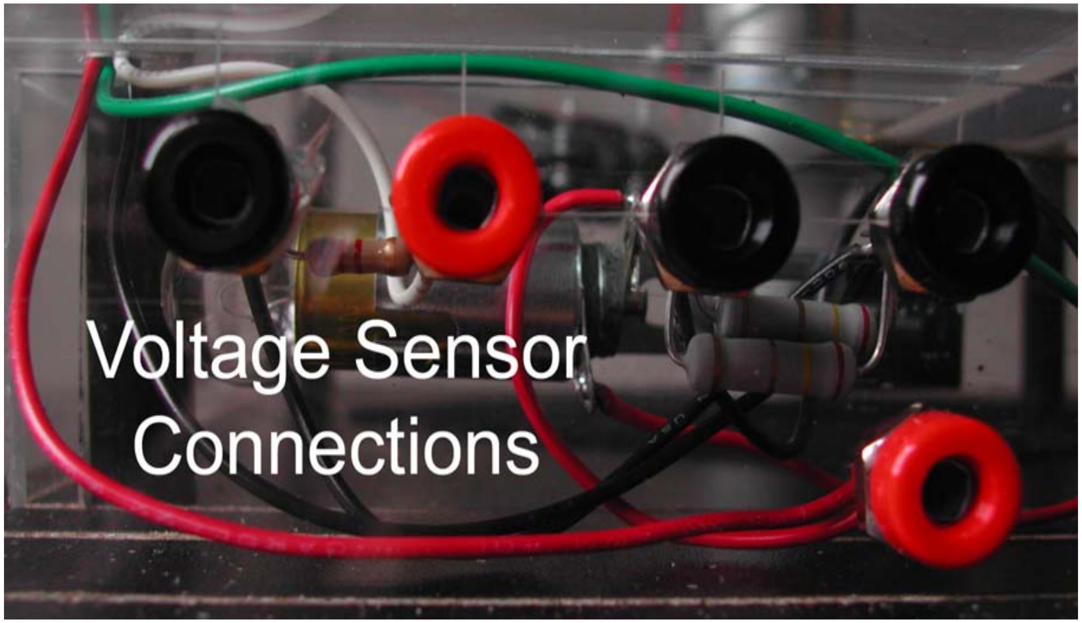


Turn the box on its side and measure the radius of the mass when $\omega = 0$:



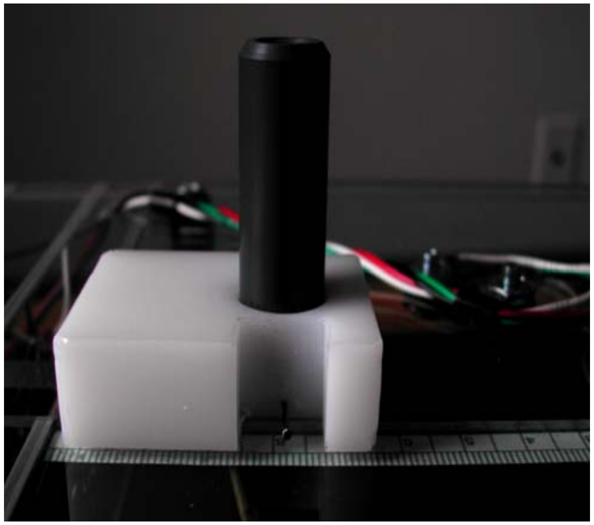
Connecting the motor

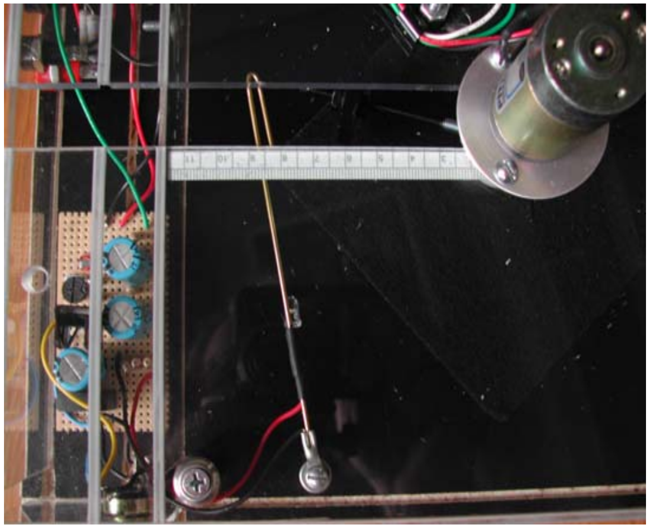
For safety, place the side panel after measuring r_0!



Connect the voltage sensor to the left side banana jacks!

Measurement of r_m





Wait for the rotation speed to stabilize.

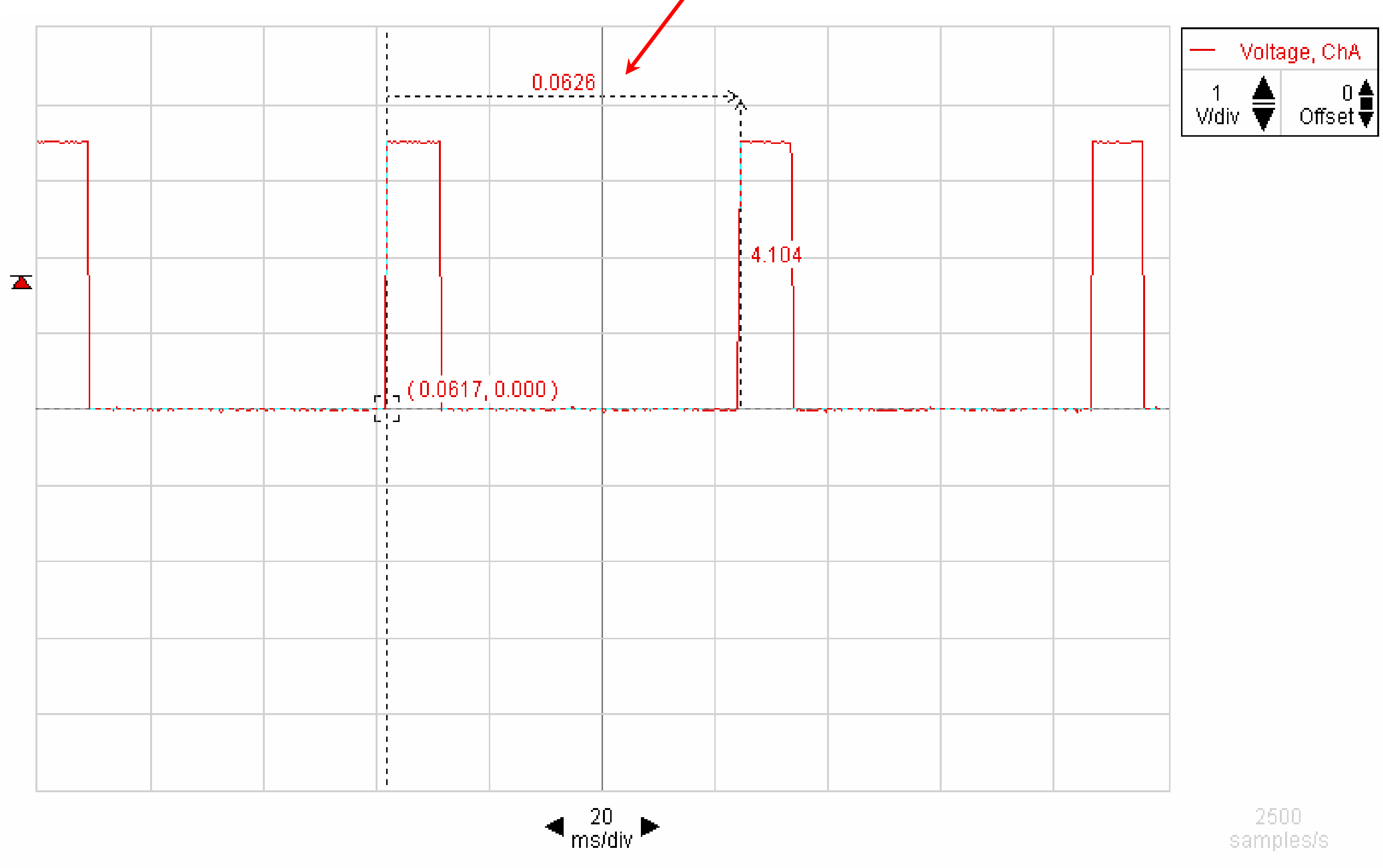
Use the "viewer" to measure r_m and the scope display to measure the period T.

(This may work better with the room lights off!)

Enter r_m and $\omega = 2\pi/T$ in a DataStudio table, with ω as the independent (left) variable.

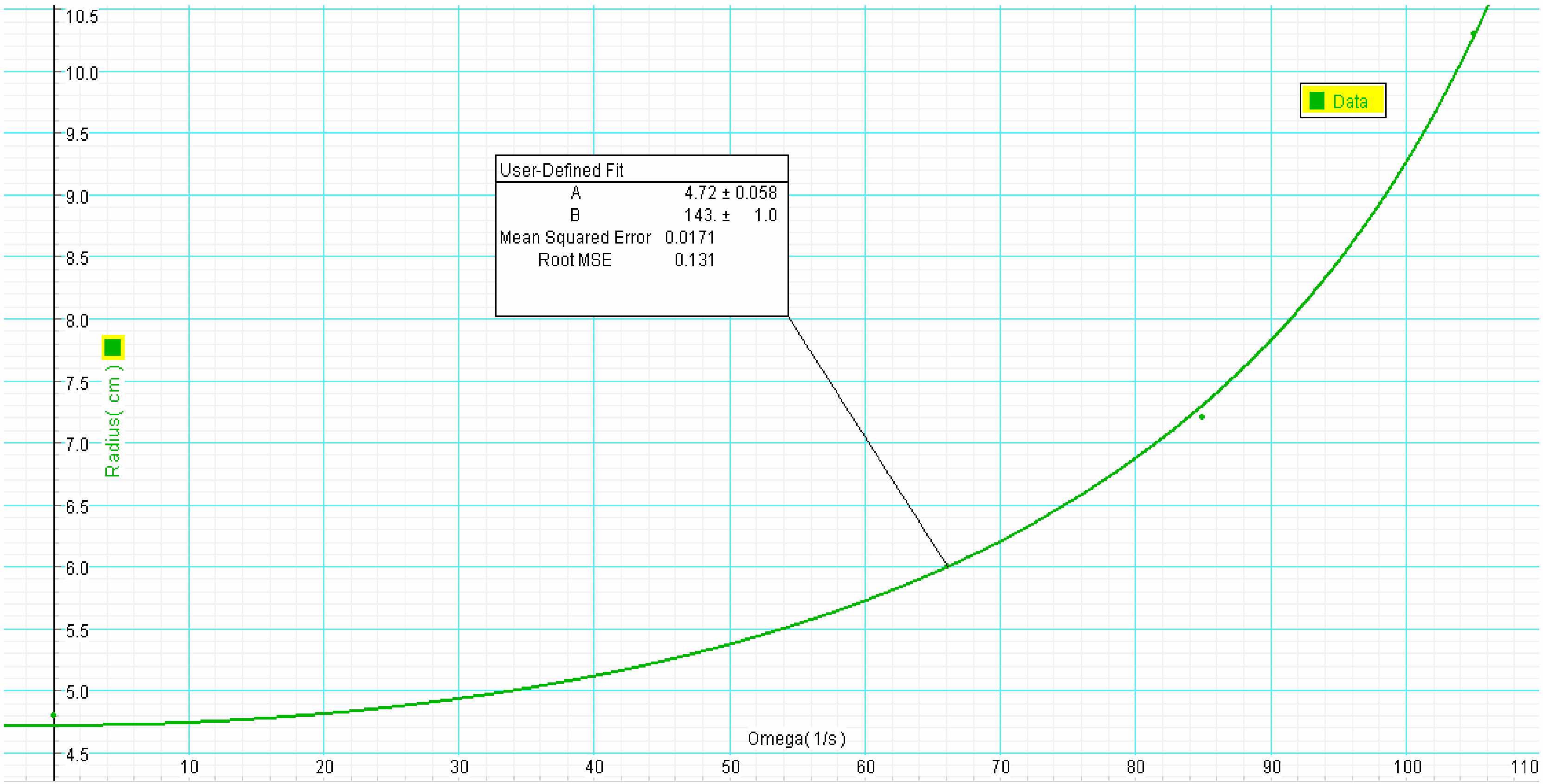
Measurement of ω

Example of scope trace to determine T: Here: T=0.0626s



Fitting

Perform a User-Defined fit to: A/(1-x*x/(B*B))



Fitting results

Defining $r_0 = r_m(\omega=0)$, compare model and fit:

$$r_m(\omega) = \frac{r_0}{1 - (\omega/\omega_c)^2} \quad r_m(x) = \frac{A}{1 - (x/B)^2}$$

Determine ω_c and k using
m=8.5g!

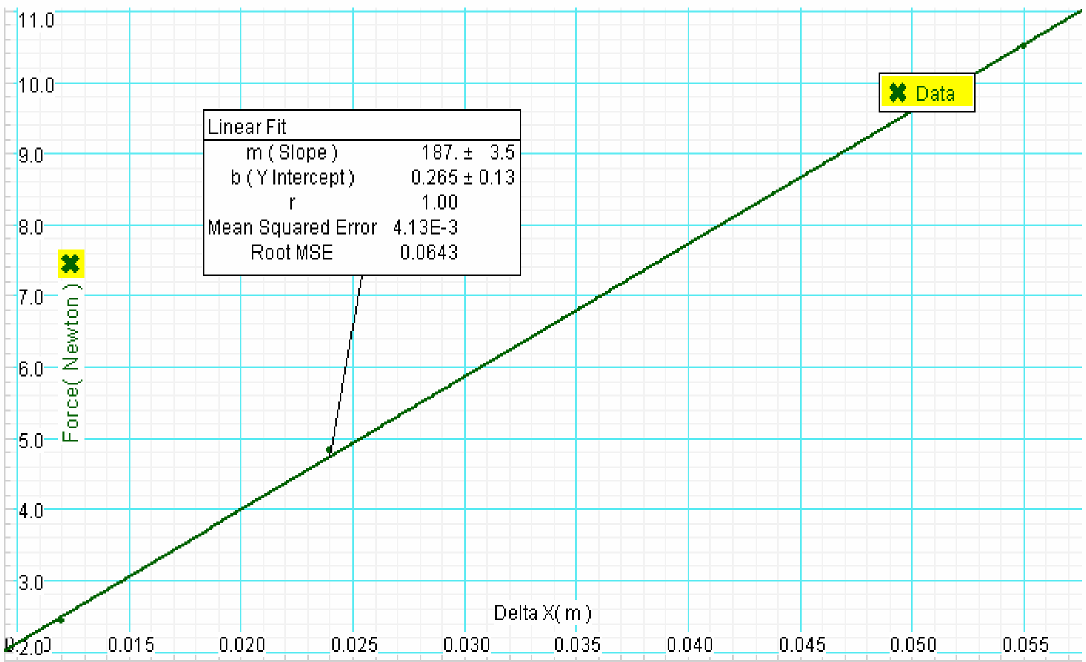
$$\omega_c = \sqrt{k/m}$$

How does A compare with your measured r_0?.

Determine k and pre-tension of spring

Enter $\Delta r = r_m - r_0$ and $F_s = r_m \omega^2$ in a DataStudio table, with Δr as the independent (left) variable.

Do a Linear Fit to find $F_s = F_0 + k\Delta r$



Determine k and pre-tension of spring

If you don't have access to a fitting program, you will make a very small mistake (because F_0 is small) if you assume $F_0 = 0$. Then finding k becomes trivial. If you do take this approach, explain to the grader that you lacked a means to fit the data outside of class and thus made this assumption. We will encourage the problem set graders not to penalize you for this.

Test Review

- One Dimensional Motion
- Two Dimensional Motion
- Force Law Problems
- Relative Inertial Reference Frames