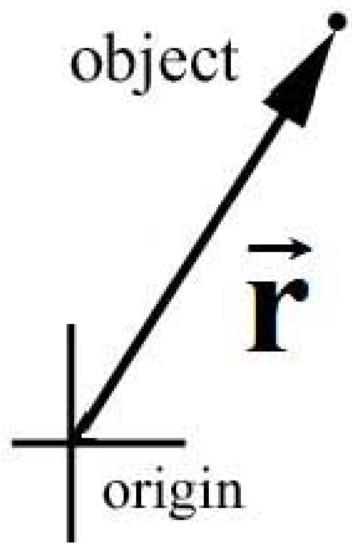
Applying Newton's Second Law

8.01T

Sept 22, 2004

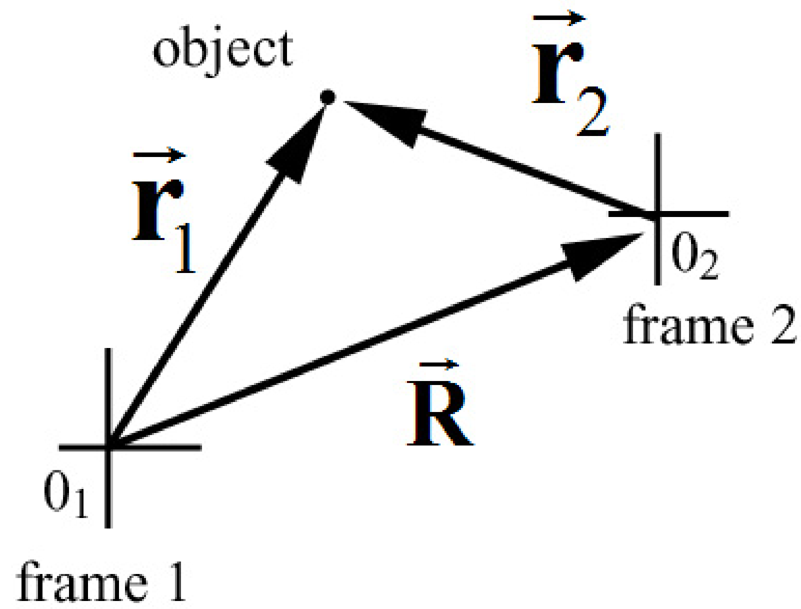
Reference Frame

Coordinate system with an observer placed at origin is a ***'reference frame'*** in which the position, velocity, and acceleration of objects are mathematically defined



Relative Reference Frames

- Two reference frames
- Origins need not coincide
- Moving object
- Position vectors in different frames



$$\vec{\mathbf{r}}_1 = \vec{\mathbf{R}} + \vec{\mathbf{r}}_2$$

Relatively Inertial Reference Frames

Relative velocity between the two reference frames is constant

$$\vec{\mathbf{V}} = d\vec{\mathbf{R}}\Big/dt$$

The relative acceleration is zero

$$\vec{\mathbf{A}} = d\vec{\mathbf{V}}\Big/dt = \vec{\mathbf{0}}$$

Frames are called *relatively inertial reference frames*

Law of Addition of Velocities

Suppose object is moving, then observers in different reference frames will measure different velocities

- velocity of the object in Frame 1: $\vec{\mathbf{v}}_1 = d\vec{\mathbf{r}}_1 / dt$
- velocity of the object in Frame 2: $\vec{\mathbf{v}}_2 = d\vec{\mathbf{r}}_2 / dt$
- velocity of an object in two different reference frames

$$\vec{\mathbf{v}}_1 = \vec{\mathbf{V}} + \vec{\mathbf{v}}_2$$

Acceleration in Relatively Inertial Reference Frames

Suppose object is accelerating, then observers in different relatively reference frames will measure the same acceleration

- acceleration of the object in Frame 1: $\vec{\mathbf{a}}_1 = d\vec{\mathbf{v}}_1 / dt$
- acceleration of the object in Frame 2: $\vec{\mathbf{a}}_2 = d\vec{\mathbf{v}}_2 / dt$
- Relative acceleration of frames is zero $\vec{\mathbf{A}} = d\vec{\mathbf{V}} / dt = \vec{\mathbf{0}}$
- acceleration of an object in two different reference frames $\vec{\mathbf{a}}_1 = \vec{\mathbf{A}} + \vec{\mathbf{a}}_2 = \vec{\mathbf{a}}_2$

Newton's First Law: Law of Inertia

Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it

Application: Newton's Second Law in either frame is

$$\vec{\mathbf{F}}_1^{\,T} = m\vec{\mathbf{a}}_1 \qquad \vec{\mathbf{F}}_2^{\,T} = m\vec{\mathbf{a}}_2$$

Relatively inertial reference frames have same accelerations.

$$\vec{\mathbf{a}}_1 = \vec{\mathbf{a}}_2$$

So use the same forces $\vec{\mathbf{F}}_1^{\,T} = \vec{\mathbf{F}}_2^{\,T}$

So it does not matter which frame you choose to describe the problem.

Principle of Relativity

For observers moving in two different reference frames, no mechanical force can distinguish which observer is at rest and which observer is moving

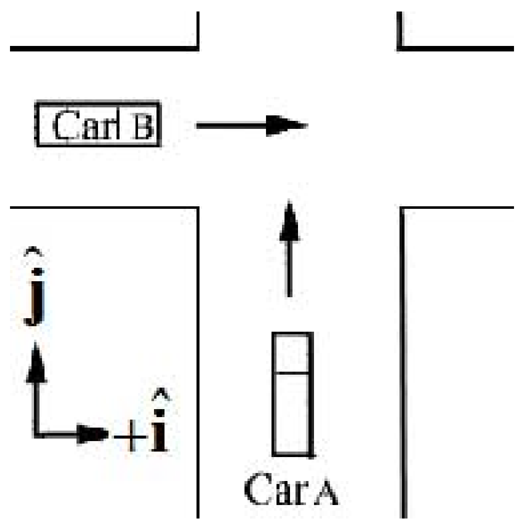
Einstein extended this to all physical phenomena.

The laws of physics are the same in all relatively inertial reference frames,

In particular, Einstein extended the Galilean principle of relativity to electromagnetism and optics which describe the theory of light

In-Class Problem 7: Law of Addition of Velocities

Suppose two cars, Car A, and Car B, are traveled along roads that are perpendicular to each other. An observer is at rest with respect to the ground. A second observer is in Car A. According to the observer on the ground, Car A is moving with a velocity $\vec{\mathbf{v}}_A = v_A\,\hat{\mathbf{j}}$, and Car B is moving with a magnitude of velocity $\vec{\mathbf{v}}_B = v_B\,\hat{\mathbf{i}}$.



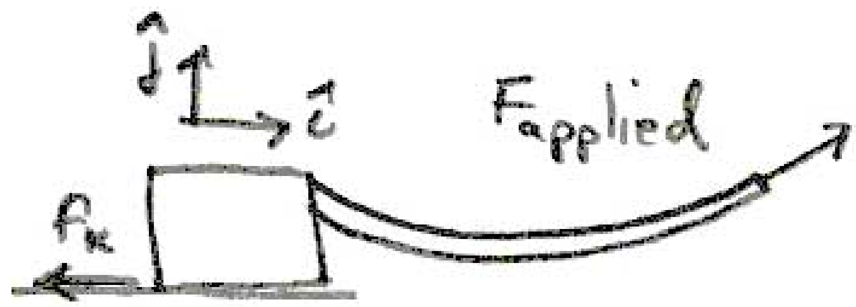
What is the velocity of Car B according to the observer in Car A? Express your answer both as components of the velocity vector, and direction and magnitude of the velocity vector.

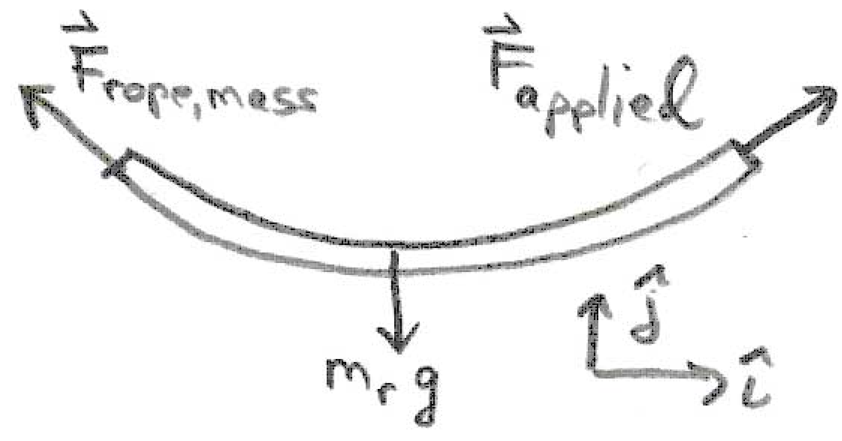
Tension

Consider a rope pulling a mass.

How do we define `tension' in a rope?

Force Diagram on Rope

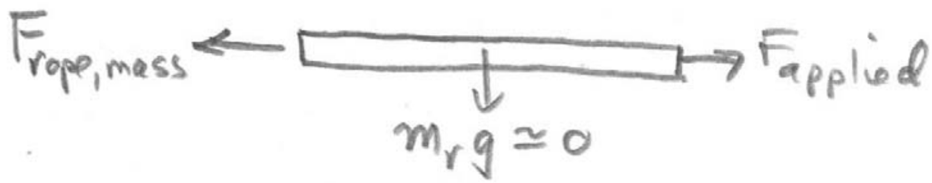




Tension: Massless Rope

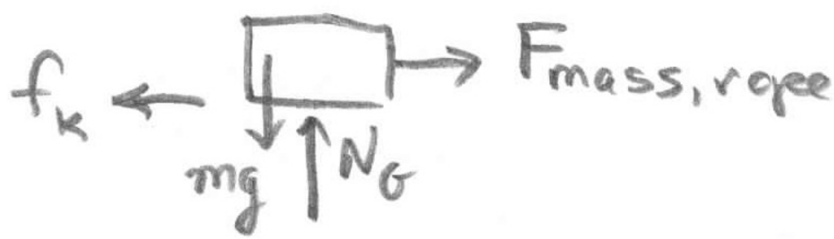
- **Assumption:** Assume that the mass of the rope is small so that we can ignore all y-components of the applied force and the force of the object on the rope.

Rope:



$$F_{applied} - F_{rope,mass} = m_r a_{r,x}$$

Object:



$$F_x^{total} = F_{mass,rope} - f_{kinetic} \qquad F_y^{total} = N - mg = 0$$

Equations of Motion

Friction Force Law: $$f_{kinetic} = \mu_k N$$

Newton's Third Law: $$\vec{\mathbf{F}}_{mass,rope} = -\vec{\mathbf{F}}_{rope,mass}$$

$$F_{rope,mass} = F_{mass,rope}$$

Newton's Second Law: $$F_{mass,rope} - \mu_k mg = ma_{m,x}$$

$$F_{applied} - F_{mass,rope} = m_r a_{r,x}$$

Inextensible Rope Assumption: $$a_{m,x} = a_{r,x} \equiv a$$

Special Cases

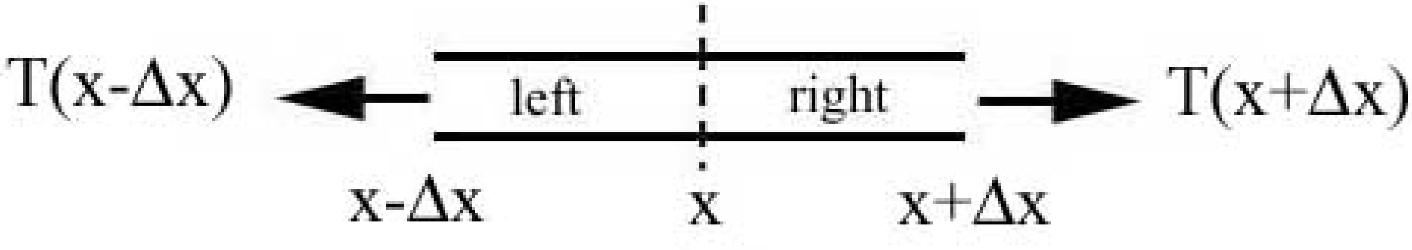
1. Pulling the object at a constant velocity: $a = 0$

2. Rope is essentially massless: $m_r \cong 0$

$$F_{applied} - F_{mass,rope} = 0$$

Pulling force is transmitted through the rope

Tension in a Rope



The *tension* in a rope at a distance x from one end of the rope is the magnitude of the action -reaction pair of forces acting at that point ,

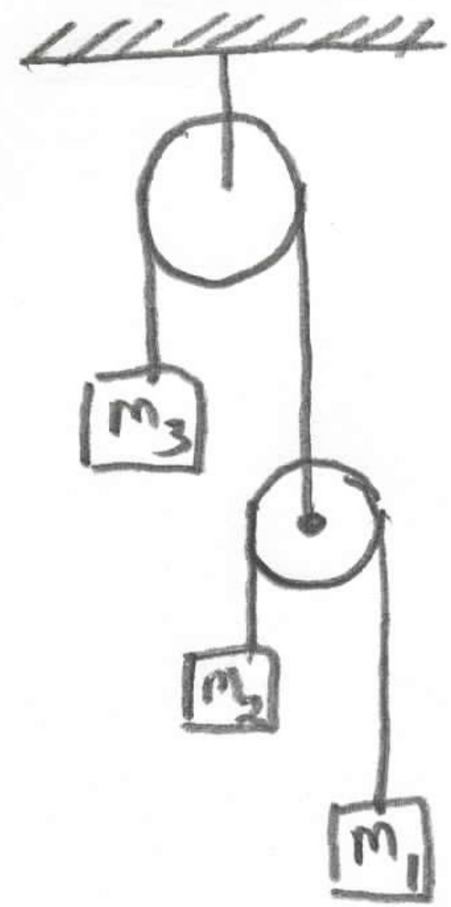
$$T(x) = \left|\vec{\mathbf{F}}_{left,right}(x)\right| = \left|\vec{\mathbf{F}}_{right,left}(x)\right|$$

Constraint Condition in Pulley Systems

Consider the arrangement of pulleys and objects shown in the figure. Draw force diagrams on each object.

How are the accelerations of the objects related?

Solve for the accelerations pf the objects and the tensions in the ropes.



Force Law: Newtonian Induction

- Definition of force has no predictive content
- Need to measure the acceleration and the mass in order to define the force
- **Force Law:** Discover experimental relation between force exerted on object and change in properties of object
- **Induction:** Extend force law from finite measurements to all cases within some range creating a **model**
- **Second Law can now be used to predict motion!**
- If prediction disagrees with measurement adjust model.

Experiment 03: Modeling Forces

Goal

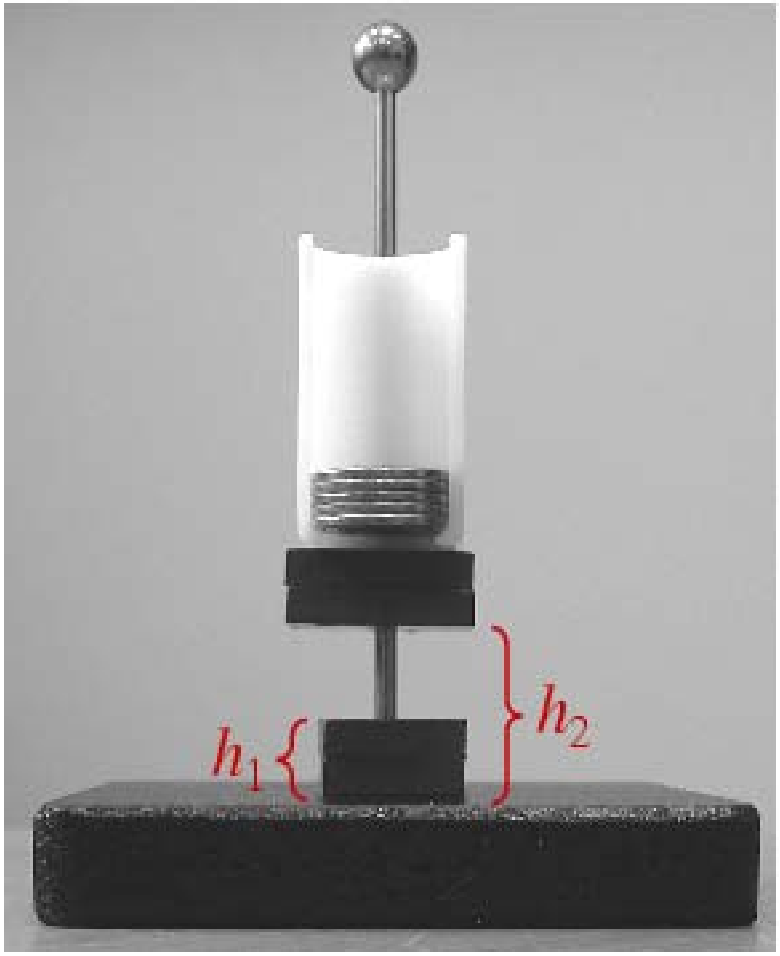
- Use DataStudio to plot and analyze the force that two magnets exert on one another as a function of the distance between them.

- Use linear, semi-log, and log-log graphs to gain some insight into how the force varies with separation.

- Find a mathematical function that describes this force, a "force law".

Experimental setup

- Measuring the magnet gap $h_2 - h_1$



- Measure heights h_1 and h_2 with your ruler, and subtract them. (h_1 will be constant.)

- The two magnets stuck together weigh 6.0 pennies. The plastic coin holder weighs 4.0 pennies.

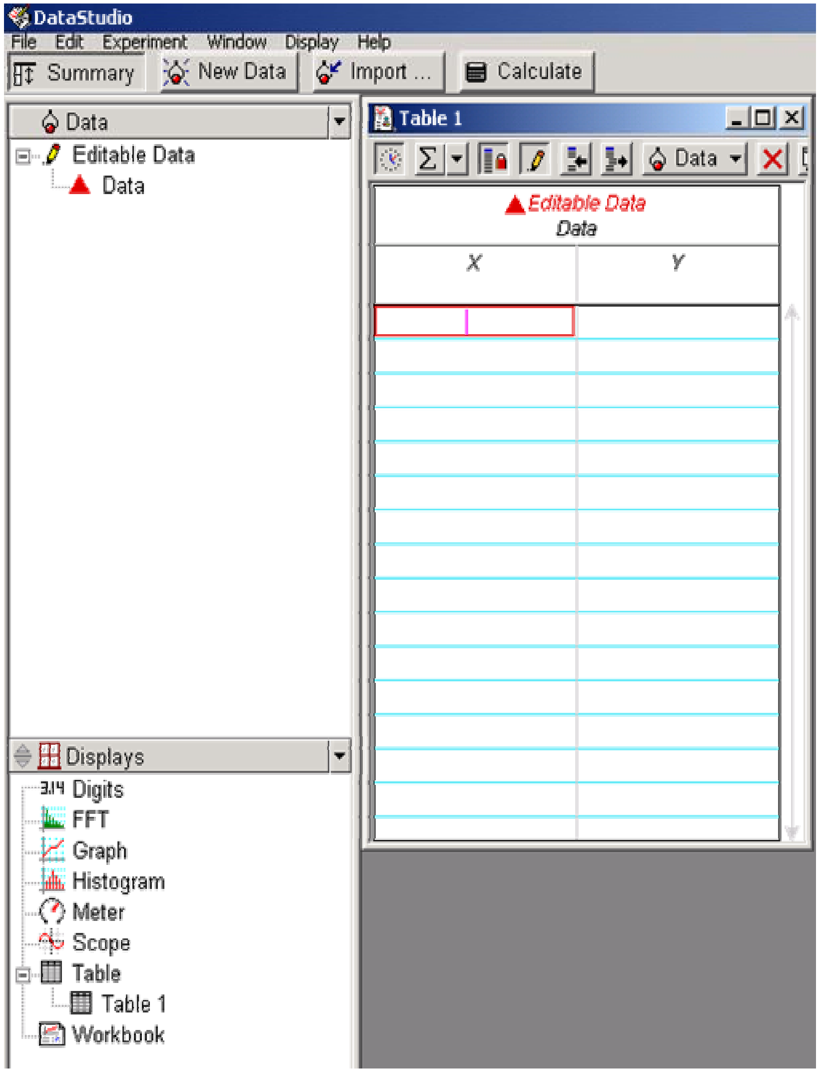
- Enter the gap (in mm) and the total weight (in pennies) into a table in DataStudio.

- The gap goes in the X (left) column of the table.

Starting DataStudio

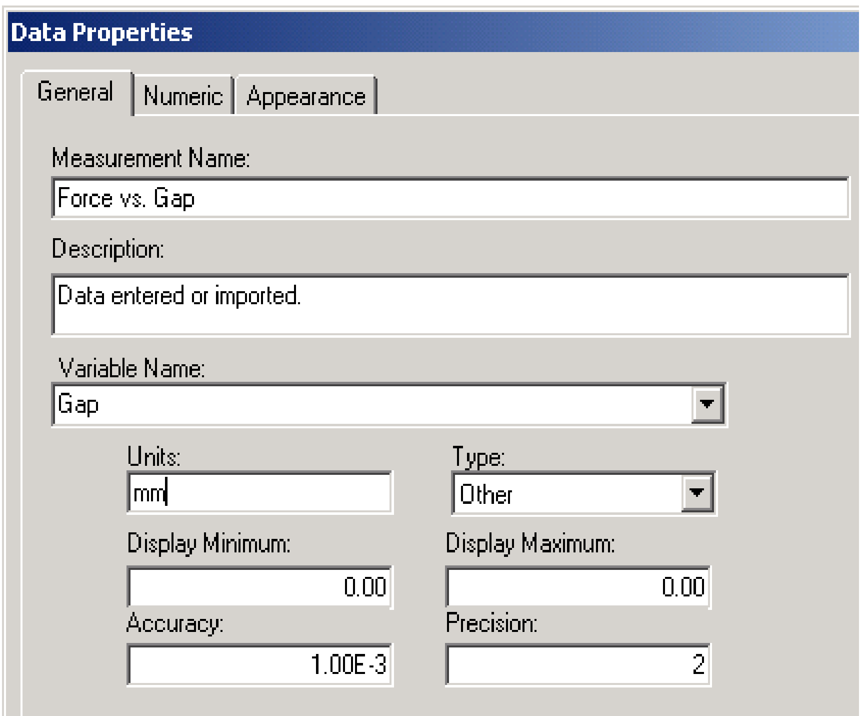
- Choose the "Enter Data" option.

Making a table I



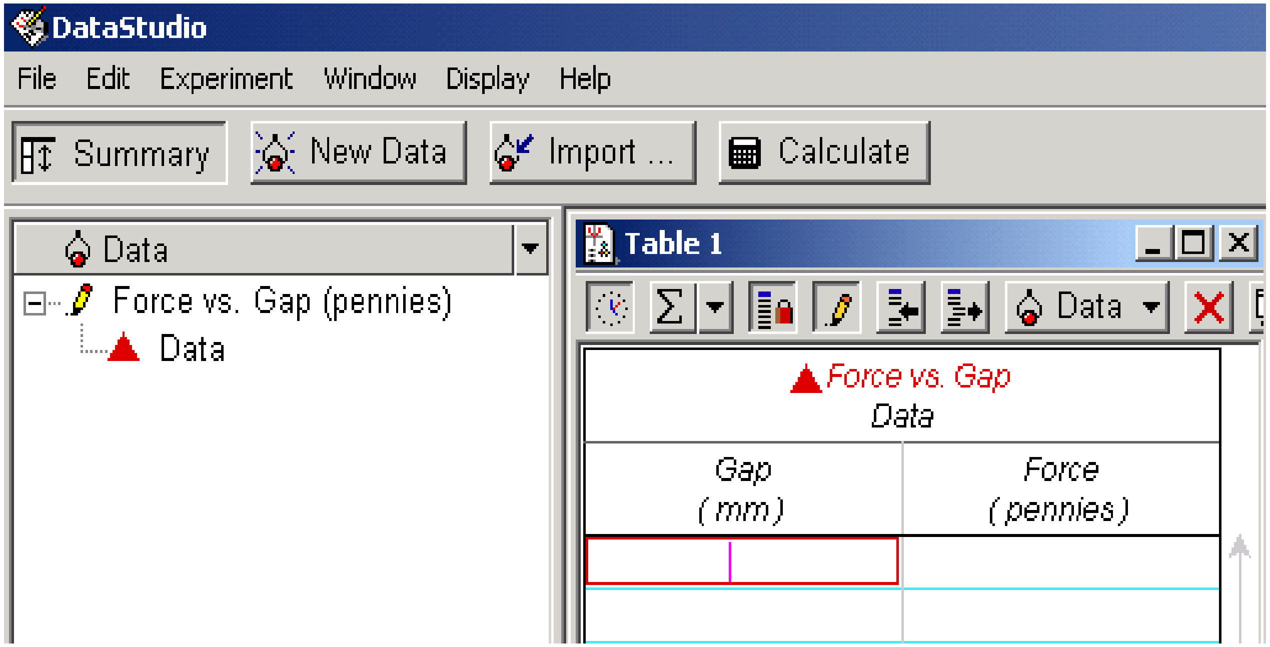
- A table and a graph will appear. Close the graph window (removes it). Drag the table borders to make it smaller.
- Click the "Summary" button to open the "Data" and "Displays" windows.
- Double-click "Editable Data" in the Data window. This opens a "Data Properties" window…

Making a table II



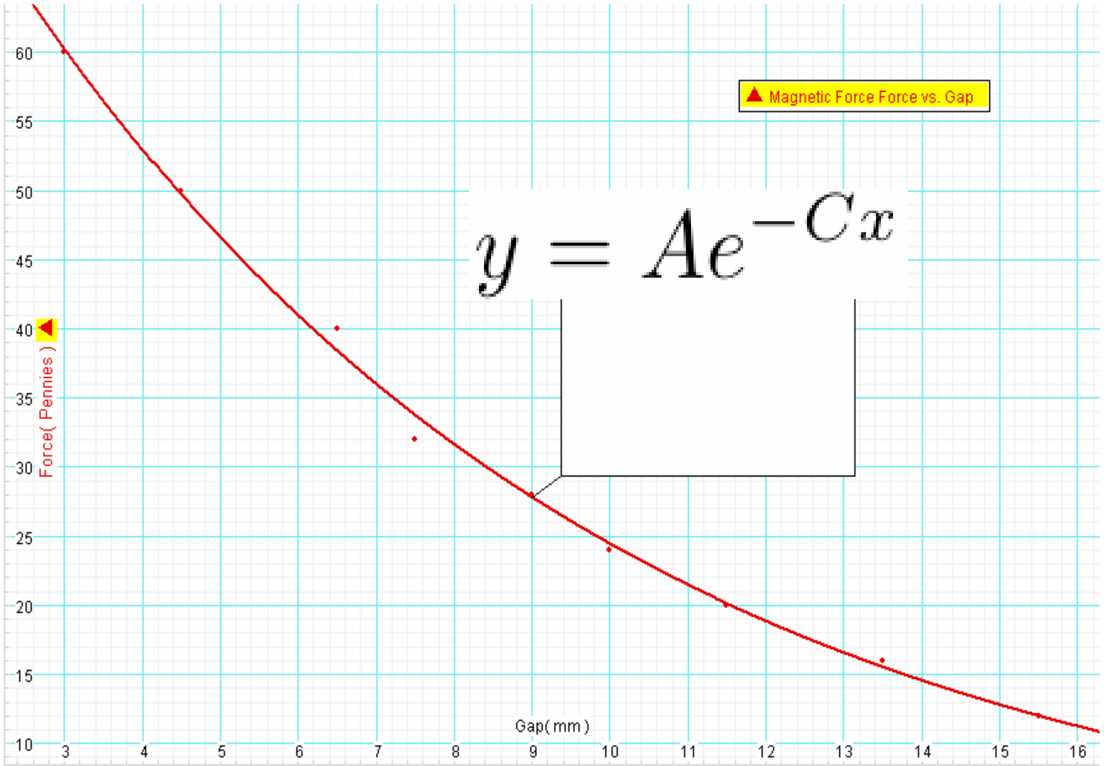
- Choose a title for the data set.
- Pick names and units of the X and Y variables.

Making a table III



- Type in your measurements, gap in the left (*X*) column and force in the right (*Y*) column.
- To plot them, drag the "Force vs.Gap" entry in the Data window onto "Graph" in the Displays window.

Exponential fit

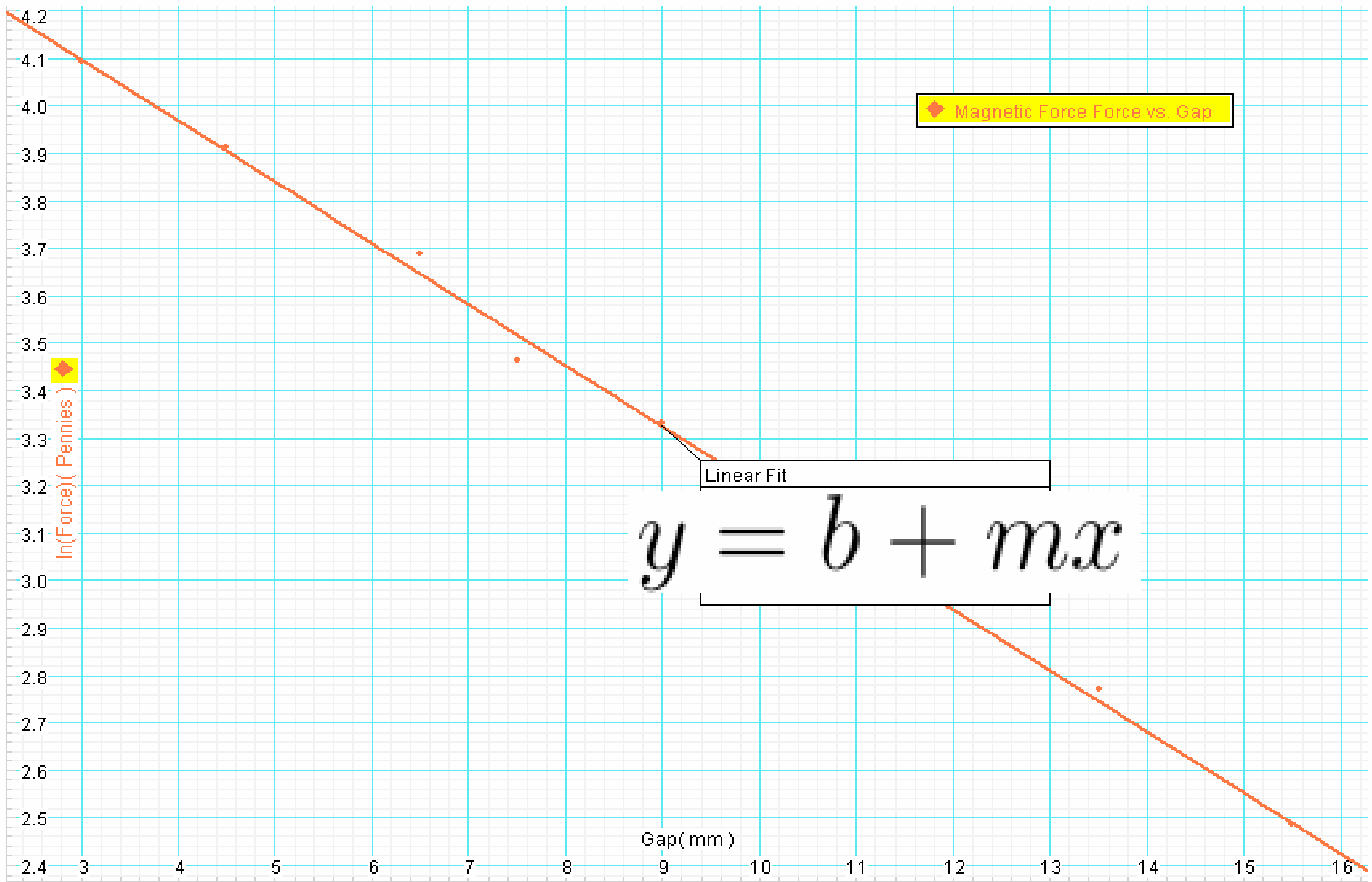


- Carry out a user-defined fit of: $y = Ae^{-Cx}$
- Record A and C for part (a) and answer question about the characteristic length l over which the force drops by a factor 1/e

Semi-log plot and linear fit I

- Click the “Calculate” button.
- In the calculator definition window type lnF = ln(y) and click Accept.
- Under the “Variables” pull-down menu choose “Data Measurement” and then your data in the yellow window that opens.
- After you click the Yes button, there should be a new Data entry “lnF = ln(y)” in the Data window.
- Make a graph of lnF vs. gap by dragging this entry onto the Graph entry in the Displays window.
- Use the Linear Fit function to see if it is a straight line.

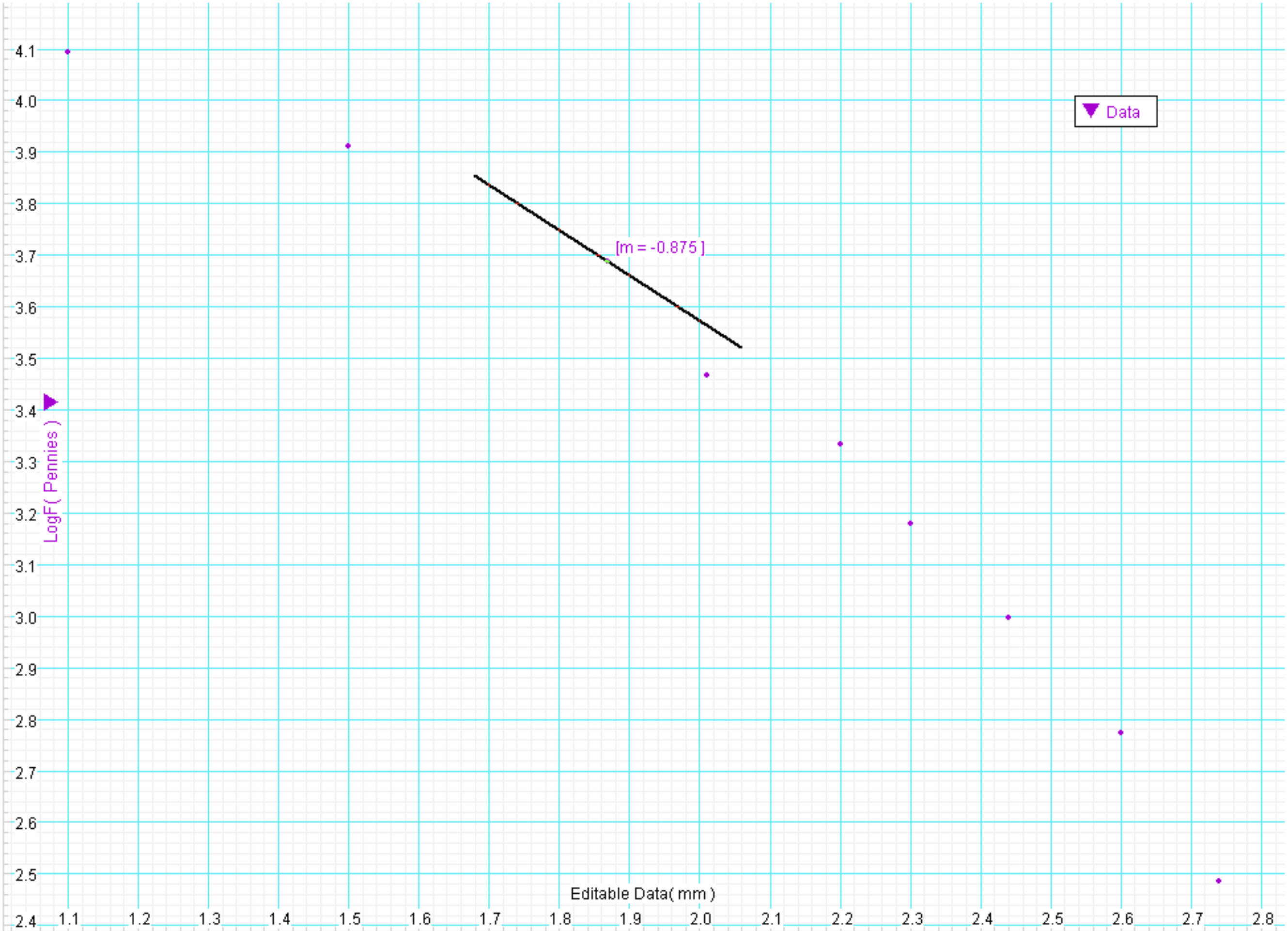
Semi-log plot and linear fit II



Log-log plot and slope tool I

- Make a new empty data table by clicking the "New Data" button.
- Using a calculator, manually enter the natural log of your gap measurements in the left (X) column.
- You may enter the corresponding force (in pennies) in the right (Y) column, then let DataStudio calculate the log as you did for the semi-log graph, <u>or</u> you may calculate the natural log of the force yourself and enter it directly in the table.
- Make a graph of ln(force) vs. ln(gap).
- Use the graph's Slope Tool to fill in the table in your report, part (b).

Log-log plot and slope tool II



Report

- Turn in one report page per group.

- There is a follow-up homework problem about this experiment.