Newton’s Laws of Motion

8.01T

Sept 20, 2004

Newton’s First Law

Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

Newton's Second Law

The change of motion is proportional to the motive force impresses, and is made in the direction of the right line in which that force is impressed.

Newton’s Third Law

To every action there is always opposed an equal reaction: or, the mutual action of two bodies upon each other are always equal, and directed to contrary parts.

Inertial Mass

- inertial mass is a ‘quantity of matter’
- standard body with mass m_s and SI units [kg]
- the mass of all other bodies will be determined relative to the mass of our standard body.
- apply the same action (force) to the standard body and an unknown body
- Define the unknown mass in terms of ratio of accelerations

$$\frac{m_u}{m_s} = \frac{a_s}{a_u}$$

Standard Kilogram

- Cylindrical alloy of 90 % platinum and 10 % iridium

$$\rho = 21.56\ \mathrm{g \cdot cm^{-3}}$$

- Density and volume of the standard kilogram,

$$V = m/\rho \cong 1000\ \mathrm{g}/22\ \mathrm{g \cdot cm^{-3}} \cong 46.38\mathrm{cm^3}$$

- Constant volume for a cylinder

$$V = \pi r^2 h$$

- The surface area

$$A = 2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{2V}{r}$$

- minimize the area with respect to the radius

$$\frac{dA}{dr} = 4\pi r - \frac{2V}{r^2} = 0$$

$$r = h/2$$

- radius is one half the height,

$$r = \left(V/2\pi\right)^{1/3} \cong 1.95\ \mathrm{cm}$$

Definition: Momentum (Quantity of Motion)

- Momentum is a vector quantity $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$

- Magnitude: product of the mass with the magnitude of the velocity

- Direction: the direction of the velocity

- In the SI system of units, momentum has units

[kg-m-s^{-1}]

Definition: Force

- Force is a vector quantity. $$\vec{\mathbf{F}} \equiv m\vec{\mathbf{a}}$$

- Magnitude of the total force acting on the object is the product of the mass and the magnitude of the acceleration .

- Direction of the total force on a body is the direction of the acceleration of the body.

- SI units for force are **[kg-m-s^{-2}]**

- Unit has been named the Newton **1 N = 1 kg-m-s^{-2}**

Superposition Principle

Apply two forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ on a single body

Total force is the vector sum of the two forces

$$\vec{\mathbf{F}}^{total} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$$

Average Impulse

Apply an average force $\overline{\vec{\mathbf{F}}}$ for an interval of time Δt

Average impulse is the product of the average force and the time interval

$$\overline{\vec{\mathbf{I}}} = \overline{\vec{\mathbf{F}}}\Delta t$$

Units for impulse are the same as for momentum

[kg-m-s^{-1}] = [N-s]

Impulse

When force is applied continuously over a time interval

$$\left[t_0, t_f \right]$$

Impulse is the integral

$$\vec{\mathbf{I}} = \int_{t_o}^{t_f} \vec{\mathbf{F}} dt$$

Newton's Second Law

The change of momentum is equal to the applied average impulse

$$\bar{\vec{\mathbf{I}}} = \bar{\vec{\mathbf{F}}}\Delta t = \Delta\vec{\mathbf{p}}$$

For an instantaneous action of the total force, force is equal to the product of mass with acceleration.

$$\vec{\mathbf{F}}^{total} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{p}}}{\Delta t} \equiv \frac{d\vec{\mathbf{p}}}{dt}$$

When the mass remains constant in time,

$$\vec{\mathbf{F}}^{total} = m\vec{\mathbf{a}}$$

Force Law: Newtonian Induction

- Definition of force has no predictive content
- Need to measure the acceleration and the mass in order to define the force
- **Force Law:** Discover experimental relation between force exerted on object and change in properties of object
- **Induction:** Extend force law from finite measurements to all cases within some range creating a **model**
- **Second Law can now be used to predict motion!**
- If prediction disagrees with measurement adjust model.

Hooke's Law

- Spring attached to a body
- Stretch or compress spring by different amounts produces different accelerations
- Magnitude: $\left|\vec{\mathbf{F}}\right| = k\Delta l$
- Direction: restoring spring to equilibrium
- Model holds within some reasonable range of extension or compression

Contact Forces Between Surfaces

Component Perpendicular to surface is called the Normal Force

$$\vec{\mathbf{N}}$$

Component tangent to the surface is called the friction force

$$\vec{\mathbf{f}}$$

Vector sum is the total contact force

$$\vec{\mathbf{C}} \equiv \vec{\mathbf{N}} + \vec{\mathbf{f}}$$

Kinetic Friction

Force Law for kinetic friction: independent of surface area of contact, independent of velocity, proportional to the normal force

Magnitude: $f_k = \mu_k N$

where different contact surfaces have different coefficients of friction μ_k

Direction: opposes motion

Static Friction

Varies in direction and magnitude depending on applied forces

Magnitude varies between $0 \le f_s \le f_{s,\max} = \mu_s N$

Just Slipping Condition: two objects in contact

- accelerations are equal $a_1 = a_2$
- Static friction is equal to it's maximum value $f_{s,\max} = \mu_s N$

Models in Physics: Fundamental Laws of Nature

Force laws are mathematical models of physical processes

Search for *fundamental forces*

- Electromagnetism
- Weak Force
- Strong Force
- Gravity
- Fifth Force?

Universal Law of Gravitation

- Gravitational force between two bodies with masses m_1 and m_2
- Direction: force points along the line connecting the bodies and is attractive
- Magnitude: proportional to the product of the masses; and inversely proportional to square of the distance, $r_{1,2}$, between the bodies

$$\vec{\mathbf{F}}_{1,2} = -G\frac{m_1 m_2}{{r_{1,2}}^2}\hat{\mathbf{r}}_{1,2}$$

- with $$G = 6.67\times10^{-11}\,\text{N-m}^2\cdot\text{kg}^{-2}$$

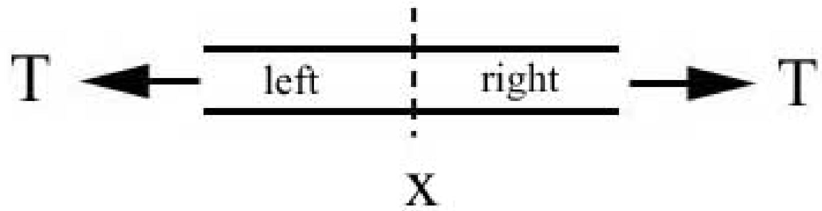
Newton’s Third Law

- To every action there is always opposed an equal reaction: or, the mutual action of two bodies upon each other are always equal, and directed to contrary parts.

$$\vec{\mathbf{F}}_{1,2} = -\vec{\mathbf{F}}_{2,1}$$

- Action-reaction pair of forces cannot act on same body; they act on different bodies.

Tension in a Rope



The *tension* in a rope at a distance x from one end of the rope is the magnitude of the action -reaction pair of forces acting at that point ,

$$T(x) = \left|\vec{\mathbf{F}}_{left,right}(x)\right| = \left|\vec{\mathbf{F}}_{right,left}(x)\right|$$

Free Body Force Diagram

- Isolate each object

- Represent each force that is acting on the object by an arrow that indicates the direction of the force.

$$\vec{\mathbf{F}}^{total} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \ldots$$

- Use free body force diagram to make these vector decompositions of the individual forces.

$$\vec{\mathbf{F}}^{total} = F_x^{\ total}\hat{\mathbf{i}} + F_y^{\ total}\hat{\mathbf{j}} + F_z^{\ total}\hat{\mathbf{k}}$$

Newton's Law in Components

- Use Experimental Force Laws and Newton's Second Law to predict the acceleration of the body

$$F_x^{total} = ma_x$$

$$F_y^{total} = ma_y$$

$$F_z^{total} = ma_z$$

Methodology for Newton's Second Law

I. Understand – get a conceptual grasp of the problem

- Think about the problem.
- Sketch the system at some time when the system is in motion.
- Choose a coordinate system and identify the position function of all objects.
- Quantify the constraint conditions.
- Draw free body diagrams for each body in the problem.

II. Devise a Plan

Draw free body diagrams for each body in the problem.

- Include the set of unit vectors
- Each force represented by an arrow indicating the direction of the force
- Choose an appropriate symbol for the force

Choosing directions for the forces

- If you solve for a force and find that the force is negative then the force points in the opposite direction as your choice of direction for the force.

II. Devise a Plan - (con't)

Apply vector decomposition to each force in the free body diagram

$$\vec{\mathbf{F}}_i = (F_x)_i\,\hat{\mathbf{i}} + (F_y)_i\,\hat{\mathbf{j}} + (F_z)_i\,\hat{\mathbf{k}}$$

Apply superposition principle to find total force in each direction

$$\hat{\mathbf{i}} : F_x^{total} = \left(F_x\right)_1 + \left(F_x\right)_2 + \ldots$$

$$\hat{\mathbf{j}} : F_y^{total} = \left(F_y\right)_1 + \left(F_y\right)_2 + \ldots$$

$$\hat{\mathbf{k}} : F_z^{total} = \left(F_z\right)_1 + \left(F_z\right)_2 + \ldots$$

II. Devise a Plan: Equations of Motion

- Application of Newton's Second Law

$$\vec{\mathbf{F}}^{total} = m\vec{\mathbf{a}}$$

- This is a vector equality so the two sides are equal in magnitude and direction

$$\hat{\mathbf{i}} : \left(F_x\right)_1 + \left(F_x\right)_2 + ... = ma_x$$

$$\hat{\mathbf{j}} : \left(F_y\right)_1 + \left(F_y\right)_2 + ... = ma_y$$

$$\hat{\mathbf{k}} : \left(F_z\right)_1 + \left(F_z\right)_2 + ... = ma_z$$

II. Devise a Plan (con't)

Analyze whether you can solve the system of equations

- Common problems and missing conditions.
- Constraint conditions between the components of the acceleration.
- Action-reaction pairs.
- Different bodies are not distinguished

Design a strategy for solving the system of equations.

III. Carry Out your Plan

Hints:

Use all your equations. Avoid thinking that one equation alone will contain your answer!

After you solve your equations of motion for the components of acceleration, you now may consider whether your kinematic models will help you solve the problem.

IV. Look Back

- **Check your algebra**
- **Substitute in numbers**
- **Check your result**
- **Think about the result:** solved problems become models for thinking about new problems.