Problem Solving Methodologies for Two Dimensional Kinematics

8.01T

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1: Understand – a get conceptual grasp of the problem

- How many objects are involved in the problem?
- How many different stages of motion occur?
- For each object, how many independent directions are needed to describe the motion of that object?

1: Understand – a get conceptual grasp of the problem

- What choice of coordinate system best suits the problem?
- What information can you infer from the problem?

2. Devise a Plan - set up a procedure to obtain the desired solution

- Sketch the problem
- For each object in the problem, choose a coordinate system
- Write down the complete set of equations for the position and velocity functions; identify any specified quantities; clean up the equations.

Equations of Motion: y-direction

- Acceleration y-component:

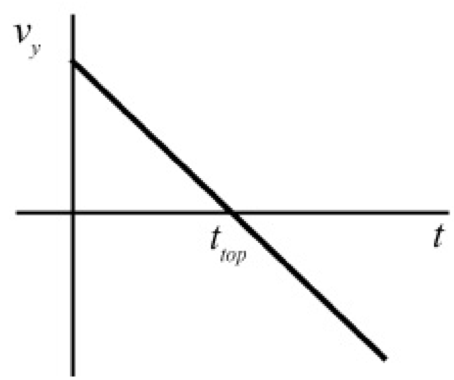
$$a_y = -g$$

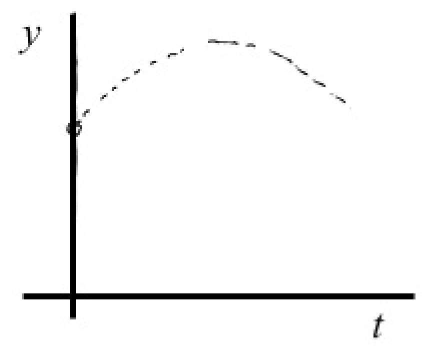
$$v_y(t) = v_{y,0} - gt$$

- Velocity y-component:

$$y(t) = y_0 + v_{y,0}t - \frac{1}{2}gt^2$$

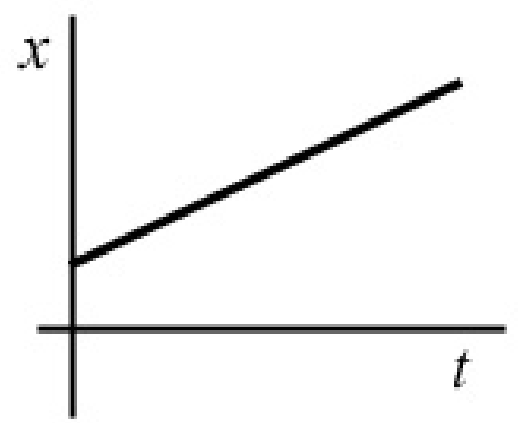
- Position y-component:





Equations of Motion: x-direction

- Acceleration x-component: $a_x = 0$
- Velocity x-component: $v_x(t) = v_{x,0}$
- Position x-component: $x(t) = x_0 + v_{x,0}t$



2. Devise a Plan - set up a procedure to obtain the desired solution

- Identify given information

- Identify 'initial state' with initial conditions

- Initial position depends on choice of origin

$$\vec{r}_0 = x_0\hat{i} + y_0\hat{j}$$

- Identify any other 'state' of the system, possibly the 'final state' with appropriate conditions for kinematic quantities

Initial Conditions: velocity

- Initial velocity

$$\vec{\mathbf{v}}_0(t) = v_{x,o}\hat{\mathbf{i}} + v_{y,o}\hat{\mathbf{j}}$$

with components:

$$v_{x,0} = v_0 \cos\theta_0$$

$$v_{y,0} = v_0 \sin\theta_0$$

- Magnitude

$$v_0 = (v_{x,0}^2 + v_{y,0}^2)^{1/2}$$

- direction

$$\theta_0 = \tan^{-1}\left(\frac{v_{y,0}}{v_{x,0}}\right)$$

2. Devise a Plan - set up a procedure to obtain the desired solution

- Design a strategy for solving the system of equations.
- You can solve a system of n independent equations if you have exactly n unknowns.
- These quantities are specified as numbers $x_0, y_0, v_{x,0}, v_{y,0}, a_x, a_y$
- While these quantities vary $x(t), y(t), v_x(t), v_y(t), t$
- Look for constraint conditions

Design a strategy for solving the system of equations.

3. Carry our your plan

solve the problem!

4. Look Back – check your solution and method of solution

- Check your algebra and units.
- Substitute in numbers.
- Check any possible limiting behavior
- Think about the result
- Solved problems act as models for thinking about new problems.

In-Class Problem 2:Throwing a Stone Down a Hill

- A person is standing on top of a hill which slopes downwards uniformly at an angle Φ θ_0 with respect to the horizontal. The person throws a stone at an initial angle θ_0 from the horizontal with an initial speed of v_0 . You may neglect air resistance. How far below the top of the hill does the stone strike the ground?

Class Problem 3

- A person is standing on a ladder holding a pail. The person releases the pail from rest at a height, h_1 ,above the ground. As second person standing some distance away aims and throws a small ball in order to hit the first ball with an initial velocity that has magnitude, v_0 , and thrown at an angle, θ_0 ,with respect to the horizontal. The second person releases the ball at a height, h_2 , above the ground and a horizontal distance, s_2 , from the line of flight of the first ball. You may ignore air resistance.

Class Problem 3: Continued

a) Find an expression for the angle that the person aims the ball in order to hit the pail as a function of the other variables given in the problem

b) If the person aims correctly, find an expression for the range of speeds that the ball must be thrown at in order to hit the pail?