Kinematics and One Dimensional Motion

8.01t

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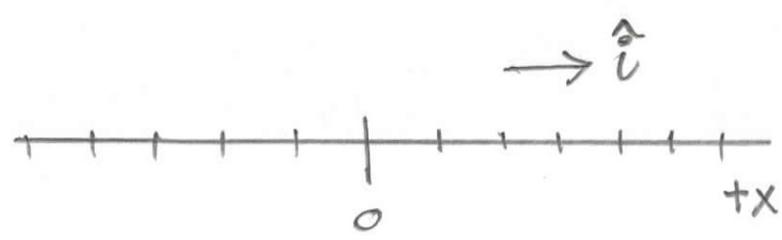
Kinematics

- *Kinema* means movement

- Mathematical description of motion
- Position
- Displacement
- Velocity
- Acceleration

Coordinate System in One Dimension

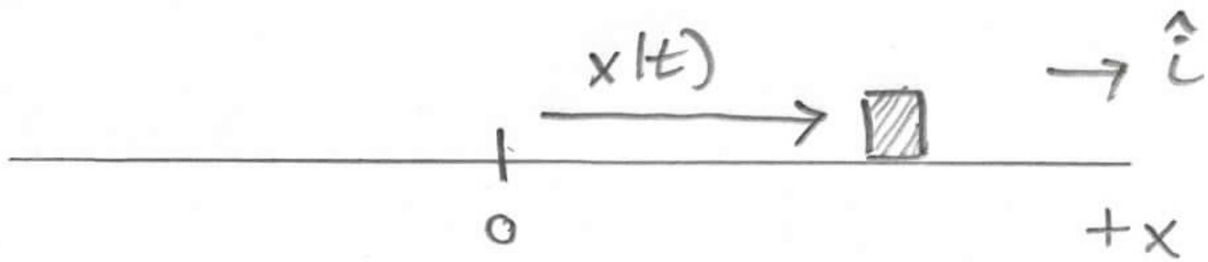
- Choice of origin
- Choice of coordinate axis
- Choice of positive direction for the axis
- Choice of unit vectors at each point in space



Position

- Vector from origin to body

$$\vec{\mathbf{x}}(t) = x(t)\hat{\mathbf{i}}$$



Displacement

- change in position coordinate of the object between the times t_1 and t_2

$$\Delta\vec{\mathbf{x}} \equiv \left(x(t_2) - x(t_1) \right)\hat{\mathbf{i}} \equiv \Delta x(t)\hat{\mathbf{i}}$$

Average Velocity

- component of the average velocity, $\overline{v_x}$,

  is the displacement Δx divided by the time interval Δt

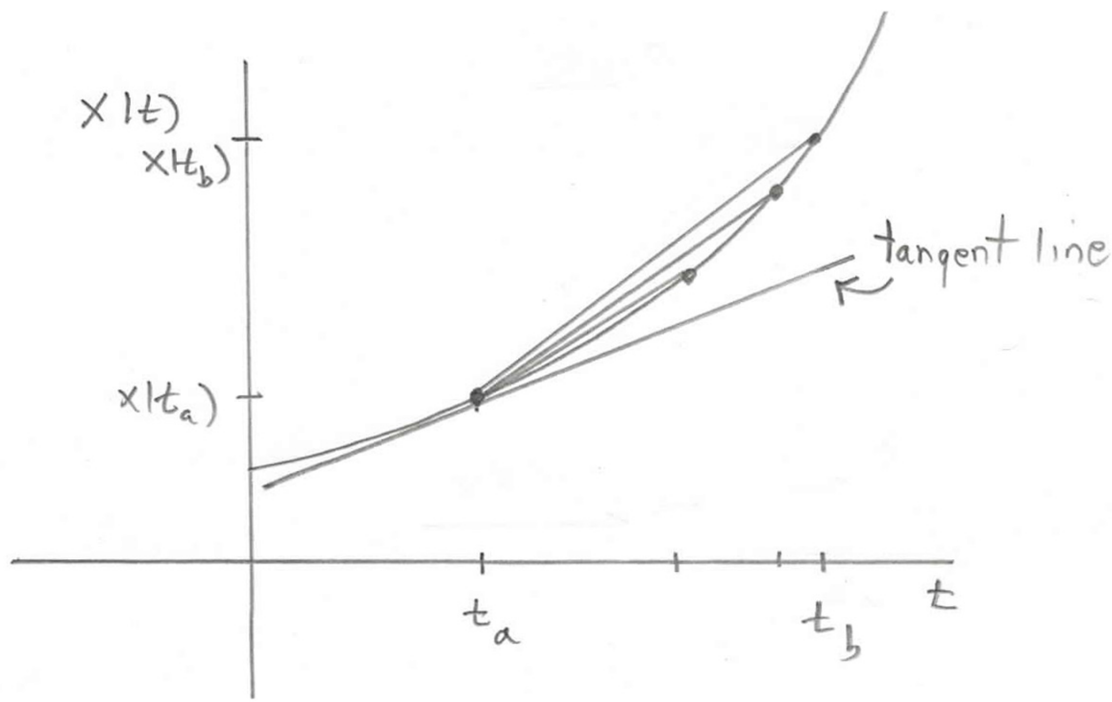
$$\overline{\vec{\mathbf{v}}}(t) \equiv \frac{\Delta x}{\Delta t}\hat{\mathbf{i}} = \overline{v_x}(t)\hat{\mathbf{i}}$$

Instantaneous velocity

- For time interval Δt, we calculate the average velocity. As $\Delta t \to 0$, we generate a sequence of average velocities. The limiting value of this sequence is defined to be the x-component of the instantaneous velocity at the time t .

$$v_x(t) \equiv \lim_{\Delta t \to 0} \overline{v_x} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{x\left(t + \Delta t\right) - x\left(t\right)}{\Delta t} \equiv \frac{dx}{dt}$$

Instantaneous velocity



Average Acceleration

- Change in velocity divided by the time interval

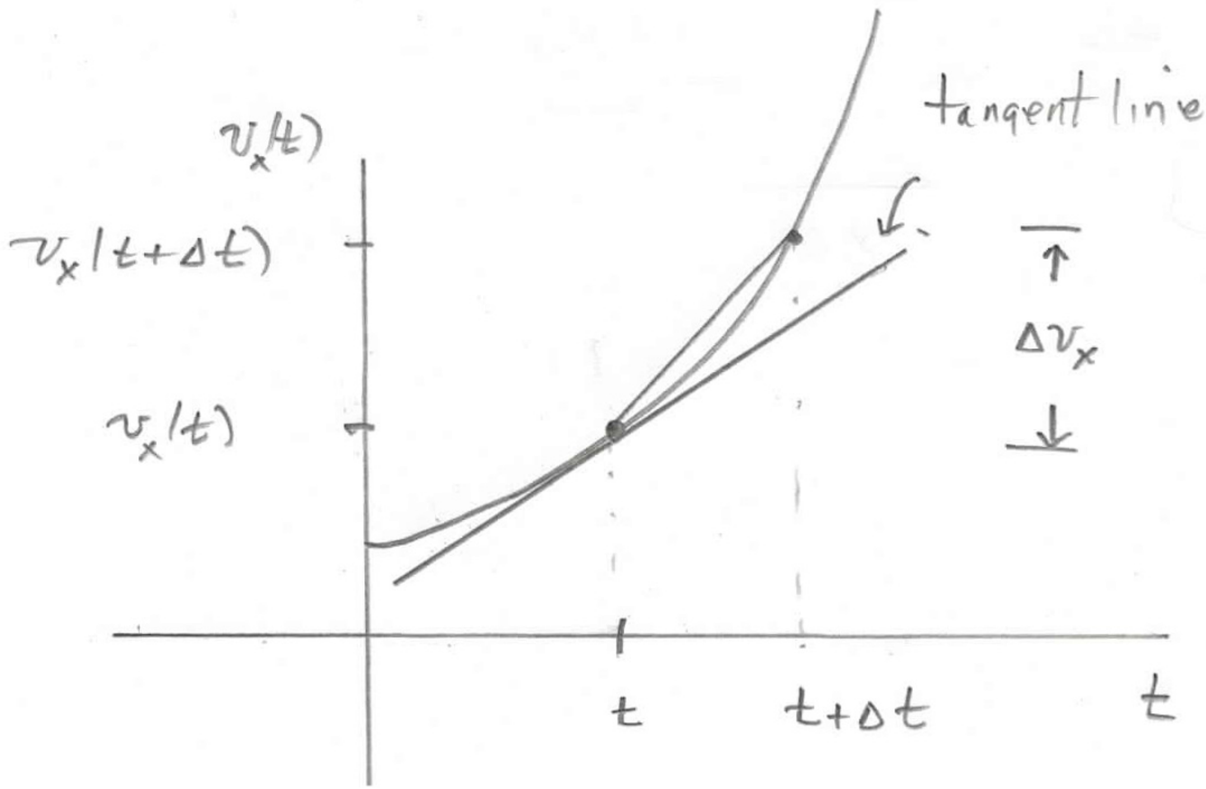
$$\overline{\vec{\mathbf{a}}} = \overline{a_x}\,\hat{\mathbf{i}} \equiv \frac{\Delta v_x}{\Delta t}\hat{\mathbf{i}} = \frac{\left(v_{x,2} - v_{x,1}\right)}{\Delta t}\hat{\mathbf{i}} = \frac{\Delta v_x}{\Delta t}\hat{\mathbf{i}}$$

Instantaneous acceleration

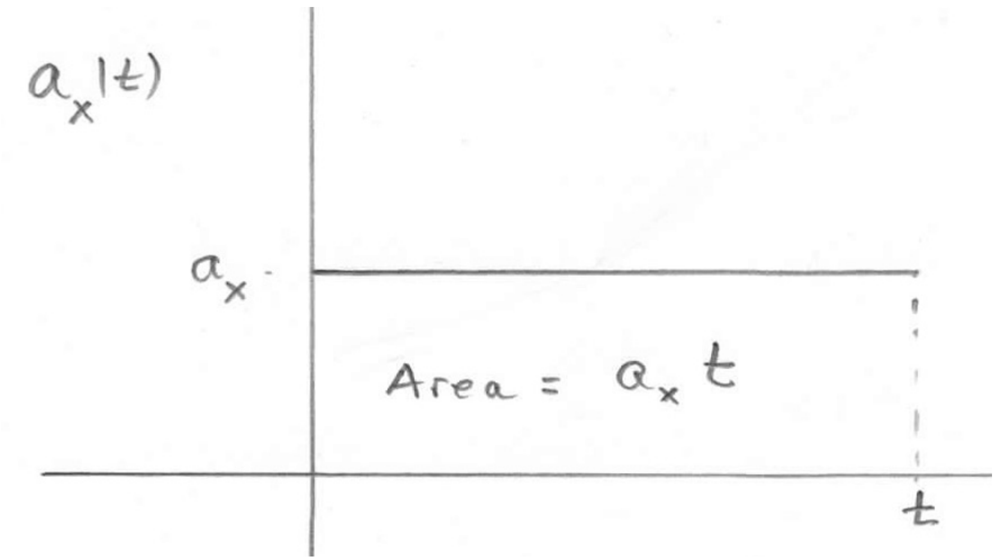
- For time interval Δt, we calculate the average acceleration. As $\Delta t \to 0$, we generate a sequence of average accelerations. The limiting value of this sequence is defined to be the x-component of the instantaneous velocity at the time t.

$$\vec{\mathbf{a}}(t) = a_x(t)\hat{\mathbf{i}} \equiv \lim_{\Delta t \to 0} \overline{a_x}\hat{\mathbf{i}} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}\hat{\mathbf{i}} = \lim_{\Delta t \to 0} \frac{v\left(t+\Delta t\right)-v\left(t\right)}{\Delta t}\hat{\mathbf{i}} \equiv \frac{dv}{dt}\hat{\mathbf{i}}$$

Instantaneous Acceleration



Constant acceleration: area under the acceleration vs. time graph

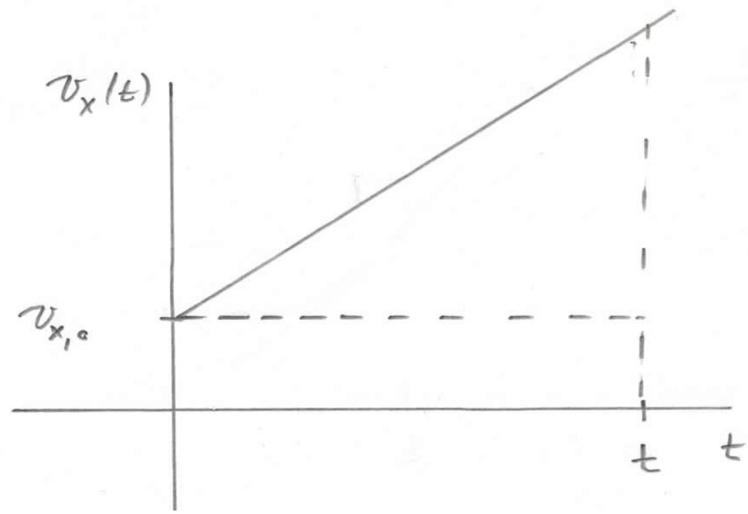


$$a_x = \overline{a_x} = \frac{\Delta v_x}{\Delta t} = \frac{v_x(t) - v_{x,0}}{t}$$

$$v_x(t) = v_{x,0} + a_x t$$

Constant acceleration: Area under the velocity vs. time graph

$$Area(v_x, t) = v_{x,0}t + \frac{1}{2}(v_x(t) - v_{x,0})t$$

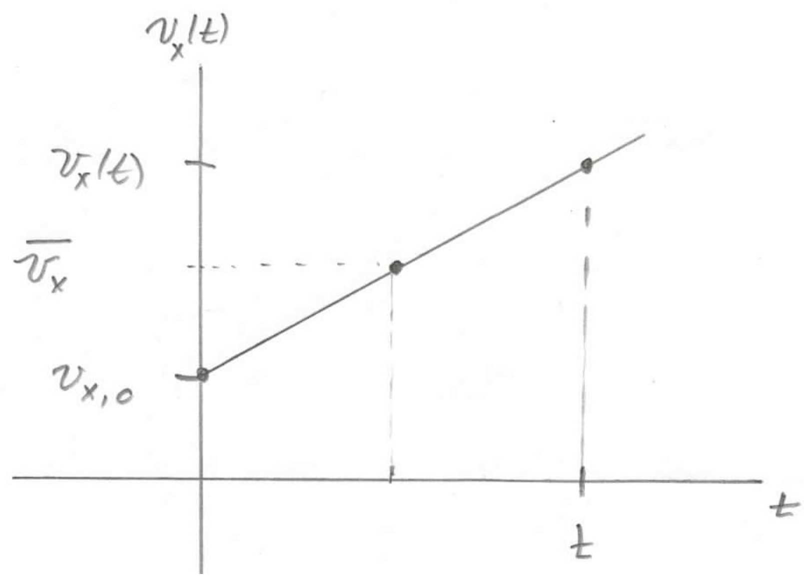


$$v_x(t) = v_{x,0} + a_x t$$

$$Area(v_x, t) = v_{x,0}t + \frac{1}{2}(v_{x,0} + a_x t - v_{x,0})t = v_{x,0}t + \frac{1}{2}a_x t^2$$

Constant acceleration: Average velocity

- When the acceleration is constant, the velocity is a linear function of time. Therefore the average velocity is



$$\overline{v_x} = \frac{1}{2}\left(v_x(t) + v_{x,0}\right)$$

$$\overline{v_x} = \frac{1}{2}\left(v_x(t) + v_{x,0}\right) = \frac{1}{2}\left(\left(v_{x,0} + a_x t\right) + v_{x,0}\right) = v_{x,0} + \frac{1}{2}a_x t$$

Constant acceleration: Area under the velocity vs. time graph

- displacement is equal to the area under the graph of the x-component of the velocity vs. time

$$\Delta x \equiv x(t) - x_0 = \overline{v_x} t = v_{x,0} t + \frac{1}{2} a_x t^2 = Area(v_x, t)$$

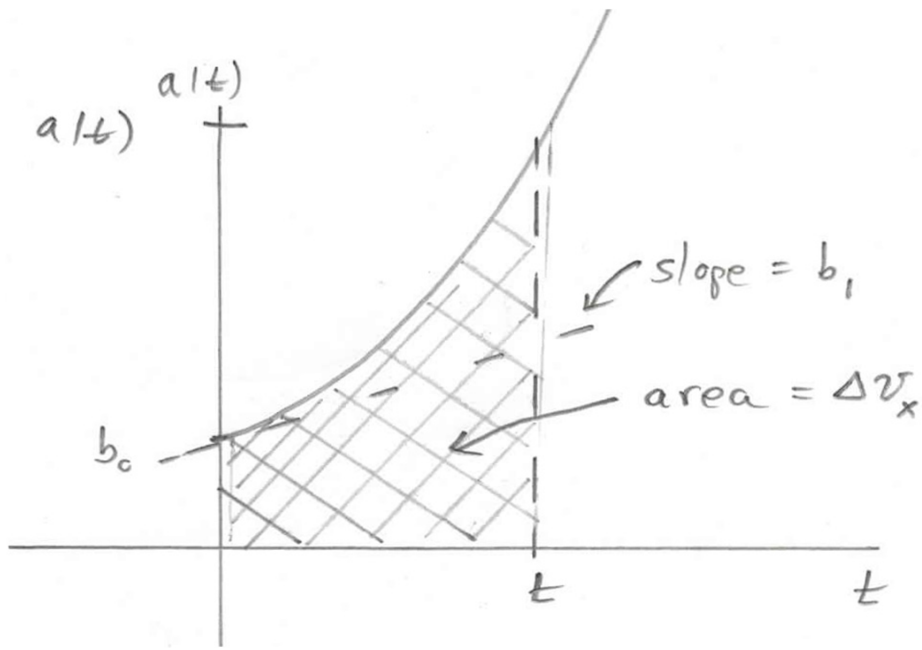
$$x(t) = x_0 + v_{x,0} t + \frac{1}{2} a_x t^2$$

Summary: constant acceleration

- Position $$x(t) = x_0 + v_{x,0}t + \frac{1}{2}a_x t^2$$

- velocity $$v_x(t) = v_{x,0} + a_x t$$

Velocity as the integral of the acceleration



Velocity as the integral of the acceleration

- *the area under the graph of the acceleration vs. time* is the change in velocity

$$\int_{t'=0}^{t'=t} a_x(t')dt' \equiv \lim_{\Delta t_i \to 0} \sum_{i=1}^{i=N} a_x(t_i)\Delta t_i = Area(a_x, t)$$

$$\int_{t'=0}^{t'=t} a_x(t')dt' = \int_{t'=0}^{t'=t} \frac{dv_x}{dt} dt' = \int_{v'_x=v_x(t=0)}^{v'_x=v_x(t)} dv'_x = v_x(t) - v_{x,0}$$

Position as the integral of velocity

- area under the graph of velocity vs. time is the displacement

$$v_x(t) \equiv \frac{dx}{dt}$$

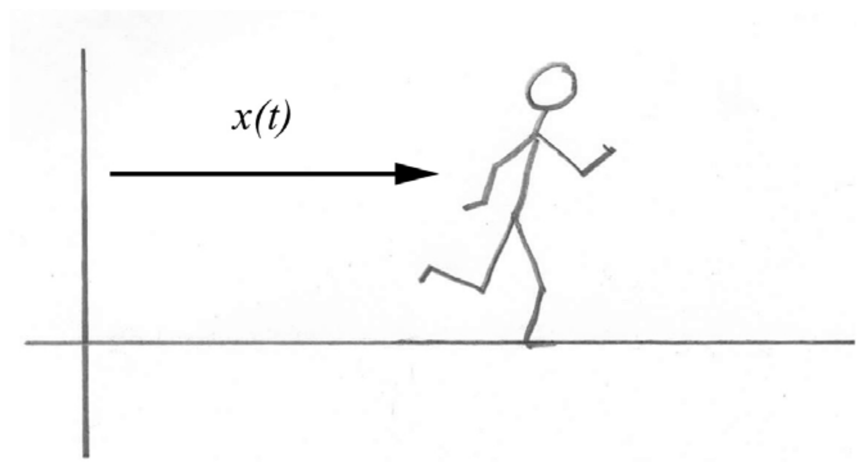
$$\int_{t'=0}^{t'=t} v_x(t')dt' = x(t) - x_0$$

Example:

- A runner accelerates from rest for an interval of time and then travels at a constant velocity. How far did the runner travel?

I. Understand – get a conceptual grasp of the problem

- stage 1: constant acceleration
- stage 2: constant velocity.



Tools:

Coordinate system

Kinematic equations

Devise a Plan:

Stage 1: constant acceleration

Initial conditions: $x_0 = 0$ $v_{x,0} = 0$

Kinematic Equations: $x(t) = \frac{1}{2} a_x t^2$ $v_x(t) = a_x t$

Final Conditions: end acceleration at $t = t_a$

position: $x_a \equiv x(t = t_a) = \frac{1}{2} a_x t_a{}^2$

velocity $v_{x,a} \equiv v_x(t = t_a) = a_x t_a$

Devise a Plan

Stage 2: constant velocity, time interval $[t_a, t_b]$

- runs at a constant velocity for the time $t_b - t_a$
- final position $x_b \equiv x(t = t_b) = x_a + v_{x,a}(t_b - t_a)$

III. Solve

- three independent equations

$$x_a = \frac{1}{2} a_x t_a{}^2$$

$$v_{x,a} = a_x t_a$$

$$x_b = x_a + v_{x,a}\left(t_b - t_a\right)$$

- Six unknowns: $x_b \quad x_a \quad v_{x,a} \quad a_x \quad t_a \quad t_b$

- Need three extra facts: for example:

$$a_x \qquad t_a \qquad t_b$$

III. Solve

- solve for distance the runner has traveled

$$x_b \equiv x\left(t = t_b\right) = \frac{1}{2} a_x t_a^{\ 2} + a_x t_a \left(t_b - t_a\right) = a_x t_a t_b - \frac{1}{2} a_x t_a^{\ 2}$$

IV. Look Back : Choose Values

runner accelerated for $t_a = 3.0\,\text{s}$

initial acceleration: $a_x = 2.0\,\text{m}\cdot\text{s}^{-2}$

runs at a constant velocity for $t_b - t_a = 6.0\,\text{s}$

Total time of running $t_b = t_a + 6.0\,\text{s} = 3.0\,\text{s} + 6.0\,\text{s} = 9.0\,\text{s}$

Total distance running

$$x_{total} = a_x t_a t_b - \frac{1}{2} a_x t_a{}^2 = \left(2.0\,\text{m}\cdot\text{s}^{-2}\right)\left(3.0\,\text{s}\right)\left(9.0\,\text{s}\right) - \frac{1}{2}\left(2.0\,\text{m}\cdot\text{s}^{-2}\right)\left(3.0\,\text{s}\right)^2 = 4.5\times10^1\,\text{m}$$

Final velocity $v_{x,a} = a_x t_a = \left(2.0\,\text{m}\cdot\text{s}^{-2}\right)\left(3.0\,\text{s}\right) = 6.0\,\text{m}\cdot\text{s}^{-1}$