

## Experiment 10: Energy Transformation

### Purpose of the Experiment:

In this experiment you will add some heat to water in a plastic jar and measure the resulting increase in temperature. In the first part of the experiment the work added will be from non-conservative mechanical work (friction), which is why this experiment is informally known as the “pot scrubber” experiment. In the second part, you will add heat from the non-conservative dissipation of electrical energy; thus you will measure the mechanical and electrical equivalents of heat. This experiment—more than those we have done previously—will show that it is not a trivial matter to measure some important quantities accurately.

### The Apparatus:



The photo shows the apparatus. A motor rotates a plastic pot scrubber against a thin metal disk that forms the bottom of the plastic reservoir that contains the water, and the work done by the friction warms the water. A thermistor immersed in the water enables you to measure its temperature.

You can attach the leads of a pocket DVM to the left pair of binding posts on the apparatus in order to measure the thermistor resistance, to an accuracy of  $\pm 0.1 \Omega$  when you use the  $200 \Omega$  scale.

The right pair of binding posts connects to a  $2.5 \Omega$  resistor you will later use to heat the water electrically.

The reservoir is supported by a plastic arm and a spring-loaded screw part way along the arm allows you to adjust the normal force between the jar and the pot scrubber. The reservoir can rotate about a vertical axis, and the frictional torque is determined by hanging a known weight and attaching it to the black pulley on this axis.

## Preparing to Do the Experiment:

You do not have to use *DataStudio* to acquire data in this experiment but it is recommended, and it makes it easier to use the program to plot and analyze your results.

In a thermistor, thermal fluctuations excite charge carriers across a semiconducting band gap; as the temperature rises more carriers are excited across the gap and the electrical resistance of the thermistor drops. The probability of this excitation varies as  $e^{-E_{\text{gap}}/k_B T}$  and so the resistance of the thermistor has a theoretical dependence on temperature of the mathematical form  $e^{T_0/T}$  when  $T$  is measured in kelvins.

Over the limited temperature range of this experiment, the thermistor resistance is well described by a simpler function

$$R = R_0 e^{-\alpha T} \quad (1)$$

where  $T$  is the temperature in degrees Celsius and  $R_0$  is the resistance at  $0^\circ\text{C}$ . We can invert this expression to find the temperature from the thermistor resistance:

$$T = \frac{1}{\alpha} \ln \frac{R_0}{R} = B - A \ln(R) \quad (2)$$

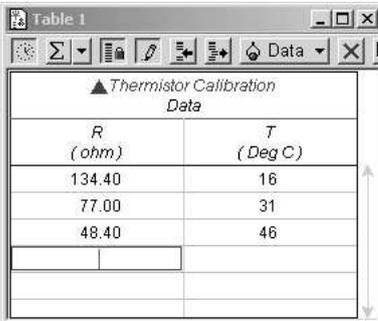
The resistance of your thermistor at three temperatures is given on the plastic jar that contains the water in your experiment. For my apparatus, the values were

<b>Resistance (<math>\Omega</math>):</b>	134.4	77.0	48.4
<b>Temperature (<math>^\circ\text{C}</math>):</b>	16	31	46

You may use *DataStudio* to determine the parameters  $B$  and  $A$  of Eq. (2) that characterize the thermistor in your apparatus. First, connect a *ScienceWorkshop 750* interface and start the program. Drag a Voltage Sensor icon to the interface in the Setup window. You will not actually measure any voltage this way, but it is required so that *DataStudio* will time your measurements, record the thermistor resistance, and calculate the temperature for you.

## Calibrating the Thermistor:

1. Go to the top menu bar and choose “Experiment” and “New Empty Data Table...” from the sub-menu. That will put an entry called “Editable Data” in the Data area of the Summary column at the upper left of the screen.
2. Double-click Editable Data and a Data Properties window will open up that allows you to give the table a name. Choose “Thermistor Calibration” or something equally descriptive. You should also use this window to give names to the variables you will enter and specify their units; these will be used when you plot them on a graph. The  $X$  variable (left column) should be  $R$  in units “ohm” and the  $Y$  variable should be  $T$  in units “Deg C”. Both variable types should be Other.

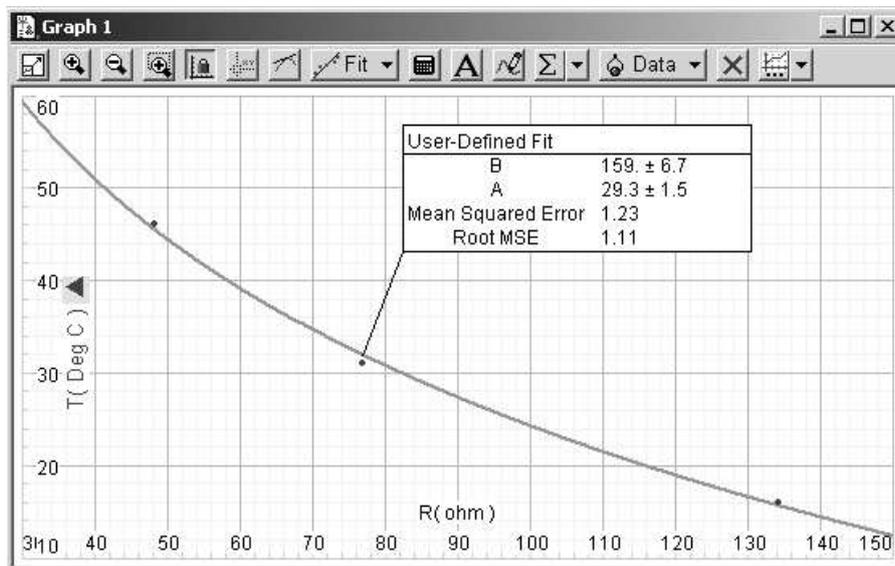


$R$ (ohm)	$T$ (Deg C)
134.40	16
77.00	31
48.40	46

Go to the table window and type the three temperatures in the  $Y$  (right) column and the three resistance values in the  $X$  column. Then you can minimize the table, or reduce its size to something convenient.

- Plot your thermistor data by dragging it from the Data window onto the Graph icon in the Displays window.

Your three data points should now be plotted on a graph with  $R$  along the  $X$  axis and  $T$  along the  $Y$  axis. Your graph will look neater if you click the Settings button  and remove the thin line connecting the data points. The next step is to select a User-Defined Fit from the graph's Fit menu. When you first do that, the text box where you expect to see the fit results appear on the graph will probably have a message that a best fit could not be found. Double-click the text box with the message and a new window with the title Curve Fit should open up. Type  $B-A*\ln(x)$  in the function definition window, make sure the Automatic button is chosen (depressed) and click the Accept button.



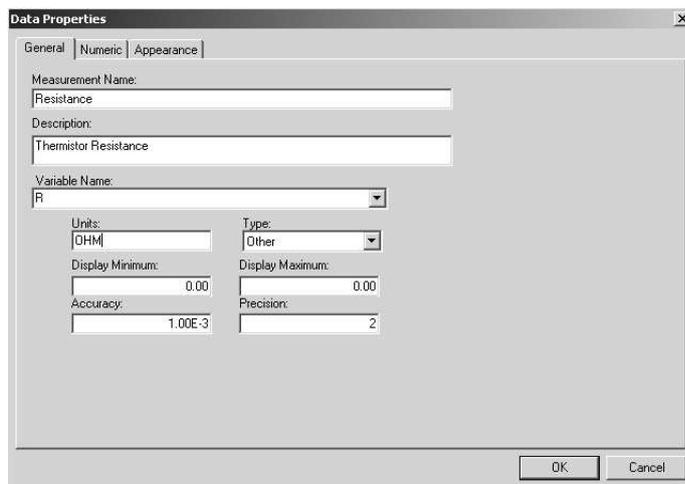
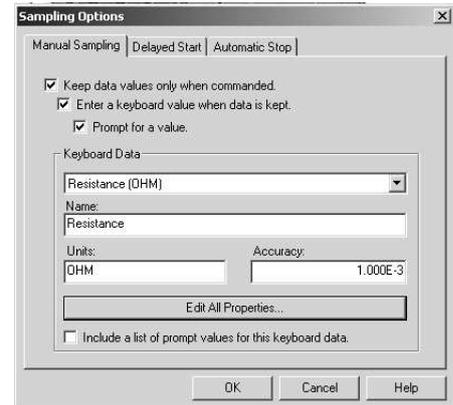
*DataStudio* should be able to find a best fit and display the results in the Curve Fit window and on the graph, like the one above. (If the program cannot find a fit, give it  $B = 150$  and  $A = 30$  for initial guesses.)

You will use these parameters in Eq. (2) to find the temperature from your measurements of the thermistor resistance. Before you do anything else, use the DVM ( $200\ \Omega$  scale) to measure the resistance of your thermistor. If the apparatus has been sitting for several hours, that should let you determine the room temperature—which you will need when you analyze your results. If the apparatus has been used in a previous class your thermistor will not be at room temperature. In that case ask your instructor what value to use for  $T_{\text{room}}$ . Fill in the table below and in your report. You will also need them for a homework problem.

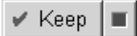
$B$ ( $^{\circ}\text{C}$ )	$A$ ( $^{\circ}\text{C}$ )	$T_{\text{room}}$ ( $^{\circ}\text{C}$ )

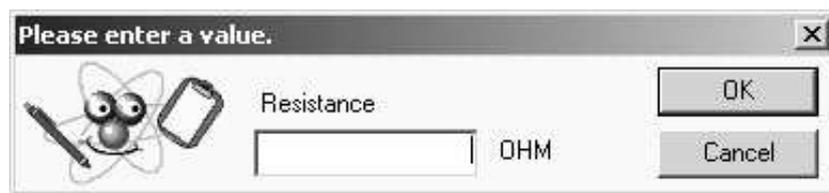
## Preparing to Make Measurements:

You can use *DataStudio*'s Manual Sampling mode to time your measurements, enter the results into a data table, calculate the temperature and analyze the results. To set this up, click the Options  button. A Sampling Options window will open. All of the boxes under the Manual Sampling tab should be checked. Click the Edit All Properties tab and a Data Properties window will open. Name the measurement "Resistance", set the variable name to *R* and the units to "ohm". Click OK on both windows.



A new entry for the Resistance will appear in the Data window.

With this setup, when you click the Start button it will change to a Keep button  with a stop button (small red square) beside it. Whenever you click the Keep button, *DataStudio* will measure the voltage at the input to the 750 interface (which will be 0) and open a window for you to type in the resistance of your thermistor.



When you click OK (or type a carriage return) the resistance value will be entered as a dependent variable into the Resistance (ohm) data set along with the time you clicked the Keep button as the independent variable. When you want to stop making measurements, click the small red stop button and the data will be saved as Run #1 under the Resistance (ohm) heading in the Data window.

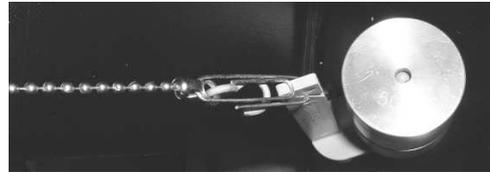
**Note:** To save its battery the DVM may shut itself off. You can turn it back on by switching to a different scale and back to the one you were using.

## Adding Heat from Friction:

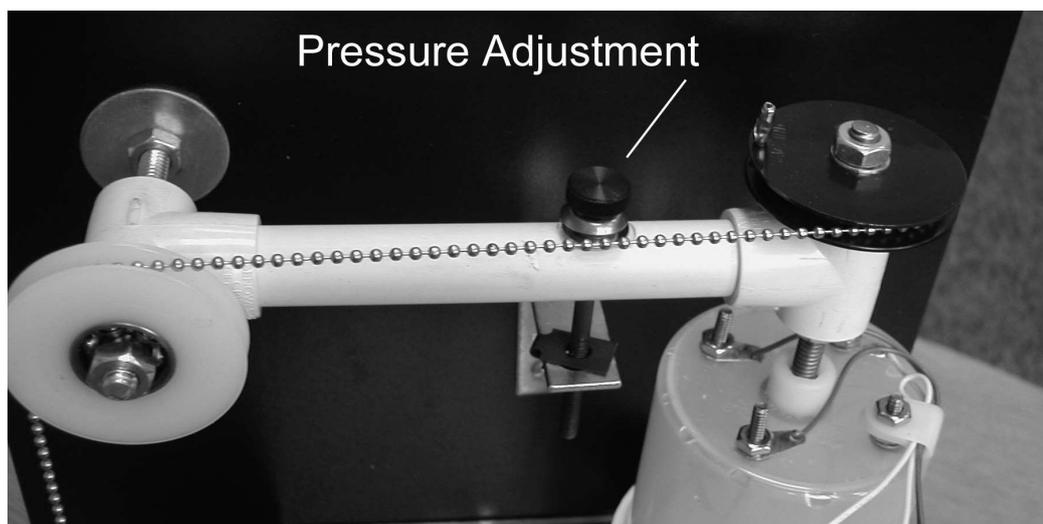
Refer back to the photo on the first page.

Pass the chain over the white pulley, as shown in the photo. The chain should wrap at least  $\frac{1}{4}$  turn around the black reservoir axis pulley (to make sure the torque moment arm is equal to the radius of the pulley) and the hook should not contact the white pulley, in order that the weights can move up and down freely.

Place three 100 gm weights on the 5 gm plastic holder (total mass 305 gm) and attach them to the hook at the end of the chain. A paper clip can be used to secure the hook to the slot in the holder.



Until the motor has been turned on and you have balanced the torques, you will find it necessary to hold the reservoir so it does not twist too far and pull on the wires attached to it. You want to make sure there is no stress on the electrical leads attached to the reservoir, as that would apply an unknown torque to the reservoir. The axis connects to a white plastic socket on top of the reservoir; if necessary you may lift the axis out of the socket to rotate the reservoir with respect to the black pulley that is fixed to the axis.



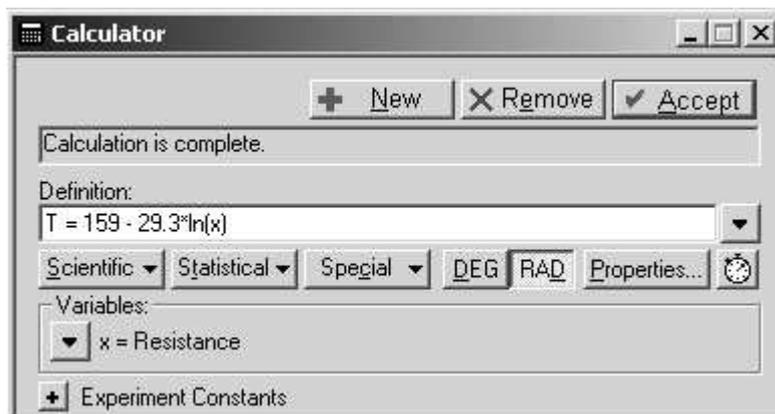
Turn on the motor and adjust the pressure adjustment screw so that the system balances with the torque from the hanging weight, and the reservoir does not rotate. The chain should wrap around the black pulley from  $\frac{1}{4}$  to  $\frac{3}{4}$  of a turn. You may need to re-adjust the screw from time to time to maintain this balance throughout your measurement.

As the motor turns, the weight will move up and down several mm and the reservoir will oscillate back and forth. That is useful because it keeps the water stirred and hence at a uniform temperature. If that does not happen with your apparatus, you will have to agitate it (as described in the Electric Heating section) to keep the water stirred.

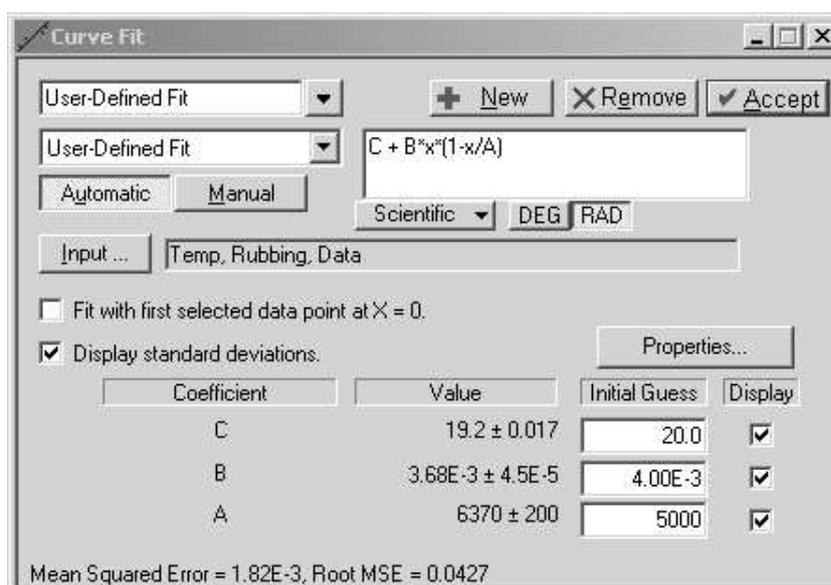
When things are stable and you are ready to take data click the Start button.

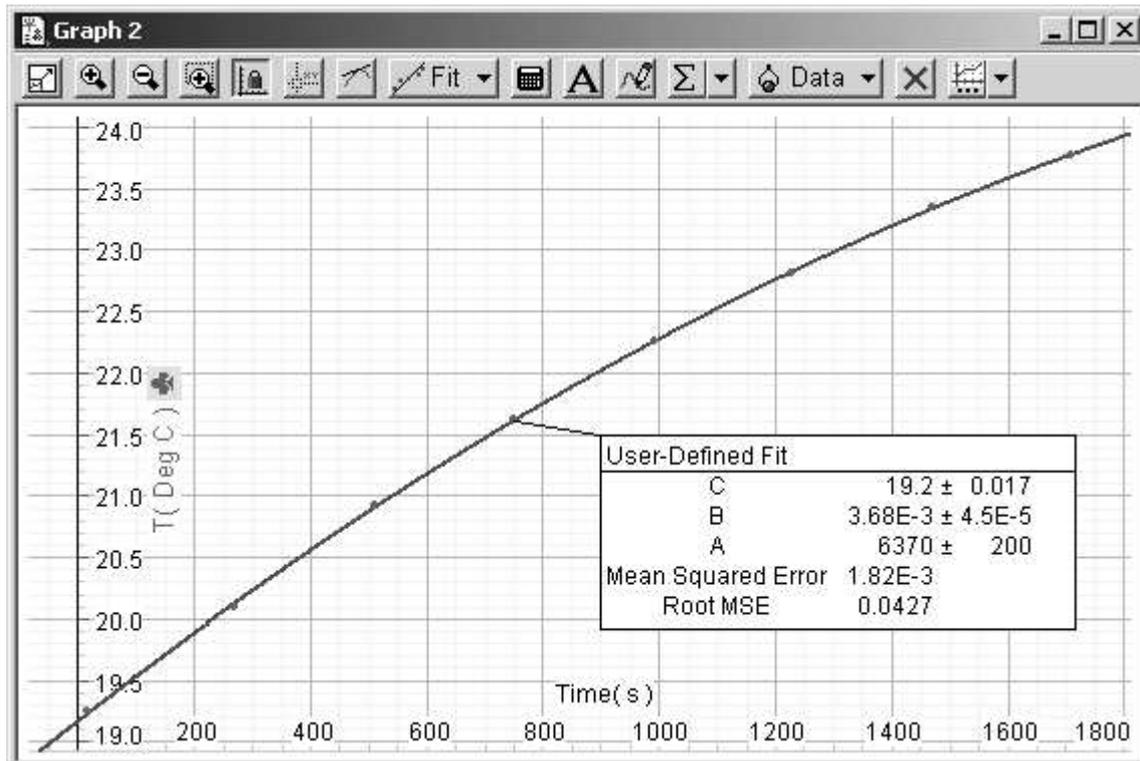
To make a measurement, click the Keep button. That will record the time in *DataStudio* and open a window for you to type in the thermistor resistance. I found it satisfactory to record thermistor resistance about every three minutes for a total time of 21 minutes. At the end of that time, click the stop button and turn off the motor.

You will now have a set of data labeled “Run #1” under the Resistance entry in the Data window. Drag it onto the Graph icon in order to make a plot of the data; it will have thermistor  $R$  on the  $Y$  axis and time on the  $X$  axis. Click the calculator button on the graph’s toolbar. That will open a window like this one.



In the Definition window type  $T=159-29.3*\ln(x)$  (use the appropriate  $A$  and  $B$  parameters for your thermistor) and click Accept. That will produce a new data set in the Data window called (surprise!)  $T=159-29.3*\ln(x)$ . You can double-click it to open a Data Properties window so you can choose variable names and units. Drag this data set onto the Graph icon to make a graph. You will then have a plot of the water temperature on the  $Y$  axis and time on the  $X$  axis. Your graph will look neater if you click the Settings button  and remove the thin line connecting the data points. The next step is to make a User-Defined Fit to these data. In the Curve Fit window, define the fitting function as  $C+B*x*(1-x/A)$  (the reason for this choice of function is explained in the Data Analysis section of these notes). When you first click Accept *DataStudio* will probably not be able to find a fit, so you should provide some help by supplying the initial guesses  $C = 20$ ,  $B = 0.004$  and  $A = 5000$ .





This graph has my fit parameters. You will need yours for your data analysis. Enter them (include the  $\pm$  standard deviations of the parameters) in the table below and in your report. You will also need them for a homework problem.

$C$ ( $^{\circ}\text{C}$ )	$B$ ( $^{\circ}\text{C}/\text{s}$ )	$A$ (s)

### Electric Heating:

In this measurement you will add heat electrically. First, remove the hanging weight, loosen the pressure adjustment screw and remove the reservoir from its position on the vertical axis. Place it on the platform above the four binding posts.

During this measurement you will have to agitate the water to keep the temperature uniform. If you rock the apparatus front to back, the water in the jar will slosh back and forth. During the experiment, do this for about 10s once a minute and do it continuously during the 30s just before you measure the thermistor resistance. Do it vigorously enough that you think the water is well mixed. You may also agitate the water by picking the jar up and rocking it. If you do that try to pick it up by the jar lid at its base, to reduce heat transfer to your hand.

Temporarily connect your DVM to the right hand pair of binding posts and measure the resistance of the heating resistor in the reservoir; it should be  $2.5 \pm 0.1 \Omega$ .

Make sure the electric power supply is turned off. Set the current knob to the midpoint of its range and turn the voltage controls all the way down (CCW).

Connect the power supply to the right hand pair of binding posts. Set the DVM scale to 20 V. Turn on the power supply and use the fine voltage control in combination with the DVM to set the voltage as close to 2.5 V as you can.

Record the heating resistance and the power supply voltage in the table below and in your report. You will also need them for a homework problem.

$R$ ( $\Omega$ )	$V$ (volts)

Connect the DVM (on the 200  $\Omega$  scale) back to the thermistor binding posts and agitate the water. You are ready to measure the electric heating curve.

Click the Start button and agitate the water as described above. Measure the thermistor resistance about every three minutes for a total time of 24 minutes.

When you have completed that, make graphs and carry out the same User-Defined Fit as you did for your friction heating measurements. Enter your results in the table below (include the  $\pm$  standard deviations) and in your report. You will also need them for a homework problem.

$C$ ( $^{\circ}\text{C}$ )	$B$ ( $^{\circ}\text{C}/\text{s}$ )	$A$ (s)

### Data Analysis:

The data analysis for this experiment is based on the following model. The specific heat of water is  $c_w = 1.00 \text{ cal}/(\text{gm } ^{\circ}\text{C})$ , so a mass  $m_w$  of water has a heat capacity  $m_w c_w \text{ cal}/^{\circ}\text{C}$ . If heat energy is added to the water at a rate  $P_{\text{net}} \text{ cal}/\text{s}$ , the water temperature will increase according to

$$\frac{dT}{dt} = \frac{P_{\text{net}}}{m_w c_w}.$$

The energy  $P_{\text{net}}$  added to the water has two parts. First, there is energy added by the non-conservative frictional or electrical work. Second, if the water in the reservoir is warmer or colder than the surrounding air in the room, energy will flow from the the water to the room, or vice versa. Because the water and room temperatures are not very different, the heat energy exchanged with the room will be proportional to the temperature difference.

$$P_{\text{room}} = -\lambda(T - T_{\text{room}})$$

where  $P_{\text{room}}$  is the heat energy added by the room to the water and  $\lambda$  is some unknown constant. We will see how to determine it presently. Let us consider the case of friction heating of the water, where the heat is added by friction at the rate  $P_f$ . Then the water temperature is described by the differential equation

$$\frac{dT}{dt} = \frac{P_f}{m_w c_w} - \frac{\lambda(T - T_{\text{room}})}{m_w c_w}.$$

Now, we can calculate  $P_f$  because it is just the friction torque multiplied by the angular velocity the motor rotates. Thus

$$P_f = W r_{\text{pulley}} \omega_{\text{motor}}$$

where  $W = 0.305 \times 9.805 = 2.99 \text{ N}$  is the weight suspended from the chain,  $r_{\text{pulley}} = 0.019 \pm 0.001 \text{ m}$  is the radius of the pulley on the vertical axis, and  $\omega$  is the angular velocity of the motor. It is specified by the manufacturer to be 200 rpm. I measured it with a strobe light and found  $\omega = 200 \pm 4 \text{ rpm} = 20.9 \pm 0.5 \text{ rad/s}$ . Using these values, I calculated  $P_f = 1.19 \pm 0.06 \text{ J/s}$ . Most of the uncertainty comes from the uncertainty in  $r_{\text{pulley}}$ .

To make further progress, we have to solve the differential equation for  $T$ . First, consider the simpler equation when  $P_f = 0$ .

$$\frac{dT}{dt} = -\frac{\lambda(T - T_{\text{room}})}{m_w c_w}.$$

If  $T_{\text{room}}$  is constant we can introduce a new variable  $\Delta = T - T_{\text{room}}$  and the equation becomes

$$\frac{d\Delta}{dt} = -\frac{\lambda \Delta}{m_w c_w} \quad \text{or} \quad \frac{d\Delta}{\Delta} = -\frac{\lambda}{m_w c_w} dt = -\frac{dt}{\tau_0}$$

which can be integrated to give

$$\ln(\Delta) = -\frac{t}{\tau_0} + \text{const} \quad \text{or} \quad \Delta(t) = \Delta_0 e^{-t/\tau_0}$$

where  $\tau_0 = m_w c_w / \lambda$  is a characteristic time or time constant, and  $\Delta_0$  is a constant equal to  $T(t=0) - T_{\text{room}} \equiv T_0 - T_{\text{room}}$ . The equation we really want to solve is the same one with a constant,  $P_f / (m_w c_w)$ , added to the right side.

$$\frac{d\Delta}{dt} = \frac{P_f}{m_w c_w} - \frac{\Delta}{\tau_0}$$

If you have had a course in differential equations you know that the solution to this equation is

$$\Delta(t) = \Delta_0 e^{-t/\tau_0} + \frac{\tau_0 P_f}{m_w c_w} (1 - e^{-t/\tau_0}).$$

If you haven't had such a course, just substitute the solution into the equation to see that it works. Returning to the original variable  $T = \Delta + T_{\text{room}}$  gives

$$T(t) = T_0 + \left( \frac{\tau_0 P_f}{m_w c_w} - T_0 + T_{\text{room}} \right) (1 - e^{-t/\tau_0}).$$

If  $t/\tau_0$  is small, the exponential can be expanded using the series

$$e^x = 1 + x + \frac{1}{2}x^2 + \dots$$

This gives

$$T(t) = T_0 + \left( \frac{P_f}{m_w c_w} - \frac{T_0 - T_{\text{room}}}{\tau_0} \right) t \left( 1 - \frac{t}{2\tau_0} + \dots \right).$$

This is the same expression you used to fit your measurements. By comparing coefficients in the two versions, you can find

$$C = T_0, \quad B = \left( \frac{P_f}{m_w c_w} - \frac{T_0 - T_{\text{room}}}{\tau_0} \right) \quad \text{and} \quad A = 2\tau_0.$$

Therefore, from the parameters obtained in your fit you can find

$$\frac{P_f}{m_w c_w} = B + \frac{2(C - T_{\text{room}})}{A}.$$

The same expression holds true for electric heating if  $P_f$  is replaced by  $P_e$ , the rate energy is added to the water by electric non-conservative work. From my measurements I found

$$\frac{P_f}{m_w c_w} = 3.79 \times 10^{-3} \text{ }^\circ\text{C s}^{-1} \quad (\pm 1.5\%)$$

and

$$\frac{P_e}{m_w c_w} = 5.25 \times 10^{-3} \text{ }^\circ\text{C s}^{-1} \quad (\pm 2.5\%)$$

### **An Historical Note:**

(From <http://www.rit.edu/~flwstv/heat.html>, also the “Special” link on the web page.) For a long time it was thought that heat was a fluid called *phlogiston* or *caloric* that flowed into bodies when they became hotter and flowed out of them when they cooled. One calorie was the amount of this fluid that flowed into 1 gm of water to increase its temperature by 1 °C. One of the first to recognize this was incorrect was Benjamin Thompson, a New Englander from Concord, N.H. (Thompson chose the losing side during the American Revolution and finished his career in Europe, where he became Count Rumford—an old name for Concord.) While observing the boring of canon barrels, Rumford concluded that mechanical energy must be converted into heat. This was an important step in developing the concept of the conservation of energy. Rumford measured the mechanical equivalent of heat—the amount of non-conservative mechanical work that equalled one calorie, but obtained a result that was too large. It was James Prescott Joule who measured it precisely and who also worked out the formula  $I^2R$  for the non-conservative work done by an electric current. The currently accepted mechanical equivalent of heat is 4.186 joules per calorie.

### How good were my results?

I have kept track of the experimental uncertainties in the quantities I measured, and now is the time to see if the mechanical equivalent of heat I obtain is as good as Joule's, or more like Rumford's.

From my rubbing experiment I calculated  $P_f$  was  $1.19 \pm 0.06$  J/s, and if  $m_w = 60$  gm (the amount of water in the reservoir I used) the fitting parameters for the heating curve give  $P_f = 0.227 \pm 0.004$  cal/s. This gives the mechanical equivalent of heat as  $5.2 \pm 0.3$  J/cal, too large!

Similarly, for my electric heating Joule's formula gives  $P_e = 1.54 \pm 0.08$  J/s (my heating voltage was 1.96 V; with 2.5 V your  $P_e$  will be different). The uncertainty comes from the  $\pm 0.1 \Omega$  uncertainty in the heater resistance. From the fitting parameters  $P_e = 0.315 \pm 0.008$  cal/s if  $m_w = 60$  gm. This gives  $4.9 \pm 0.3$  J/cal, again too large.

How can this discrepancy, clearly greater than the estimated measurement errors, be explained? The most likely explanation is that more mass than just the 60 gm of water in the reservoir is being heated. The heat lost to the room has been allowed for in the model, but the plastic jar, its metal lid, and few other small items are also being heated. If the jar and the other items had the same heat capacity as 10 gm of water—which seems very reasonable to me—the measured equivalent for electric heating would be 4.2 J/cal.

In the case of friction heating, the pot scrubber and its mount provide some additional heat capacity; if that were the same as 4 gm of water, the friction result would also be 4.2 J/cal.

It is difficult, but very important, in measuring the mechanical equivalent of heat to know the heat capacity of the mass you are heating by non-conservative work. Perhaps Rumford had a similar problem.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Physics Department

Physics 8.01T

Fall Term 2004

**Part of Problem Set 12**

**Section and Group:** \_\_\_\_\_

**Your Name:** \_\_\_\_\_

**Part One: Calibration**

For the thermistor the temperature is calculated from the resistance using  $T = B - A \ln(R)$ . Enter the room temperature during your measurements and the parameters  $A$  and  $B$  for your thermistor in the table below.

$B$ ( $^{\circ}\text{C}$ )	$A$ ( $^{\circ}\text{C}$ )	$T_{\text{room}}$ ( $^{\circ}\text{C}$ )

If you can measure the thermistor resistance with an uncertainty of  $\pm 0.1 \Omega$ , what is the uncertainty in the temperature you measure?

**Part Two: Analysis**

If the temperature of the water in the reservoir obeys the differential equation

$$\frac{dT}{dt} = \frac{P_f}{m_w c_w} - \frac{\lambda(T - T_{\text{room}})}{m_w c_w} = \frac{P_f}{m_w c_w} - \frac{T - T_{\text{room}}}{\tau_0}$$

and you were to run the experiment for a time long compared to  $\tau_0$ , what would the temperature of the water be (in terms of the quantities in the equation)?

**Friction Heating:** When you fit your heating curve to  $T(t) = C + Bt(1 - t/A)$ , what did you find for the parameters?

$C$ ( $^{\circ}\text{C}$ )	$B$ ( $^{\circ}\text{C}/\text{s}$ )	$A$ (s)

What mass of water  $m_w$  was in your jar?

What did you calculate for the mechanical equivalent of heat using this  $m_w$ ?

What value did you need to use for  $m_w$  to get the result 4.2 J/cal?

If you ran your apparatus for a time long compared to  $A$ , what temperature would the water come to?

**Electric Heating:** What did you measure for the heater resistance and applied voltage?

$R$ ( $\Omega$ )	$V$ (volts)

When you fit your heating curve to the expression  $T(t) = C + Bt(1 - t/A)$ , what did you find for the parameters?

$C$ ( $^{\circ}\text{C}$ )	$B$ ( $^{\circ}\text{C}/\text{s}$ )	$A$ (s)

What mass of water  $m_w$  was in your jar?

What did you calculate for the mechanical equivalent of heat using this  $m_w$ ?

What value did you need to use for  $m_w$  to get the result 4.2 J/cal?

If you ran your apparatus for a time long compared to  $A$ , what temperature would the water come to?