MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.01T   Fall Term 2004

Experiment 09: Angular Momentum

Purpose of the Experiment:

In this experiment you investigate rotational collisions and the conservation of angular momentum in rigid body rotational dynamics. It is the rotary counterpart of Experiment 07 in which you investigated linear collisions. It is also more difficult.

The heart of the experiment is a high quality DC motor to spin a rotor up to several hundred radians per second. When power to the motor is shut off, it serves as a tachometer-generator whose output voltage is proportional to the angular velocity of the rotor; thus the angular velocity of the rotor can be determined by measuring the output voltage. When you hold down the red pushbutton switch on the apparatus, power is applied to the motor; when you release it, the rotor coasts and the output voltage can be read by the *DataStudio* program. This experiment will give you practice in

- measuring and calculating moments of inertia,
- calculating rotational kinetic energy and non-conservative rotational work, and
- using several other concepts from our study of rotational dynamics.

Setting Up the Experiment:

Plug the rotary motion apparatus into its power supply; you should see the LED in the plastic pipe elbow come on. Connect the phototransistor and generator output voltages to inputs A and B on the *ScienceWorkshop 750* interface box using the leads from two voltage sensor plugs. (The generator output terminals are the two farthest from the power input connector.) The best *DataStudio* sampling options depend on what you are going to measure and will be given in a table for each section of the experiment. Before you can carry out any rotational collision measurements, you must first calibrate the tachometer-generator and measure the moment of inertia of the rotor.

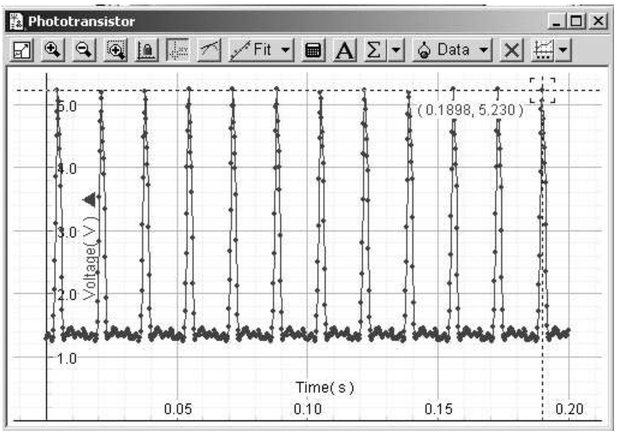
Calibrating the Generator:

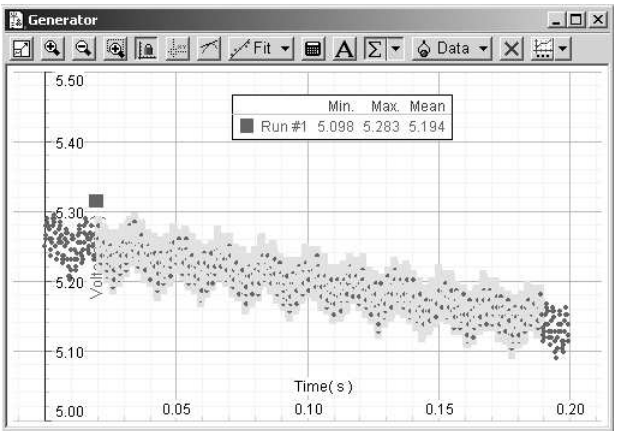
Stick a black sticker or a small piece of black tape on the white plastic centerpiece of the rotor so that it will be illuminated by the LED and the reflected light detected by the phototransistor. The voltage output will be about 5 V when the LED is reflected from the black tape and 1 V when it is reflected from the white plastic. Set up the *DataStudio* sampling options as follows.

Voltage Sensitivity	Voltage Sample Rate	Delayed Start	Automatic Stop
Low (1X)	5000 Hz	None	0.25 sec

Set up graphs to plot both the phototransistor output and the generator output as a function of time. Spin the motor up for several seconds, release the button and allow it to coast for about a second, then click the *DataStudio* start button.

You should get plots something like these.





The voltage peaks on the left graph correspond to the black tape passing in front of the LED. The number of peaks you see depends upon how fast the rotor was spinning when you did your measurement; anywhere from five to ten peaks in the 0.250 s plot time should be OK. Use the graph Smart Tool (locked on to the points) to find the time for, say, six revolutions of the rotor. The generator output in the right graph is noisy, but you can select the data corresponding to the time period between the peaks you counted in the left graph and use the Σ tool for the generator graph to find the average output voltage for the period. Use these results to find the angular velocity and voltage generator output. (You should find that 1 V corresponds to about 70 radians/sec; I obtained 71.8 (rad/s) V^{-1}.) Record your value in your report.

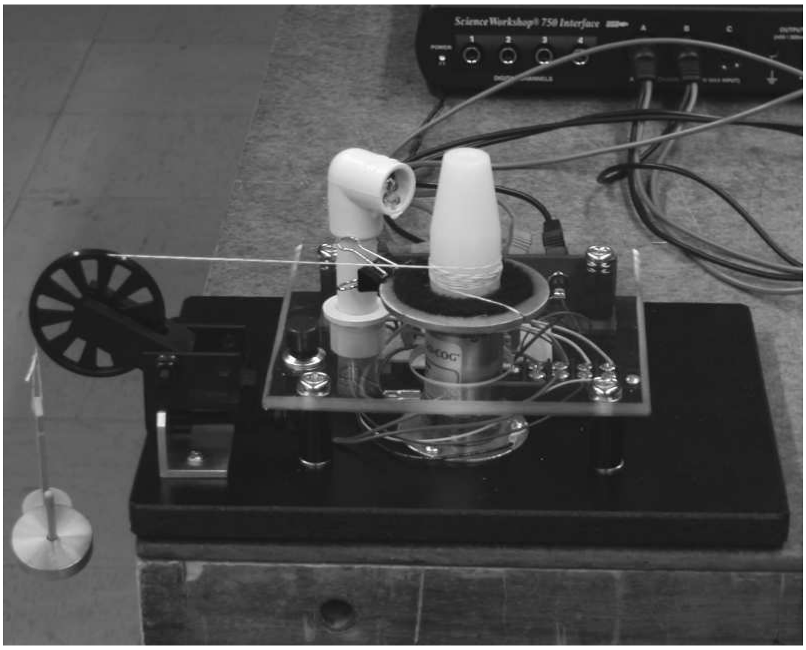
If you were fussy, you could repeat the measurement for several different rotation speeds, but I checked it and found it to be quite linear—so there is no real need.

Remove the black sticker or tape. In the remaining measurements you will only use the generator output voltage and can remove the graph of phototransistor output.

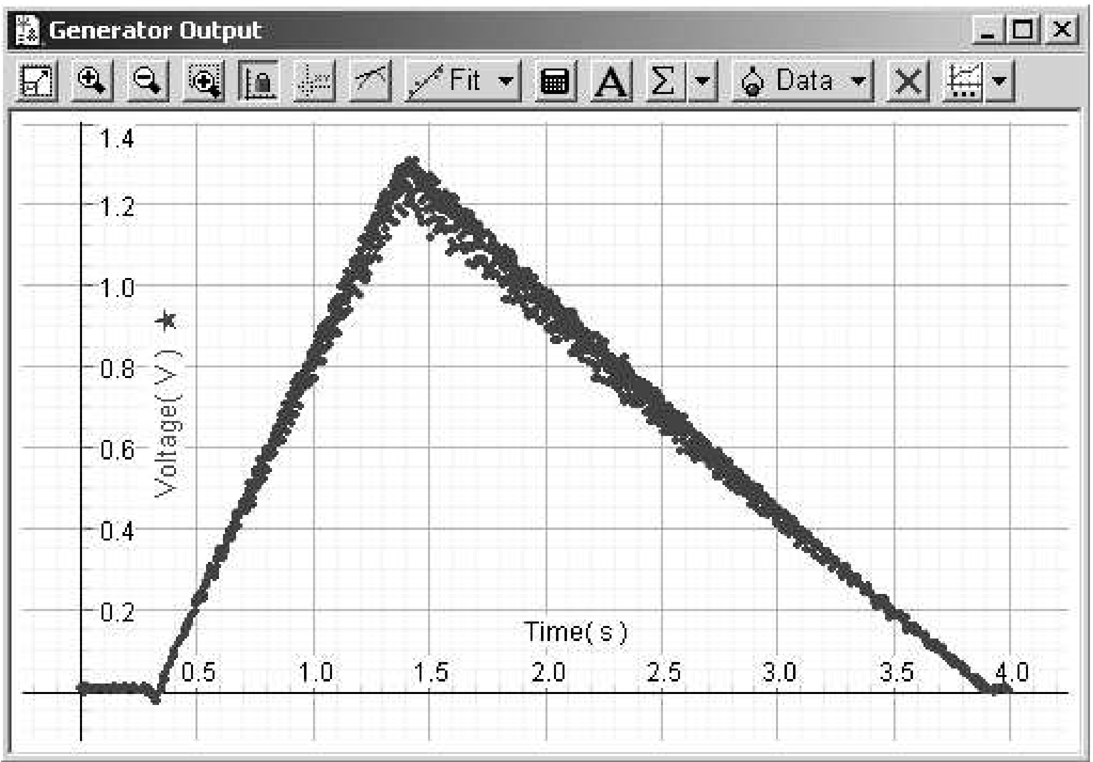
Finding the Rotor Moment of Inertia:

Next, you must measure the moment of inertia I_R of the rotor. To do this you will use a 55 gm weight (50 gm brass weight plus 5 gm plastic holder) to accelerate the rotor. Tie a loop in one end of the string and use it to suspend the weight over the pulley, as shown in the photo at the top of the next page. Use a string just long enough to reach from the weight (when it is on the floor) over the pulley to the axis of the rotor. Tie a knot at the other end of the string and insert the string into the kerf cut into the brass washer on the rotor and wind the string around the constant diameter portion of the white plastic. (This part has a diameter of 1.00 inch or a radius of 12.7 mm, which you will need to calculate the torque.) Keep the string away from the Velcro on the washer that is part of the rotor as you wind it. Set up the *DataStudio* sampling options as follows.

Voltage Sensitivity	Voltage Sample Rate	Delayed Start	Automatic Stop
Low (1X)	500 Hz	None	4.0 sec

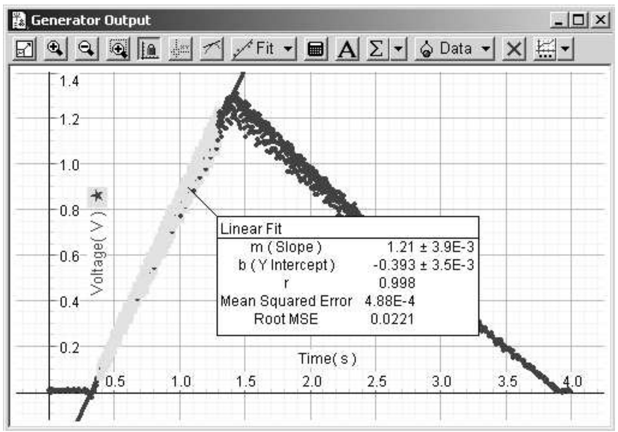


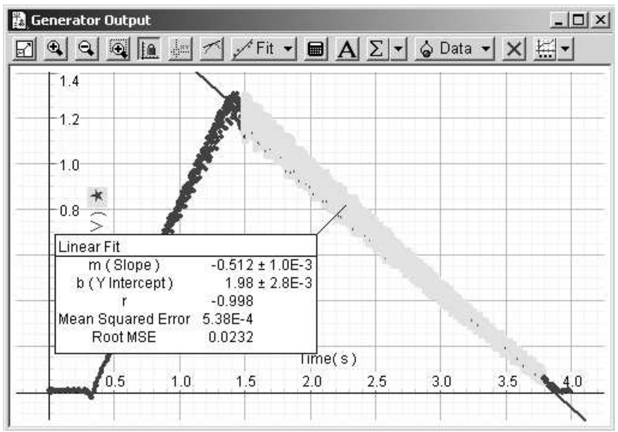
Click the *DataStudio* start button and release the weight. You should get a graph that resembles the one below.



The graph has two feaures. First the voltage (ω of the rotor) increases linearly, showing constant angular acceleration α. Then it shows a constant (but smaller in magnitude) angular deceleration. The change from one to the other comes when the string pulls out of the kerf.

The constant deceleration is produced by a constant friction torque τ_f, and the acceleration is produced by the torque from the 55 gm weight. Of course friction also acts when the falling weight is accelerating the rotor, so you will have to know τ_f in order to find the moment of inertia of the rotor. A Linear Fit separately to the rising and falling parts of the graph will give the answers.





You may be unlucky and the string will not pull cleanly out of the kerf and allow the rotor to coast. In that case you can still obtain the angular acceleration $\alpha_{\rm up}$ for the rising curve on the left in your graph but you should make a second measurement to get the angular acceleration $\alpha_{\rm down}$ when the rotor is slowing down under the torque of the bearing friction. To do that, remove the string from the rotor, spin up the motor with the red power button, release the button so it begins to coast, and start *DataStudio* again. You will get a nice graph whose downward slope $\alpha_{\rm down}$ you can measure.

For my graph the slope for $\alpha_{\rm up}$ was 1.21 V/s corresponding to 71.8 (rad/s) $\text{V}^{-1} \times 1.21$ V/s= 86.9 rad/s^2. For $\alpha_{\rm down}$ I obtained -0.512 V/s or -36.8 rad/s^2. Friction causes the only torque that produces $\alpha_{\rm down}$, so

$$I_R|\alpha_{\rm down}| = |\tau_f| \,. \tag{1}$$

The linear acceleration of the falling weight is given by $a = r\alpha_{\rm up}$ where $r = 0.0127$ m is the radius of the rotor the string was wound around. The tension in the string must then be $T = m(g - a)$ where $m = 0.055$ kg is the mass of the falling weight. The string therefore produces a torque $\tau_{\rm up} = rT$ on the rotor. The net torque produces $\alpha_{\rm up}$, thus

$$I_R\alpha_{\rm up} = \tau_{\rm up} - |\tau_f| \,. \tag{2}$$

Enter your measured values for $\alpha_{\rm up}$ and $\alpha_{\rm down}$ into the following table.

$\alpha_{\rm down}$	$\alpha_{\rm up}$	a	T	$\tau_{\rm up}$

The other values in the table may be calculated later as part of your report. Save them for a problem that is part of Problem Set 11, due November 23 and attached as the last pages of this document.

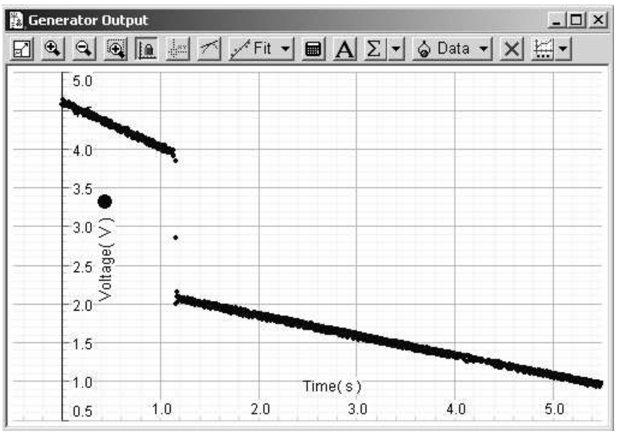
A Fast Inelastic Collision:

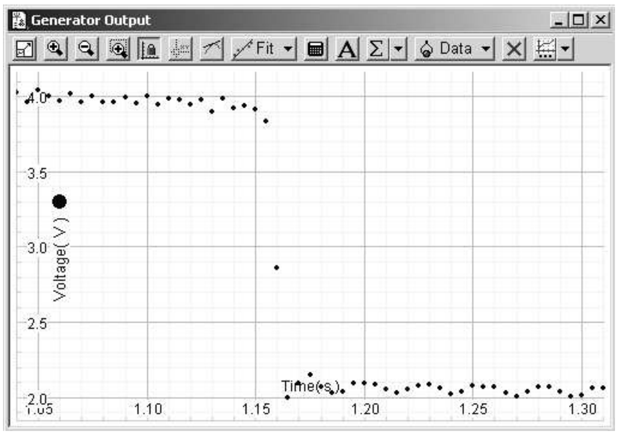
Now you have the apparatus calibrated and can do a fast angular collision. As you observed in the previous measurement, there is a significant friction torque τ_f acting on the rotor. This torque is always present and must be taken into account when you analyze your results. A "fast" collision is one in which the angular impulse of τ_f is too small to affect the conservation of angular momentum during the collision (§ 12.1.8 of course notes).

Take a large brass washer with one smooth side and one with Velcro glued onto it. If you drop it Velcro side down onto the spinning rotor, it will stick to the Velcro on the rotating washer and make a very short duration collision. For this measurement, set up the *DataStudio* sampling options as follows.

Voltage Sensitivity	**Voltage Sample Rate**	**Delayed Start**	**Automatic Stop**
Low (1X)	200 Hz	1.0 sec	Falls below 0.5 V

There is no need to retain data before the delayed start. Spin up the rotor to its maximum speed and release the red button. Then click the *DataStudio* start button. Wait a second or two and drop the washer as accurately as you can over the axis of the spinning rotor. You should obtain a graph something like the one at the left below.





The graph at the right shows the collision region expanded. From this graph you can use the Smart Tool to find the angular velocities before and after the collision, ω_1 and ω_2, and also estimate the duration of the collision δt. (I estimated δt between 5 and 10 ms from my graph). The values in this table need to be filled in while you do the experiment, as you will need them to complete your report or for the problem set.

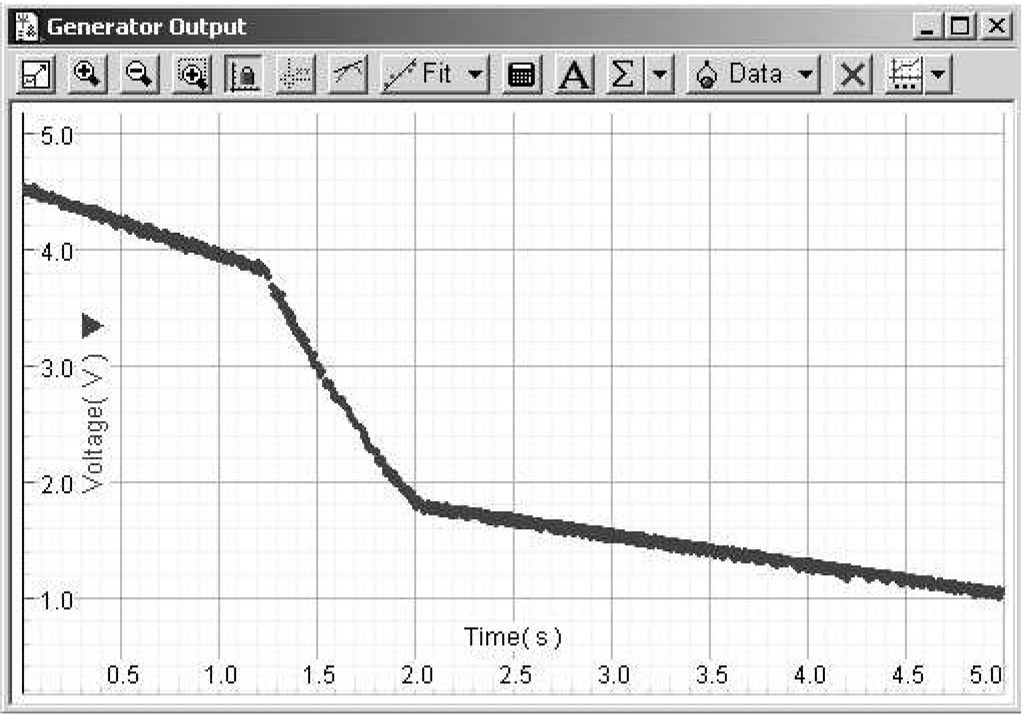
ω_1	ω_2	δt

You will need to calculate the moment of inertia of the washer you dropped to create the collision. It is given by

$$I_W = \frac{1}{2} M_W (r_o^2 + r_i^2)$$

where $r_o = 0.032\,\text{m}$ and $r_i = 0.0135\,\text{m}$. The mass of the washer, M_W, is written on the washer.

A Slow Collision:
This is a repeat of the previous measurement, except you drop the washer smooth side down. You should obtain a graph something like this.



The main difference from the previous experiment is that the collision lasts long enough that τ_f has a significant impulse; thus the angular momentum is reduced during the course of the collision. You can measure one new parameter, the angular acceleration α_c of the rotor during the collision. If you look closely, you can see that α_c is not constant during the collision. However the average value will be good enough for your analysis. You may find it two ways: either from $(\omega_2 - \omega_1)/\delta t$ or by a Linear Fit to the data points during the collision. Use the method you like best.

ω_1	ω_2	δt	α_c

Again, the values in this table need to be filled in while you do the experiment, as you will need them to complete your report and for the problem set.

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Part of Problem Set 11

Section and Group: ______________________________

Your Name: ______________________________

Part One: Rotor Moment of Inertia

Enter the results from your experiment report into the table below.

α_{down}	α_{up}	a	T	τ_{up}

Using

$$\begin{aligned} I_R \alpha_{\text{up}} &= \tau_{\text{up}} - |\tau_f| \\ &= rm(g - r\alpha_{\text{up}}) - I_R|\alpha_{\text{down}}|\,, \end{aligned}$$

($m = 0.055\,\text{kg}$ is the mass of the weight, $r = 0.0127\,\text{m}$, and α_{up} and α_{down} are obtained from your measurements) derive the relationship

$$I_R = \frac{mr(g - r\alpha_{\text{up}})}{\alpha_{\text{up}} + |\alpha_{\text{down}}|}$$

What is your numerical value for I_R ?

Part Two: Fast Inelastic Collision

Write your measurement results into the table below.

ω_1	ω_2	δt

What is your numerical value for I_W ?

1. Use the moments of inertia I_R and I_W along with ω_1 and ω_2 to calculate the angular momenta before and after the collision and compare them.

2. Use the values you found for τ_f and δt to estimate the angular impulse of τ_f during the collision. Compare it to the angular momentum difference you just calculated.

3. Calculate the rotational kinetic energies $K_1 = \frac{1}{2} I_R \omega_1^2$, before, and $K_2 = \frac{1}{2}(I_R + I_W)\omega_2^2$, after the collision.

Part Three: Slow Inelastic Collision

Fill in the table below with the values you measured in your experiment.

ω_1	ω_2	δt	α_c

1. Use the moments of inertia I_R and I_W along with ω_1 and ω_2 to calculate the angular momenta before and after the collision and compare them.

2. Use the values you found for τ_f and δt to estimate the angular impulse of τ_f during the collision. Compare it to the difference in angular momenta before and after the collision.

3. Use the value you found for α_c to estimate the total torque τ_c on the rotor during the collision.

4. The torque τ_c is made of two parts: the friction torque τ_f from the bearings and the torque τ_{RW} the washer you dropped exerts on the rotor. By the 3$^{\text{rd}}$ law, the rotor exerts an equal and opposite torque on the washer. Since you know τ_f, subtract it from τ_c to find an estimate for τ_{RW}.

Part Four: Nonconservative Work in the Slow Collision

You can find τ_{RW} a different way, because it produces the angular acceleration of the dropped washer, whose average value is $\alpha_W = \omega_2/\delta t$. Use this relation to estimate τ_{RW} and compare it to the previous estimate you made:

The torque τ_{RW} comes from the sliding friction between the washer on the rotor and the washer you dropped. Thus there must be some non-conservative work. You may calculate it if you know the angular "distance" the washer slides over the rotor before it reaches the same angular velocity as the rotor, and you do have enough information to find that. The angle the rotor rotates through during the collision is (analogous to linear motion with constant acceleration)

$$\Delta\theta_R = \omega_1 \delta t - \frac{1}{2}|\alpha_c|\delta t^2$$

while the dropped washer rotates through an angle

$$\Delta\theta_W = \frac{1}{2}|\alpha_W|\delta t^2.$$

Thus the non-conservative work done by the sliding friction between the two washers will be

$$W_{NC,W} = \tau_{RW}(\Delta\theta_R - \Delta\theta_W).$$

You can also calculate the non-conservative work done by the bearing friction during the collision

$$W_{NC,B} = \tau_f \Delta\theta_R.$$

Calculate these two amounts of non-conservative work and compare their sum to the change in rotational kinetic energy $\frac{1}{2}I_R\omega_1^2 - \frac{1}{2}(I_R + I_W)\omega_2^2$ during the collision.