

### Experiment 06: Work, Energy and the Harmonic Oscillator

#### Purpose of the Experiment:

In this experiment you allow a cart to roll down an inclined ramp and run into a spring that is attached to a force sensor. You will measure the position and velocity of the cart and the force exerted by the spring while it is compressed. Then you will attach the cart to the force sensor by a spring and make a harmonic oscillator. Finally, you will replace the spring by a rubber band and see the effect of a non-ideal spring. You will do the following things in this experiment:

- You will investigate experimentally the work–kinetic energy theorem, how potential energy in a gravity field converts to kinetic energy which is then converted into the potential energy of a compressed spring.
- You will observe and quantify the effect of non-conservative forces and estimate the work done by these forces at various stages of the cart’s motion up and down the ramp.
- You will measure how well a harmonic oscillator spring obeys Hooke’s Law, test the formula for the resonant frequency of a simple harmonic oscillator,  $\omega = \sqrt{k/m}$ , and explore the idea of lissajous patterns to study the relationship between variables.
- You will observe that a rubber band does not obey Hooke’s Law, and the significant energy loss when a rubber band is stretched and released.

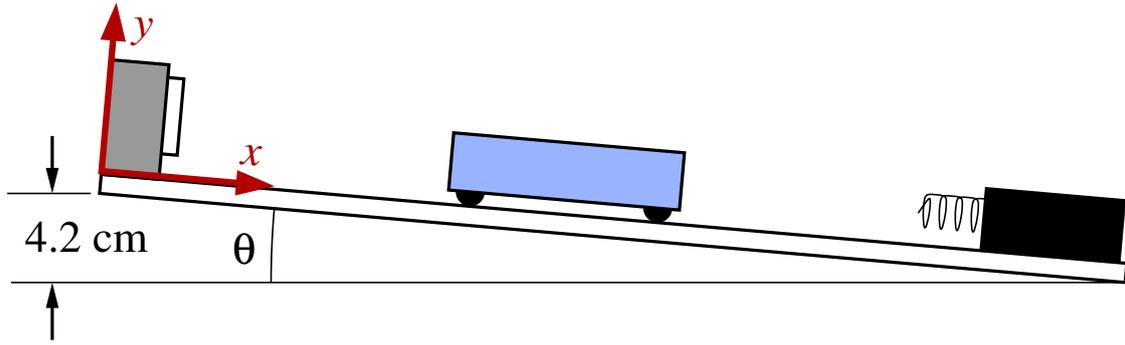
#### Setting Up the Experiment:

Refer to the photo to the right and the figure at the top of the next page. A force sensor should be mounted at the end of the track that has an adjustable support screw—which should be screwed in enough that the end of the track can lie flat on the table.

Clip the motion sensor to the other end of the track and raise it by placing a short piece of  $2 \times 4$  under the motion sensor where it clips onto the track, as you can see in the photo to the right. This should raise the end of the track 4.2 cm above the table (measure it); you can find the slope  $\theta$  as the track is 122 cm long.

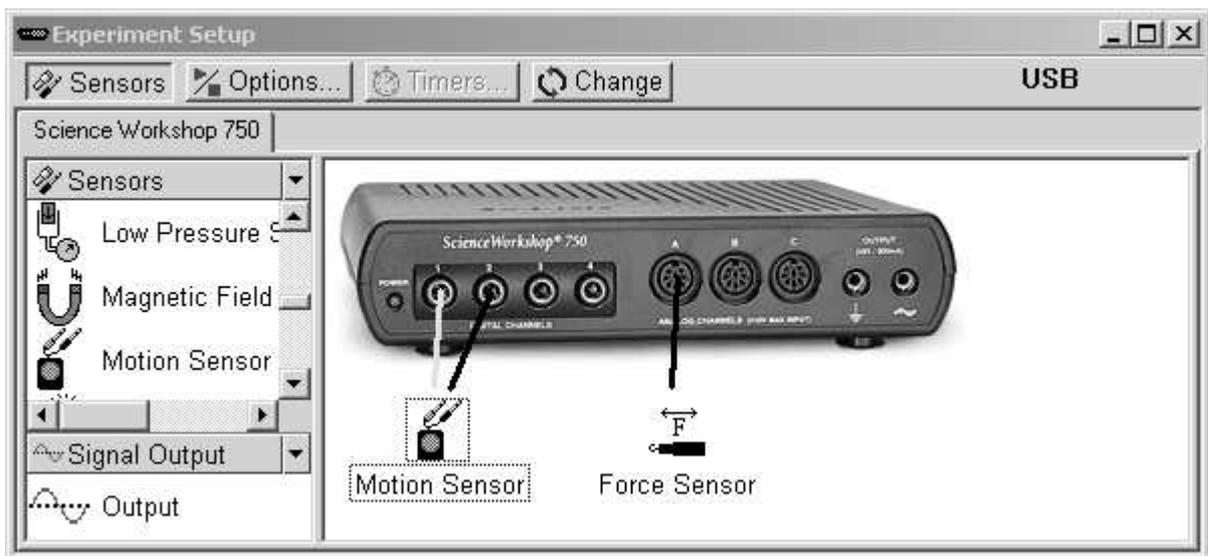


The heavier of the two springs available should be screwed into the force sensor.

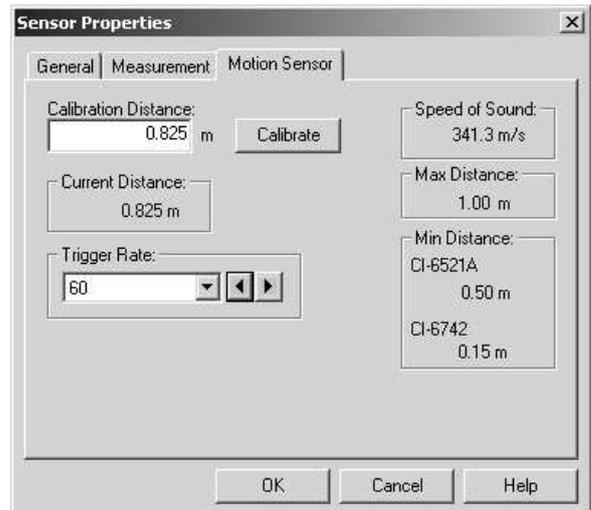
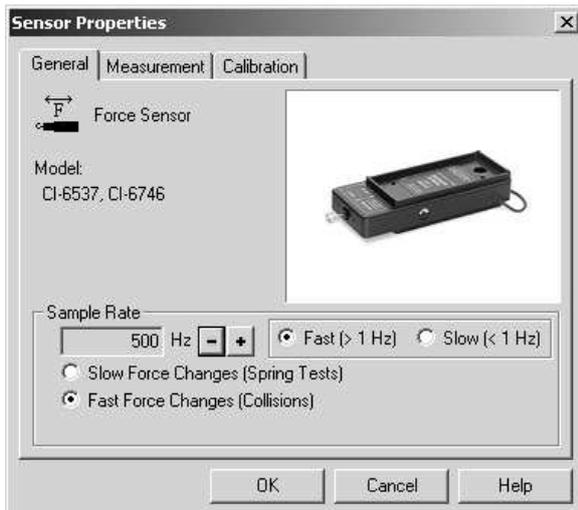


Place a cart on the track with the end having the Velcro™ patches facing the motion sensor. Put two 250 gm weights in the cart, which will bring its total mass to 750 gm. (The extra mass reduces vibrations and gives less noisy measurements.) Place the cart about half way up the track from the force sensor and release it. It will roll down the track, bounce most of the way back up, and repeat that several times. You may notice the track slides when the cart runs into the spring. This is an example of conservation of momentum. To prevent the track from sliding, place your thumb on the end of the track resting on the table and press it firmly against the table.

**Setting Up *DataStudio*:**

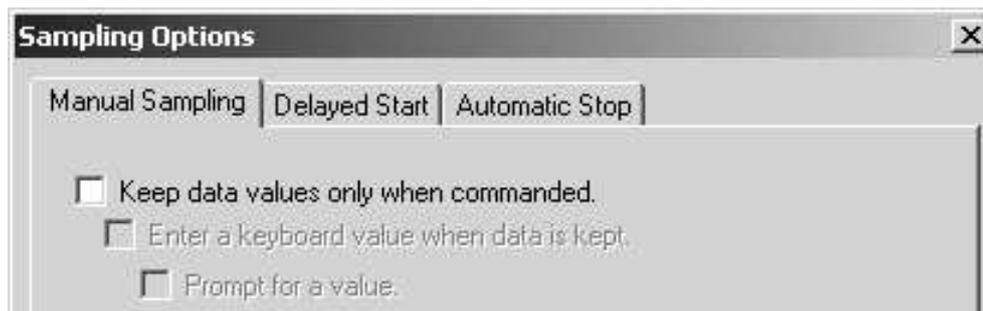


Connect the motion sensor (yellow plug into jack 1) and the force sensor to the 750 interface. The slide switch on top of the motion sensor should be set to the narrow beam position. Drag the appropriate icons to the 750 in the experiment Setup window. Double-click the force sensor icon to open the Sensor Properties window.

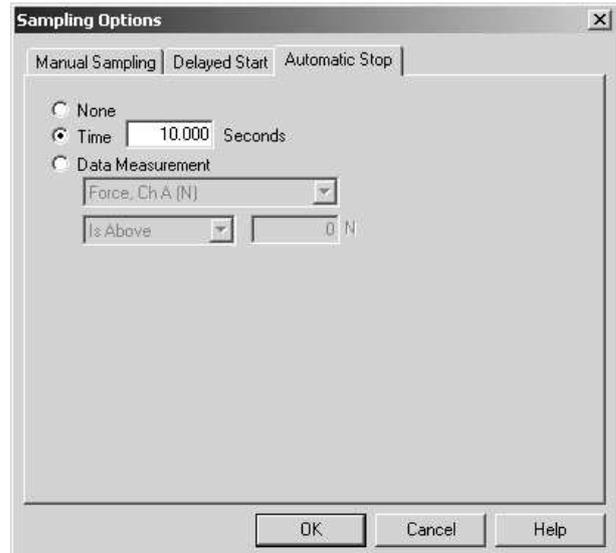
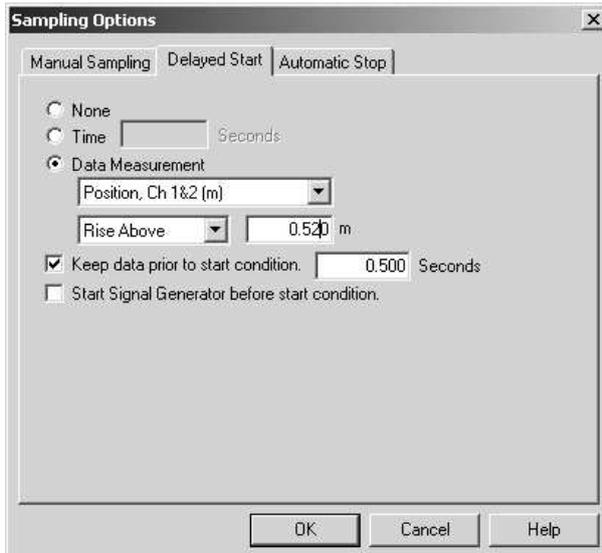


You don't need to calibrate the force sensor, but set it to Low Sensitivity under the Calibration tab and be sure to tare it before making measurements. Under the General tab set the force sensor Sample Rate to 500 Hz and click OK. Return to the Experiment Setup window and double-click the motion sensor icon. Under the Measurement tab, check the boxes so that position, velocity and acceleration will be measured. You should calibrate the motion sensor because the speed of sound varies slightly from day to day. Rest the cart against the spring on the force sensor and measure the distance between the motion sensor and the end of the cart closest to it. The motion sensor works best if it is angled up slightly rather than pointing directly at the cart. (That reduces the effect of sound waves that bounce off the track before hitting the cart.) Select the Motion Sensor tab and type the distance you measured into the Calibration Distance window and click the Calibrate button. This will be the point  $x_0$  where the cart and relaxed spring make contact. Set the Trigger Rate to 60 Hz and click OK.

Next, set the start and stop conditions. Click the Options  button. None of the boxes under the Manual Sampling tab should be checked.



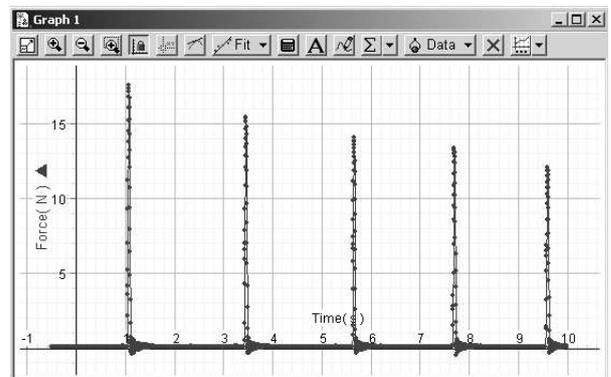
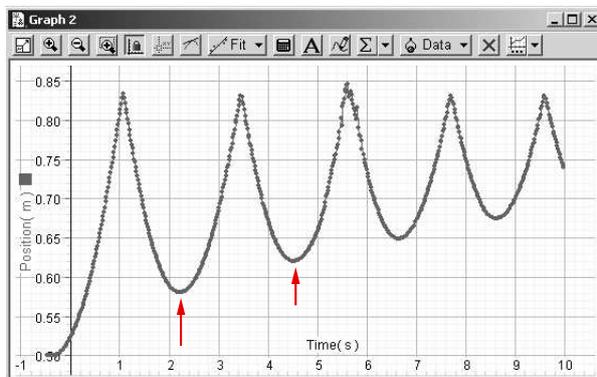
In the experiment you will let the cart roll into the spring starting from rest about 30 cm up the track from the point  $x_0$  where it first touches the spring. A convenient way to start the experiment is to hold the car on the track about 30 cm above the spring and measure the position with the motion sensor. (In my experiment that was 0.52 m.) Set the Automatic Start condition to begin measurements when the distance from the motion sensor first rises above this distance (see next page). To make a measurement, you can hold the cart 1 or 2 cm up the track from this position, click the Start button, and release the cart.



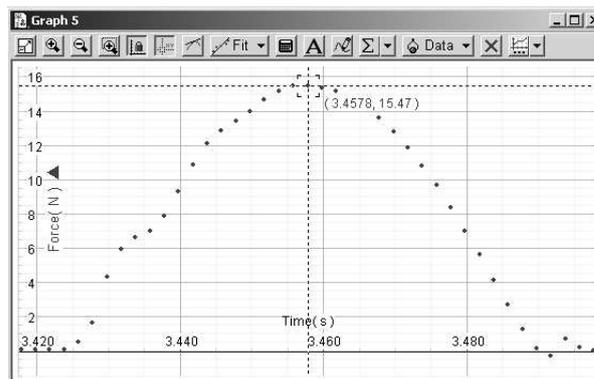
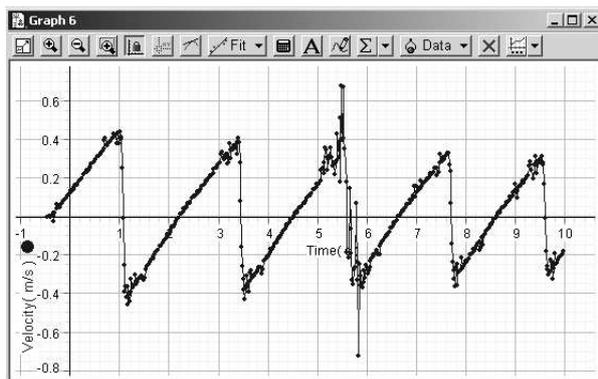
Under the Delayed Start tab click the radio button for Data Measurement, choose “Position, Ch 1&2 (m)” from the pull-down list, set the start condition to Rise Above 0.52 m (or the appropriate number for your experiment), and keep data from 0.5 s before the start condition. Then under the Automatic Stop tab click the Time radio button and type in 10 s.

Prepare to plot your measurements by dragging the Force and Position entries from the Data window onto the Graph icon in the Displays window. That will make two graphs.

Tare the force sensor, hold the cart at a position 1 to 2 cm above the point you chose for the Delayed Start condition, click the Start button, and when you see yellow numbers in the counter window release the cart. You should see the position and force plotted on your graphs as the cart bounces up and down the track.



These graphs contain a wealth of information. The position graph shows the sharp reversal of direction that occurs when the cart collides with the spring (and the force graph shows a corresponding spike in the force). You can see the slower reversal of direction as the cart coasts to high turning points on the track (the first two are marked by arrows) and then starts back down. You can see that the high turning point becomes lower each time as mechanical energy is lost to non-conservative forces. To learn more, you can expand the scale by selecting some of the points and using the “Scale to Fit” button  at the left of the graph toolbar.



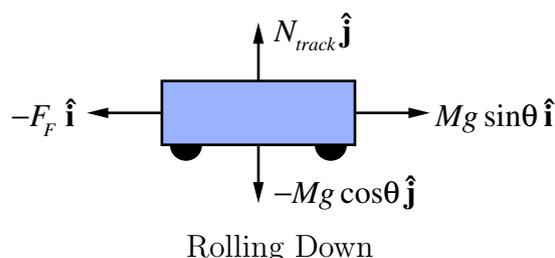
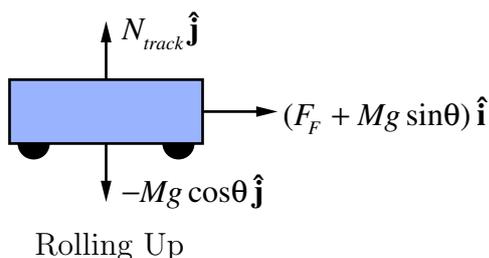
The curve above left is the velocity as a function of time and the one to the right is the force peak on the second bounce from the spring, expanded using the point selection and scale-to-fit operations on the force graph. A way to test the work-energy theorem might be to measure the velocity when the cart first hits the spring, or when it just leaves it, to calculate the kinetic energy, and to compare it with the potential energy it had at the high turning point on the track. Unfortunately the velocity measurements are too noisy at these points for this to work. The fluctuations in velocity measurements are about  $\pm 0.02$  m/s and, as we shall see presently, the friction reduces the velocity by only about 0.02 m/s from the ideal frictionless situation.

Here is a method that works. Use the Smart Tool on the position graph to find the position at the high turning points either side of the second bounce; these are marked by arrows on my graph and I call them  $h_1$  and  $h_2$ . Calculate the potential energy at  $h_1$  and  $h_2$  relative to  $x_0$ , the position where the cart just touches the spring, that you measured earlier. Enter your results in the table below and in your report. The table has my values as an example, but yours will likely be different as they depend on how you set up the apparatus and the point you chose to release the cart.

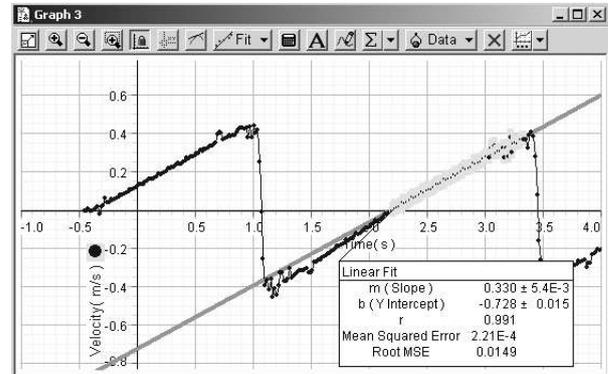
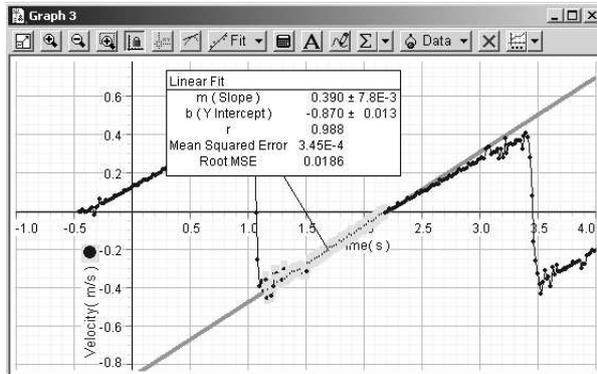
| Quantity:   | $x_0$   | $h_1$   | $h_2$   | $U(h_1)$ | $U(h_2)$ | $F$     |
|-------------|---------|---------|---------|----------|----------|---------|
| My value:   | 0.825 m | 0.620 m | 0.580 m | 61.9 mJ  | 51.8 mJ  | 22.5 mN |
| Your value: |         |         |         |          |          |         |

Knowing the cart's mass  $M = 0.750$  kg and the angle  $\theta = \tan^{-1}(4.2/122) = 1.97^\circ$  you can calculate the potential energy lost from turning point  $h_1$  to  $h_2$ . You also know the total distance the cart traveled is  $d = 2x_0 - h_1 - h_2$ . If you assume the friction force is constant, you can now calculate it. Do so, and put the result in your report and the table above.

You *can* get useful information from the velocity graph. The two figures below are free body diagrams (in the coordinate system from the top of page 2) for the cart rolling up and down the track.



From the free body diagrams, you can see that the acceleration should be different in the two cases. On your velocity graph use the Settings Button  to set the  $x$ -axis scale to plot time only up to shortly after the 2nd bounce (about 4s in my case). Then select only the data points between the 1st and 2nd bounce where the velocity is negative, and do a linear fit to them, as in my graph on the left below. The slope will give the upward acceleration  $a_{up}$ . Enter your result in the table below and in your report.

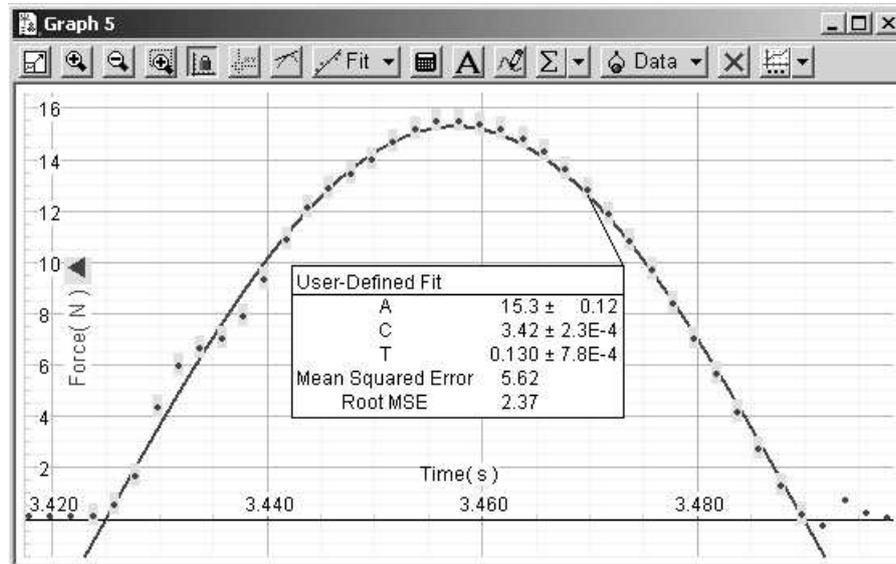


Then select only the data points between the 1st and 2nd bounce where the velocity is positive, and do a linear fit to them, as in my graph on the right above. The slope will give the downward acceleration  $a_{down}$ . Enter your result in the table below and in your report. Save these notes and your values for  $a_{up}$  and  $a_{down}$ ; you will need them for a problem on problem set 07 (included as the last page of these notes).

| Quantity:   | $a_{up}$              | $a_{down}$            |
|-------------|-----------------------|-----------------------|
| My value:   | $0.390 \pm 0.008$ m/s | $0.330 \pm 0.005$ m/s |
| Your value: |                       |                       |

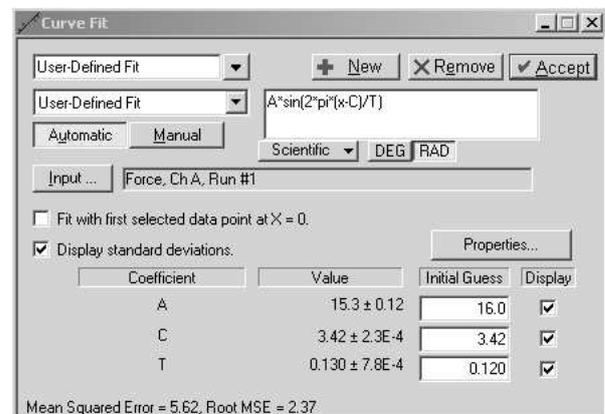
### Force Analysis:

As the final analysis of your data for this part of the experiment, you should make a new graph (not a new measurement) of the force as a function of time, then use a combination of point selection and the scale-to-fit  tool so that the force data for the second bounce fill the graph window. You should make an expanded time scale graph like this one.



Select the data points where the force is not zero, and choose a User-Defined Fit. Double-click the text window in the graph to open up a Curve Fit window and define the function as  $A \cdot \sin(2 \cdot \pi \cdot (x - C) / T)$ .

*DataStudio* will still not be able to find a fit after you click Accept, but this time places on the window will open where you can type your initial guesses for the parameters. Choose the peak height for  $A$ , the time when the force first becomes non-zero for  $C$ , and twice the width of the peak for  $T$ . With these initial guesses, a fit will be found.



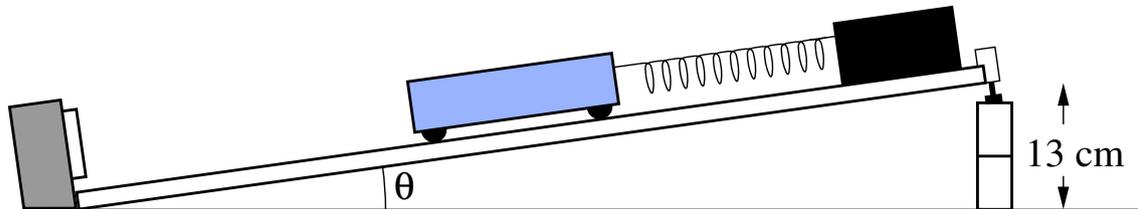
Enter your result in the table below and in your report.

| Quantity:   | $A$                       | $T$                         |
|-------------|---------------------------|-----------------------------|
| My value:   | $15.3 \pm 0.12 \text{ N}$ | $0.130 \pm 0.001 \text{ s}$ |
| Your value: |                           |                             |

Save these notes and your values for  $A$  and  $T$ ; you will need them for a problem on problem set 07. A copy of this problem is included as the last page in these notes.

### The Harmonic Oscillator:

In this part of the experiment you will make a harmonic oscillator and explore some of its properties. Remove the cart from the track and unclip the motion sensor from the end. The end of the track where the motion sensor was will now become the low end and rest on the table. Raise the other end by placing two pieces of  $2 \times 4$  on top of each other and resting the adjustable support screw of the track on top of them, as in the figure below.



This is a somewhat precarious set up, so be careful not to knock it over. Adjust the support screw so the end of the track is 13 cm above the table. Place the motion sensor on the table and slide it up to touch the low end of the track. (It is less likely to be destroyed by a runaway cart if it can slide on the table.) You may have to aim the motion sensor again; it should point up an angle somewhat greater than  $\theta$ . Replace the spring on the force sensor with the hook, and hook one end of the harmonic oscillator spring over it.

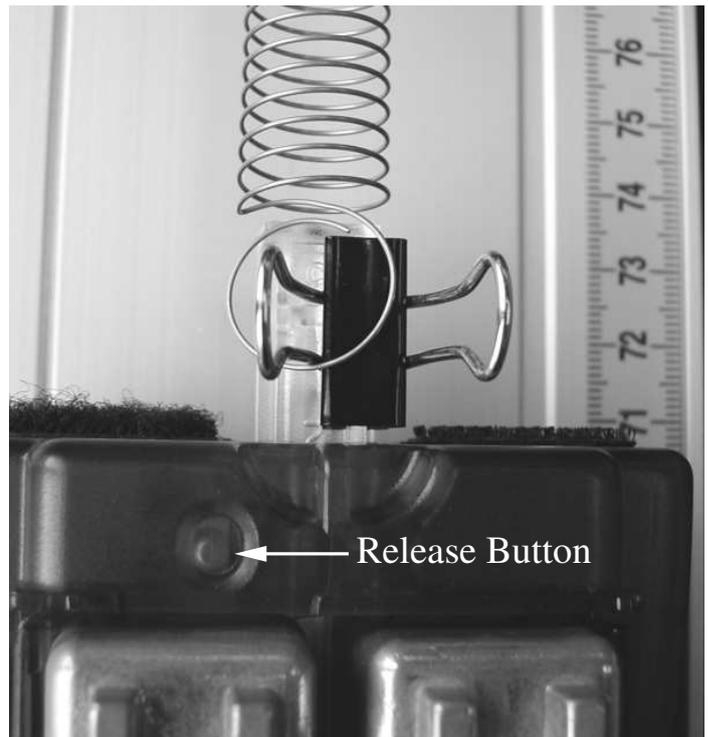
At the end of the cart with the Velcro™ patches there is a square clear plastic plunger that will slide out if you press the release button. Clip a small ( $\frac{3}{4}$  in) binder clip to the plunger as shown in the photo and push it back in.

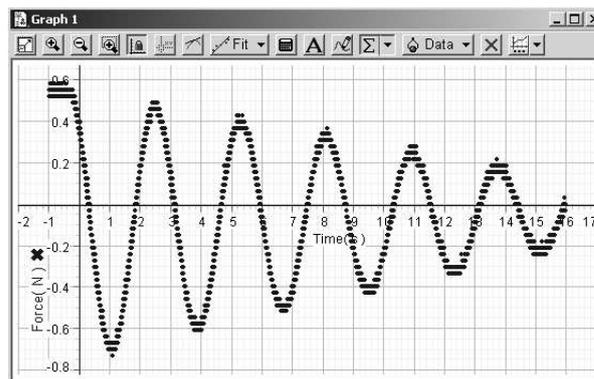
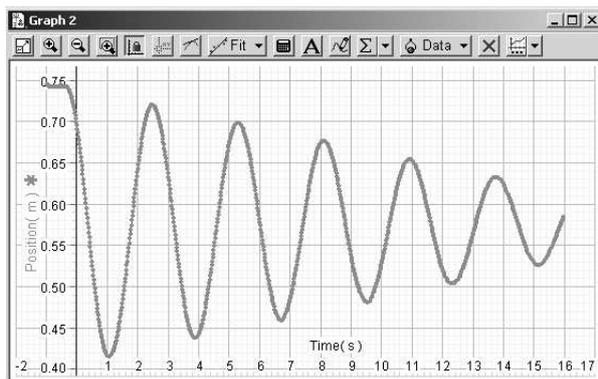
Place the cart back on the track and hook the other end of the harmonic oscillator spring over a handle of the binder clip. Carefully roll the cart down the track to its equilibrium position and add the two 250 gm weights.

With the cart at rest at its equilibrium position, re-calibrate the motion sensor.

Change the Delayed Start to begin at a position 20 cm above the equilibrium position and also change the condition from “Rise Above” to “Fall Below”.

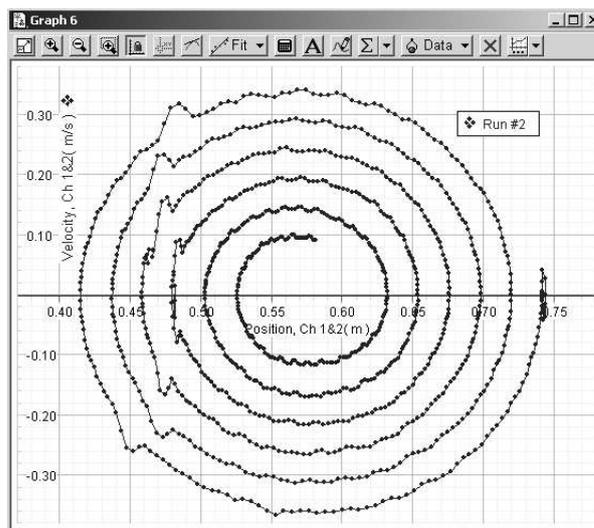
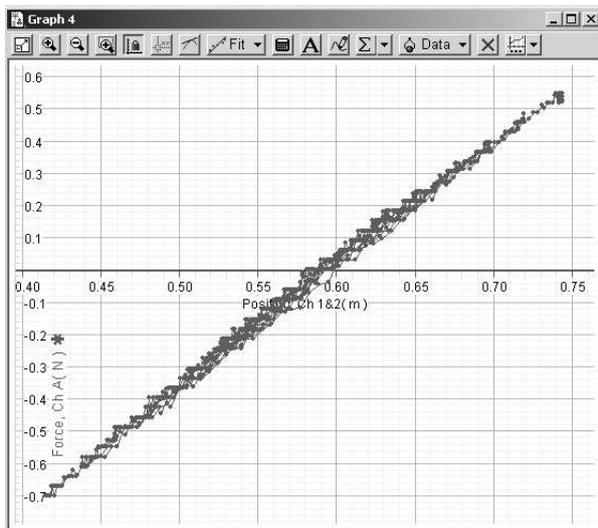
Change the Automatic Stop time from 10 s to 16 s. Unhook the spring, tare the force sensor, and hook it up again. To make a measurement hold the cart 1 or 2 cm above the start condition point, click the Start button, and release the cart after the clock figures start counting in yellow.





You should see position and force graphs like these. You can see from the position graph that the motion appears to be sinusoidal, but damped by friction. The force is also sinusoidal, as you would expect. You should measure the period  $T$  of the oscillation on the position graph (Smart Tool?) because you will need it for your report.

It is interesting to plot force on the  $y$ -axis against position on the  $x$ -axis. To do that, drag the force data into the Displays window to make a new graph, then drag the corresponding position data onto the  $x$ -axis of the new graph. (Make sure that the dotted lines surround just the  $x$ -axis, not the whole graph, before you release the drag.) If you adjust the scale and graph window borders, you can get a plot like mine on the left below.



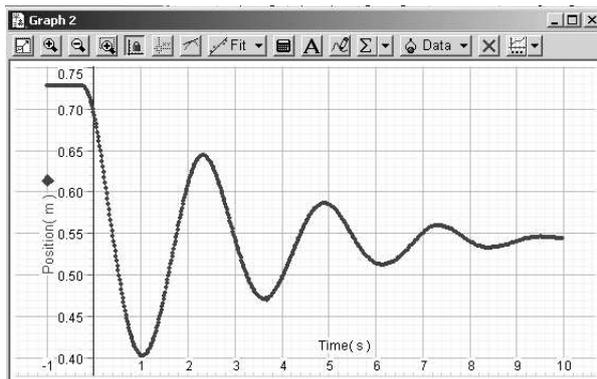
The fact that the force-position graph is linear confirms Hooke's law, and you can obtain the harmonic oscillator spring constant by a Linear Fit. You will need it for your report. (I found  $3.79 \pm 0.01$  N/m.) Another interesting graph has velocity on the  $y$ -axis and position on the  $x$ -axis. You should make one of these, too. Mine is at the right, above.

These are both examples of what are called Lissajous figures, after the French physicist Jules Antoine Lissajous. Long before the development of electronics, Lissajous used the method to compare the frequencies of sounds. He used the sounds to vibrate a mirror in two orthogonal directions and observed the motion of a beam of light reflected from the mirror. Interestingly, his technology has returned, modernized, in the form of laser light shows.

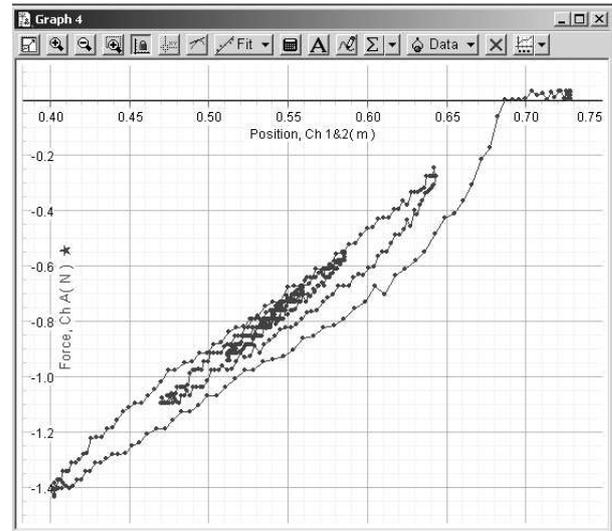
The two figures above are simple lissajous figures because the waves compared have the same frequency. The force and position form a straight line because the two are in phase. There is a  $90^\circ$  phase difference between the velocity and position, which is why the open circular pattern is formed. The inward spiral reflects the loss of mechanical energy to friction. A similar thing happens to the force-position graph, but is hidden because the phase difference is zero.

### Appendix: Rubber Band

Unhook the harmonic oscillator spring and replace it with two rubber bands joined end-to-end. Repeat the harmonic oscillator measurement. You will observe that the damping of the oscillation is significantly greater. Here is my position graph.



This extra damping must come from internal friction in the rubber band, as everything else is the same as it was with the spring harmonic oscillator.



The most interesting plot is the force-position lissajous pattern at the right above. I can make three comments about this.

- The rubber band does not follow Hooke's Law closely (paths are curved).
- The force from the rubber band is not in phase with the position (the pattern is somewhat open).
- The integral

$$\int \mathbf{F} \cdot d\mathbf{x}$$

over a cycle is the non-conservative work done during that cycle. (As you will learn when you study circuits in 8.02, this is related to the previous comment.)

You can demonstrate internal friction in rubber to yourself. Take a (clean) rubber band, stretch it quickly, and touch it as rapidly as you can to your lips; you should be able to feel its warmth. (This is easier to detect with a fat rubber band than the skinny ones we are using.)

Rubber has a number of interesting properties. The force required to stretch it increases with temperature, and a stretched rubber band will cool if you allow it to relax! (You might want to see if you can detect that with your lips, too.) This can be explained using elementary statistical mechanics, except the energy lost to internal friction over a stretch-relax cycle is difficult to predict quantitatively.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Physics Department

Physics 8.01T

Fall Term 2004

**Part of Problem Set 07**

**Section and Group:** \_\_\_\_\_

**Your Name:** \_\_\_\_\_

You may work together as a group to solve this problem, but each group member should turn in a copy of this answer with problem set 07. If you like, just fill in the information above, complete this page, and attach it to your homework solutions.

- a) Enter the values from your experiment into the table below.

| Quantity:          | $a_{\text{up}}$ (m/s <sup>2</sup> ) | $a_{\text{down}}$ (m/s <sup>2</sup> ) | $U(h_1)$ (mJ) | $U(h_2)$ (mJ) |
|--------------------|-------------------------------------|---------------------------------------|---------------|---------------|
| <b>Your value:</b> |                                     |                                       |               |               |

Use these numbers, along with  $M = 0.75$  kg, to calculate the friction force on the cart. Compare it with the value you obtained from the potential energy difference between the turning points  $h_1$  and  $h_2$ .

- b) Explain briefly in words why it makes sense to fit the function  $A \sin[2\pi(x - C)/T]$  to the force peaks when the cart bounces off the spring on the force sensor.

- c) Enter the results of your fit to the second bounce into this table

| Quantity:          | $A$ (N) | $T$ (s) | $k$ (N/m) | $A^2/2k$ (mJ) |
|--------------------|---------|---------|-----------|---------------|
| <b>Your value:</b> |         |         |           |               |

and use them to find the force constant of the spring. Compute the potential energy stored in the compressed spring,  $A^2/2k$ , and compare it with the average of  $U(h_1)$  and  $U(h_2)$  from the table above.