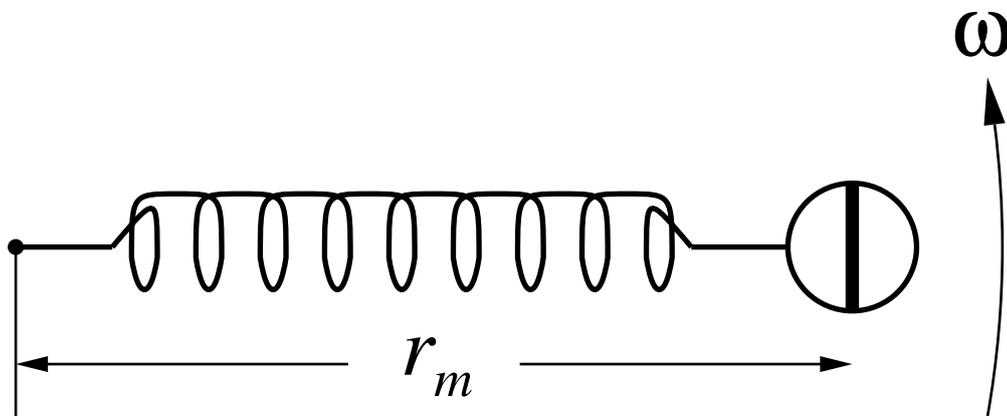


Experiment 04: Uniform Circular Motion

Purpose of the Experiment:

The direct goal of this experiment is to study a “conical pendulum” like the one in your homework assignment this week. A fishing sinker (the “mass”) is attached to a rotating shaft by a spring and you will adjust the angular velocity of the shaft rotation, and measure the radius r_m of the circular motion of the mass, calculate the centripetal force, and use the results to find the force constant of the spring. To simplify the analysis of your results, assume that the mass of the spring can be neglected.



You should learn at least the following things from this experiment:

- An experimental verification of the force required to produce centripetal acceleration.
- This system has an instability at a critical angular frequency, ω_C , for the shaft rotation; you will observe the approach to this instability and understand what causes it.
- You will get more experience in analyzing your measurements to find properties you may want to know (the spring force constant) and you may find that the spring has a property (pre-tension) that we have not discussed before.

Setting Up the Experiment:

The apparatus includes a viewer to facilitate measuring the radius of the mass’s motion, r_m . It is a white teflon block with a black viewing tube and it slides in a slot in the top cover of the apparatus. A small nail protrudes down into the slot and should be placed to engage the loop of the brass wire that has a LED taped to it; that is so the LED will move to illuminate the mass as you try to read its position. (This all works better if the room lights are off.) Another nail protrudes into the viewing tube and can be lined up over the black stripe around the mass; the same nail also allows you to read the radius on the scale.

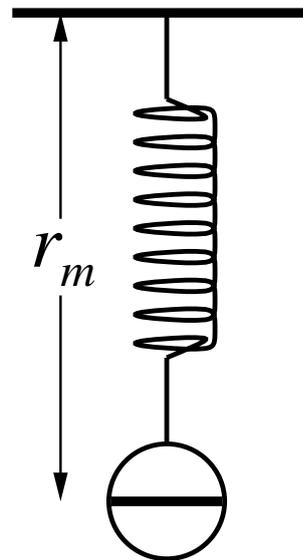
First you need to know the mass m_m of the fishing sinker. The one I used had $m_m = 9.3$ gm, but you should ask your instructor for the right mass for your apparatus (most of them seem to have a $m_m = 8.5$ gm.)

Next, attach the mass and spring to the hook on the shaft inside the box.

Before you turn on the apparatus, you should measure r_0 , which is the value that r_m would have if the spring were not stretched at all. Stand the box on its side and hold the counterweight and the shaft so that the spring and mass hang down along the scale. Use the viewer to measure the value of $r_0 = r_m(\omega = 0)$ when the mass is hanging still; you can use r_0 to calculate how much the spring stretches when it is rotating.

Next connect the apparatus to its 12V power supply via the connector on the side.

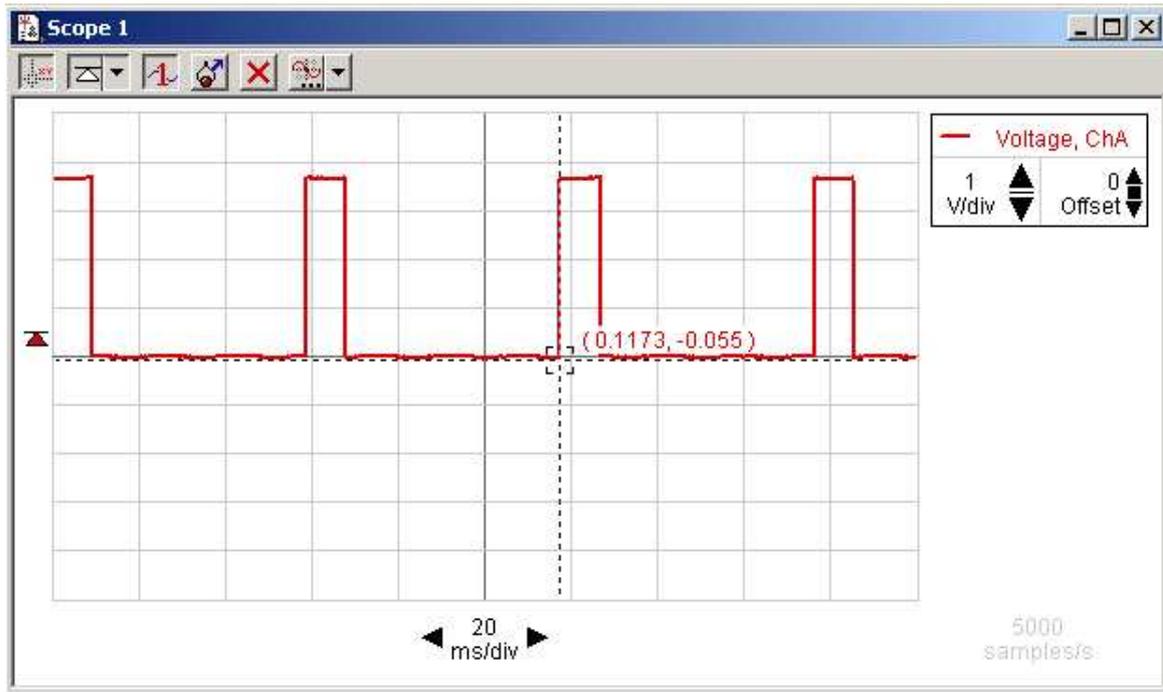
You should replace the side cover to the box before you turn on the motor. The lead sinker moves at fairly high speed and would give a nasty whack to your finger if it were hit by it. Turn on the circular motion apparatus and try different speeds to see how it behaves. You will use *DataStudio* to measure the rotation period, T , of the mass, and from that calculate the angular rotation frequency $\omega = 2\pi/T$.



As the motor rotates, a magnet in the counterweight triggers a reed relay on the top cover of the box and makes a voltage pulse every time the magnet passes under it.

A voltage sensor should be connected from the *ScienceWorkshop 750* interface to the two banana jacks that are farthest away from the speed control knob.

In *DataStudio* drag a voltage sensor to the appropriate input on the 750 interface; double-click the voltage sensor and set it to low sensitivity and a sample rate of 5000 Hz in the window that opens. Set the sampling options (either from the “Experiment” main pull-down menu or the “Options” button of the “Experiment Setup” window) to “none” for both automatic start and stop. After that, *DataStudio* will make measurements continuously when you click the start button. To display the voltage pulses, drag the Voltage in the “Data” window to the Scope icon in the “Display” window. That will open up a scope display. Set it to 1 V/div on the vertical scale and 20 ms/div on the horizontal (time) scale. The ◀ symbol to the left of the ms/div label on the x -axis reduces the time to sweep a division (increases the sweep rate) and the ▶ symbol does the opposite. Slide up the trigger level symbol $\overline{\Delta}$ on the y -axis of the graph to about 0.5V; this will give a more stable scope pattern. You should see voltage pulses like this on the scope display after you click the Start button.



Making Measurements:

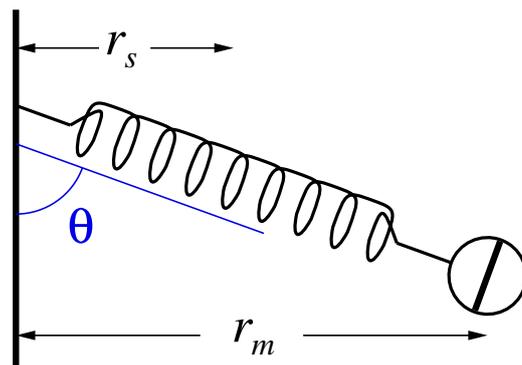
Measure the radius $r_m(\omega)$ of the mass's circular motion for three different angular velocities to give values of r_m between 5 and 10 cm. Allow the speed to stabilize before you make your measurement. To find the rotation period on the scope display, you may find it helpful to click on the single trace button at the top of the display graph and use the smart tool. For each measurement you need to determine $r_m(\omega)$ and the rotational period T .

Your goal is to complete a table like the one below for $\omega = 0$ and for three or four values of $\omega > 0$. The values for T and r_m (shown in bold type) should be entered as you do the experiment; all the other values will be calculated from them when you do the analysis.

T	r_m	ω	ΔX	$m_m r_m \omega^2$
∞	4.8 cm	0 s^{-1}	0 m	0 N
0.095 s	6.0 cm	66.1 s^{-1}	0.012 m	2.44 N
0.074 s	7.2 cm	84.9 s^{-1}	0.024 m	4.83 N
0.060 s	10.3 cm	105 s^{-1}	0.055 m	10.5 N

The table contains the values I measured, but you may have a different spring and fishing sinker mass and so may get something quite different. You calculate $\omega = 2\pi/T$ and find the amount ΔX the spring stretched using $\Delta X = r_m - r_0$.

A note about gravity: if you have been thinking as you read this, you should wonder why I am not considering the effect of gravity. Because of gravity, the spring and mass will hang down at some angle θ , as shown exaggerated at the right. The spring and mass will actually sweep out a conical path inside the box; that's why this apparatus is often called a conical pendulum.



If you ignore the mass of the spring and simply analyze a free body diagram of the sinker, you will find that $\tan \theta = r_m \omega^2 / g$. For the smallest non-zero $r_m \omega$ in my table, $\tan \theta \simeq 250$. Thus θ is very close to 90° ($\theta > 89.75^\circ$) and it is a good approximation to ignore gravity and pretend the motion is in a horizontal plane.

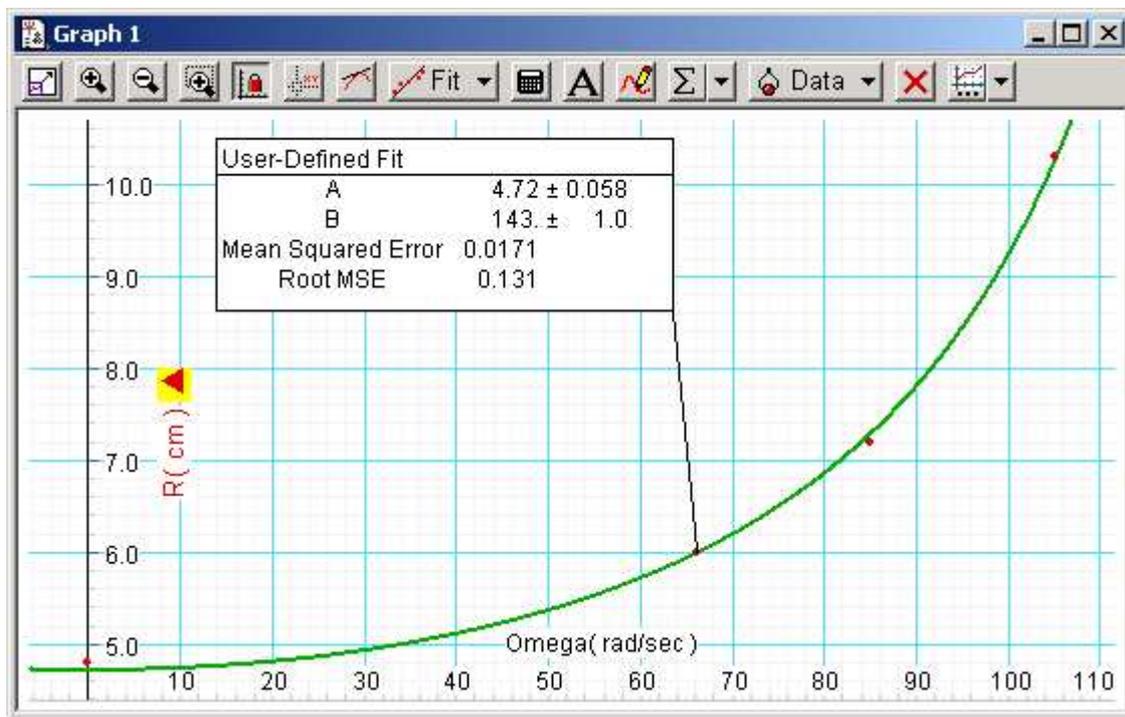
This apparatus shows that it is relatively easy to generate centripetal accelerations from 250 to 1000 times that of gravity in rotating machinery. In a laboratory ultra-centrifuge, accelerations of 700 000 g can be attained with $\omega \simeq 10\,000 \text{ s}^{-1}$ (90 000 rpm).

Approaching the Instability:

To begin this analysis, enter your values for ω in the X (left) column and r_m in the Y (right) column of a new data table in *DataStudio*. To create a data table in *DataStudio*, choose “New Empty Data Table” from the main “Experiment” pull-down menu. (If you want to give a title to the data and units for the X and Y variables, double-click the table entry in the “Data” window, and a window will open for you to do this.)

▲ <i>R vs Omega</i> Data	
Omega (rad/sec)	R (cm)
0.0	4.8
66.1	6.0
84.9	7.2
105.0	10.3

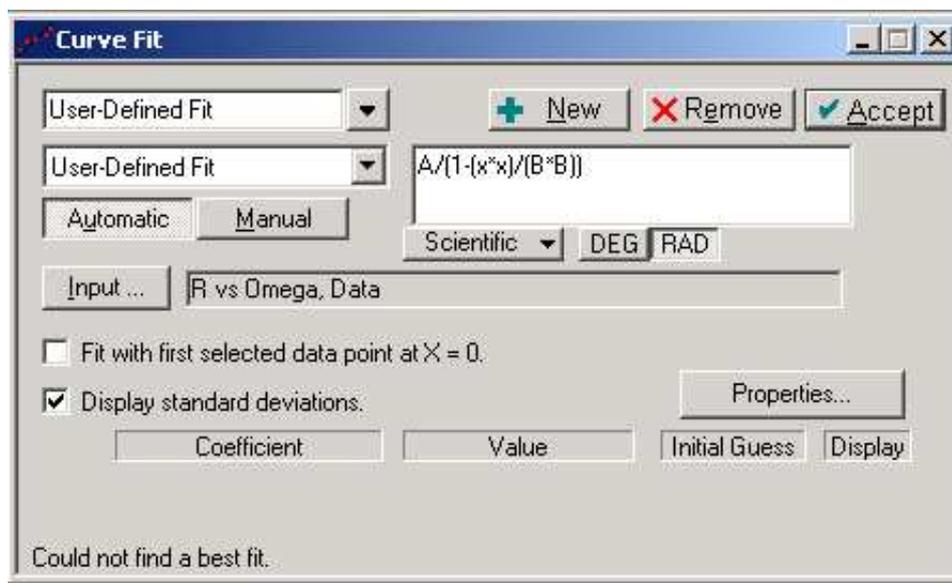
When your data have been entered, you will get a table that looks like the one above. Next, plot them on a graph by dragging the entry from the Data window onto Graph in the Displays window. Here is my plot. There is also a fitted line, which I discuss on the next page.



You should use *DataStudio* to fit your $r_m(\omega)$ data to the expression

$$y = \frac{A}{1 - (x/B)^2}$$

This function is not in *DataStudio*'s library of functions, so under the "Fit" button of the graph's toolbar, pick the User-Defined Fit option. When you do that, probably nothing will happen except a new entry called "User-Defined Fit" will appear in the Data window. If you double-click it, that should open a window like the one below. If the Automatic button is not depressed, click it so that it becomes depressed; that means *DataStudio* will try to find the values of A and B that provide the best fit to your data points. Type $A/(1-(x*x)/(B*B))$ into the window to define your function and click the Accept button. *DataStudio* will probably complain that it could not find a fit; that is a sign the program needs some help from you. Type some reasonable values into the Initial Guess fields that have opened up (Good guesses would be $A = 5 \text{ cm}$ and $B = 150 \text{ s}^{-1}$) and click Accept again. The fitted parameter values should appear in the window, the fit curve should be plotted on your graph and a text box should open on the graph that also lists the fit results.



As you can see for my data, the above expression is a very good fit. Let's see why I expected that. We know that the centripetal force is given by

$$F_c = m_m r_m \omega^2$$

while the force exerted by the spring, where I use the symbol r_0 for $r_m(\omega = 0)$, is

$$F_{spring} = (r_m - r_0)k$$

If I equate the two forces and solve the resulting equation for r_m I find

$$r_m = \frac{r_0}{1 - m_m \omega^2 / k}$$

This is the expression I fit above; you can see that the parameter A should be the quantity you measured for $r_0 = r_m(\omega = 0)$. You can also see that the mass will “escape” ($r_m \rightarrow \infty$) when

$$\omega = \omega_C = \sqrt{k/m}$$

You can understand why this instability occurs if you differentiate the two force equations.

$$\frac{dF_c}{dr_m} = m_m \omega^2$$

and

$$\frac{dF_{spring}}{dr_m} = k.$$

For $\omega < \sqrt{k/m}$ the centripetal force required to keep the mass in a circular orbit grows more slowly with r_m than the restoring force provided by the spring does; the spring can always stretch a bit more to provide enough force to hold the mass in its circular path. For $\omega \geq \sqrt{k/m}$, the opposite is true and stretching the spring—so long as it obeys Hooke’s law—only makes the situation worse. Something catastrophic will happen. To prevent damage, the power supply in your apparatus should have been adjusted to keep $\omega < \omega_c$. If it was not, you may have been able to turn it up enough to reach ω_c . That usually manifests itself by a loud clattering noise when the sinker hits the walls of the plastic box.

This instability shows up in some other circumstances. You may have experienced it while blowing up party balloons by mouth; it is especially obvious with long skinny balloons. In that case, the stretched rubber provides a restoring force (analogous to the spring) to counterbalance the air pressure in the balloon. Above a critical pressure, the force required to contain the pressure increases more rapidly with the balloon’s radius than the rubber restoring force does. The balloon does not usually pop, however; instead it suddenly inflates to a larger radius and remains whole because the highly stretched rubber provides an even greater restoring force (it has a larger rate constant). The balloon does not obey Hooke’s law.

An analogous instability can occur in arteries weakened by disease and often leads to an aneurism, often fatal when the artery ruptures.

If you want a challenge, see if you can derive an equation for the radius as a function of pressure for a balloon or artery (a cylindrical tube) that follows Hooke’s law.

Finding the Spring Rate Constant:

Note: you don't have to do this fit in order to turn in your report for this experiment. However, it is required for a problem on your next problem set (which is attached as the last page of this document). There should be enough time to do the fit in class and you can fill in the relevant numbers on the last page.

The springs supplied for this experiment have been wound to have a “pre-tension.” That means a certain force F_0 must be applied to the spring before the spring will begin to stretch, and the force law is given by $F_{spring} = F_0 + k\Delta X$. (In 8.01T we will usually call k the spring force constant, but if you look at the literature of companies that manufacture springs, they use the term “rate constant”.) When you analyze your results to find the rate constant k you will need to use this expression:

$$F_{spring} = F_0 + k\Delta X .$$

For example, from my table I need to satisfy the equations.

$$2.44 = F_0 + 0.012k \quad (1)$$

$$4.83 = F_0 + 0.024k \quad (2)$$

$$10.5 = F_0 + 0.055k \quad (3)$$

Here are three equations and two unknowns, so the coefficients F_0 and k that I want to find are overdetermined; a consistent set of values does not likely exist.

Whenever you have more measurements than adjustable parameters, which is always a desirable situation, you need to find the parameters F_0 and k that represent some kind of “best fit” compromise.

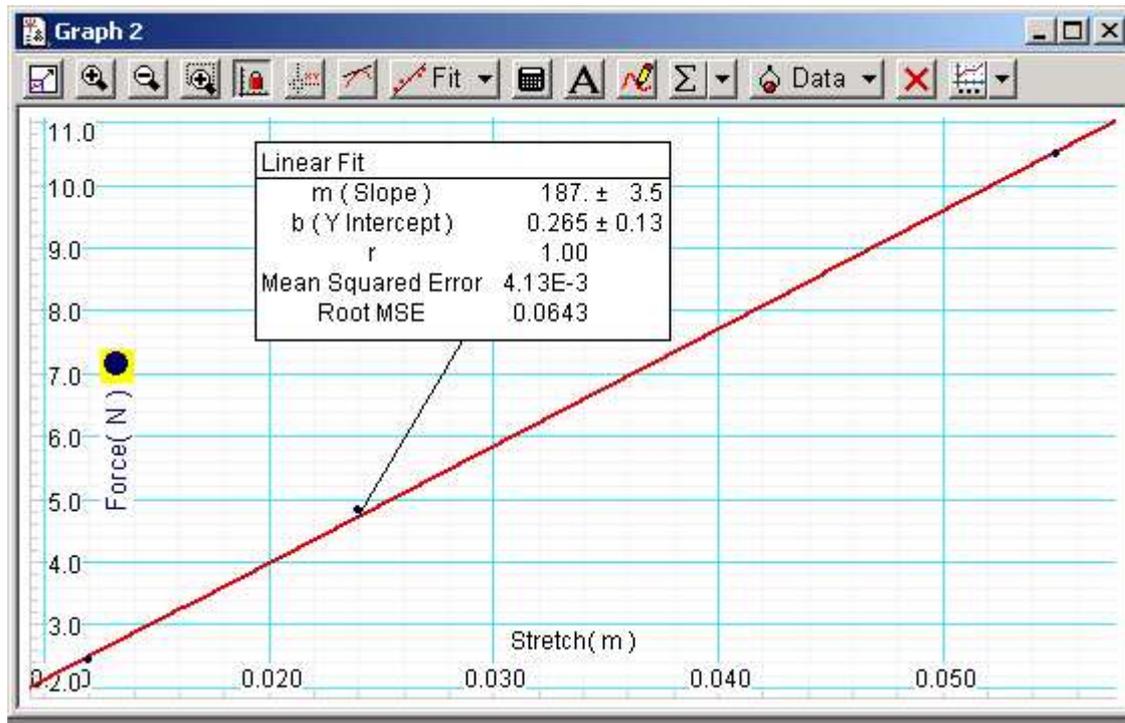
You should type the numbers into a *DataStudio* table, plot them, and use the program's Linear Fit function. Create a new table as you did for the r_m and ω data.

● Force vs. Stretch Data	
Stretch (m)	Force (N)
0.012	2.44
0.024	4.83
0.055	10.50

Type your $m_m r_m \omega^2$ values into the Y-axis (right) column and your ΔX values into the X-axis (left) column. When the table is complete, plot the data by dragging them from the “Data” window to the Graph icon in the “Displays” window.

You can also export the data as an ASCII text file (e.g., to the desktop of your computer) which you could send to yourself by email if you want a machine-readable file for later analysis with *Xess*. (With only three points, I'd use a pencil and paper and type them into *Xess* by hand.)

Select the Fit tab on the graph toolbar and ask *DataStudio* to do a linear fit. Here is what I obtained.



The results are $F_0 = 0.27 \pm 0.13 \text{ N}$ and $k = 187 \pm 4 \text{ N/m}$. As r_m could only be measured to two significant figures, I should quote k only to two significant figures as well. The pre-tension comes from subtracting two almost equal numbers and could well be zero. (A negative pre-tension would make no sense because ΔX is defined to be the distance the spring is stretched from the length it has when the applied force is zero.) You can measure the period T rather accurately, so the main source of error is r_m .

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.01T

Fall Term 2004

Part of Problem Set 04

Section and Group: _____

Your Name: _____

You may work together as a group to solve this problem, but each group member should turn in a copy of this answer with problem set 04. If you like, just fill in the information above, complete this page, and attach it to your homework solutions.

Fill out the following table for $\omega = 0$ and at least three other values of ω .

T	r_m	ΔX	ω	$m_m r_m \omega^2$
∞		0 m	0 s^{-1}	0 N

1. What was your spring's force constant from the fit? How does it compare with the value you calculated from ω_c ?
2. What was your spring's pre-tension?
3. Suppose the mass of the spring were to be included in your data analysis. (When I did this, I found it to be at most a 10% effect.) Would this mean you would find a larger or smaller value for k ? (Explain.)