Massachusetts Institute of Technology
Department of Physics

8.01T **Fall 2004**

Study Guide Final Exam

The final exam will consist of a two sections.

Section 1: multiple choice concept questions. There may be a few concept questions on time and special relativity but no analytic questions.

Section 2: analytic problems with some concept questions requiring written responses. The analytic questions will be divided into part A covering the material since the third test. Part B will cover problems from the entire year. Examples of section 2 questions are given below. Test questions will not use numbers so you do not need a calculator.

Part A: Kinetic Theory, First Law of Thermodynamics, Heat Engines

Problem 1 ***Energy Transformation, Specific Heat and Temperature***

Suppose a person of mass $m = 6.5 \times 10^1\, kg$ is running at a speed $v = 3.8\, m/s$ and is expending $9.45 \times 10^2\, W$ of power during a $1.0 \times 10^1\, km$ workout. Suppose the runner converts 20% of the internal energy change into mechanical work. The rest of the energy goes into heat. If the specific heat of the runner is $c = 4.19 \times 10^3\, J/kg - K$, how much would the body temperature rise after running the $10\, km$?

Problem 2 Kinetic Theory

An ideal gas has a density of 1.78 kg/m^3 is contained in a volume of $44.8 \times 10^{-3}\ m^3$. The temperature of the gas is $273\,K$. The pressure of the gas is 1.01×10^5 Pa. The gas constant $R = 8.31\, J \cdot K^{-1} \cdot mole^{-1}$.

a) What is the root mean square velocity of the air molecules?

b) How many moles of gas are present?

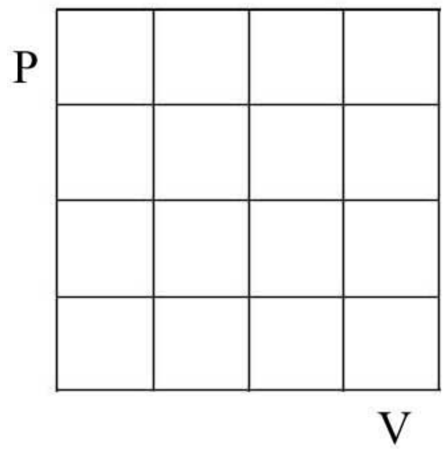
c) What is the gas?

d) What is the internal energy of the gas?

Problem 3: Carnot Cycle of an Ideal Gas

In this problem, the starting pressure P_a and volume V_a of an ideal gas in state a, are given. The ratio $R_V = V_c / V_a > 1$ of the volumes of the states c and a is given. Finally a constant $\gamma = 5/3$ is given. You do not know how many moles of the gas are present.

a) Read over steps (1)- (4) below and sketch the path of the cycle on a $P - V$ plot on the graph below. Label all appropriate points.



(1) In the first of four steps, a to b, an ideal gas is compressed from V_a to V_b while no heat is allowed to flow into or out of the system. The compression of the gas raises the temperature from an initial temperature T_1 and to a final temperature T_2 . During this process the quantity $PV^{\gamma} = \text{constant}$, where $\gamma = 5/3$.

a) What is the pressure P_b and volume V_b of the state b of the gas after the compression is finished?

b) What is the change in internal energy of the gas during this change of state?

c) What is the work done by the gas during this compression?

(2) The gas is now allowed to expand isothermally from b to c, from volume V_b to volume V_c .

d) Express the work done by the gas in this process W_{cb} and the amount of heat Q_{cb} that must be added from the heat source at T_2 in terms of P_a, V_a, T_2, T_1, and V_c.

Is this heat positive or negative? Explain whether it is added to the system or removed.

e) What is the pressure P_c of the gas after the expansion is finished?

(3) When the gas has reached point c is expands from V_c to V_d while no heat is allowed to flow into or out of the system. The expansion of the gas lowers the temperature and pressure from an initial temperature T_2 to a final temperature T_1. During this process the quantity $PV^{\gamma} = \text{constant}$.

f) What is the pressure P_d and volume V_d of the state d of the gas after the expansion is finished?

g) What is the change in internal energy of the gas during this change of state?

h) What is the work done by the gas during this expansion?

(4) The gas is now compressed isothermally from d to a at constant T_1 from volume V_d back to V_a.

i) Find the work done by the system on the surroundings W_{ad} and the amount of heat Q_{ad} that flows between the system and the surroundings. Are these quantities positive or negative? Explain whether heat is added to the system or removed from the heat source at T_1.

Total Cycle:

j) What is the total work W_{cycle} done by the gas during this cycle?

k) What is the total heat $Q_{cycle}(\mathit{from}\, T_2)$ drawn from the higher temperature heat source during this cycle?

l) What is the efficiency of this cycle $\varepsilon_{\max} = W_{cycle} / Q_{cycle}(\mathit{from}\, T_2)$?

Problem 4 Heat pump

A reversible heat engine can be run in the other direction, in which case it does negative work W_{cycle} on the world while "pumping" heat $Q_{cycle}(\mathit{into}\, T_2)$ into a reservoir at an upper temperature, T_2, from a lower temperature, T_1. The heat gain of this cycle, defined to be

$$g \equiv Q_{cycle}(into\ T_2)/W_{cycle} = (1/\varepsilon_{\max})$$

where $\varepsilon_{\max} = (T_2 - T_1)/T_2$ is the maximum thermodynamic efficiency of a heat engine. The refrigerator performance is defined to be

$$K \equiv Q_{cycle}(\,from\ T_1)/W_{cycle} = T_1/(T_2 - T_1)$$

Consider that you have a large swimming pool and plan to heat your house with a heat pump that pumps heat from the pool into your house. A large plate in the water will remain at $0\ ^oC$ due to the formation of ice. You pick T_2 to be $50\ ^oC$, which will be the temperature of the (large) radiators used to heat your house. Assume that your heat pump has the maximum efficiency allowed by thermodynamics.

a) What is the heat gain and the refrigerator performance for this cycle? Be careful to use units of Kelvin for temperature.

b) If your house formerly burned 1200 gallons of oil in a winter (at $2.00/gallon), how much will the electricity cost (at $0.10 per kilowatt-hour) to replace this heat using the heat pump? A gallon of oil has mass 3.4 kg and contains $1.4\times10^8\ \mathrm{J\cdot gal^{-1}}$.

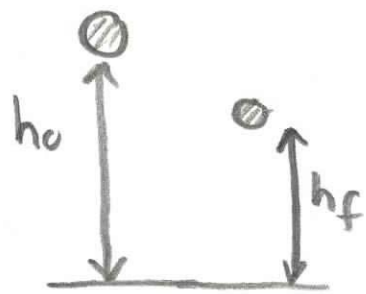
c) The ice cube that appears in your pool over the winter will be how many meters on each side? (It takes $3.35\times10^6\ \mathrm{J}$ to melt one kg of ice; it takes up this much heat when freezing.) The density of ice is $0.931\times10^3\ \mathrm{kg\cdot m^{-3}}$.

This would be great for cooling your house in the summer – even if the pool warmed up enough to swim in it, you could still cool your house by running the heat pump in reverse as an air conditioner! More practically, you might be able to use ground water (and the dirt around it) as the heat sink.

Part Two: Earlier Material

Problem 1: ***(Momentum and Impulse)***

A superball of $m_1 = 0.08kg$, starting at rest, is dropped from a height falls $h_0 = 3.0m$ above the ground and bounces back up to a height of $h_f = 2.0m$. The collision with the ground occurs over $\Delta t_c = 5.0ms$.



a) What is the momentum of the ball immediately before the collision?

b) What is the momentum of the ball immediately after the collision?

c) What is the average force of the table on the ball?

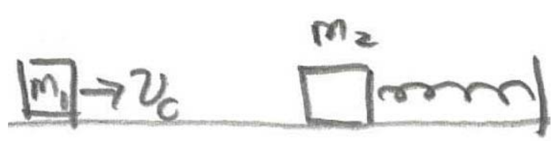
d) What impulse is imparted to the ball?

e) What is the change in the kinetic energy during the collision?

f) Assume that the rubber has a specific heat capacity of $c_r = 0.48 cal \cdot g^{-1} \cdot {}^0C^{-1}$ and that all the lost mechanical energy goes into heating up the rubber. What is the change in temperature of the superball?

Problem 2: ***(Conservation of Energy and Momentum)***

An object of mass $m_1 = 1.5kg$ is initially moving with a velocity v_0. It collides completely inelastically with a block of mass $m_2 = 2.0kg$. The second block is attached to a spring with constant $k = 5.6 \times 10^3 N \cdot m^{-1}$. The block and spring lie on a frictionless horizontal surface. The spring compresses a distance $d = 2.0 \times 10^{-1} m$.



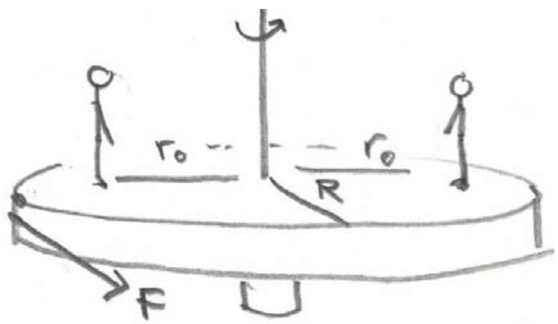
a) What is the velocity of the object of mass m_1 and the block immediately after the collision?

b) What is the initial velocity of the object of mass m_1 immediately before the collision?

c) If the block were attached to a very long string and hung as a pendulum, how high would the block and object of mass m_1 rise after the collision? Let $g = 9.8 m \cdot s^{-2}$.

Problem 3: ***(Angular Dynamics)***

A playground merry-go-round has a radius of $R = 4.0m$ and has a moment of inertia $I_{cm} = 7.0 \times 10^3 kg \cdot m^2$ about an axis passing through the center of mass. There is negligible friction about its vertical axis. Two children each of mass $m = 25kg$ were standing on opposite sides a distance $r_0 = 3.0m$ from the central axis. The merry-go-round is initially at rest. A person on the ground applied a constant tangential force of $F = 2.5 \times 10^2 N$ at the rim of the merry-go-round for a time $\Delta t = 1.0 \times 10^1 s$.



a) What was the angular acceleration of the merry-go-round?

b) What was the angular velocity of the merry-go-round when the person stopped applying the force?

c) What average power did the person put out while pushing the merry-go-round?

d) What was the rotational kinetic energy of the merry-go-round when the person stopped applying the force?

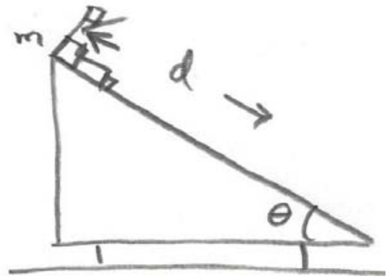
The two children then walked inward and stop a distance of $r_1 = 1.0m$ from the central axis of the merry-go-round.

e) What was the angular velocity of the merry-go-round when the children reached their final position?

f) What was the change in rotational kinetic energy of the merry-go-round when the children reached their final position?

Problem 4: ***(Energy, Force, and Kinematics)***

A child's playground slide is $d = 5.0m$ in length and is at an angle of $\theta = 2.0 \times 10^1 deg$ with respect to the ground. A child of mass $m_b = 2.0 \times 10^1 kg$ starts from rest at the top of the slide. The coefficient of sliding friction for the slide is $\mu_k = 0.2$.



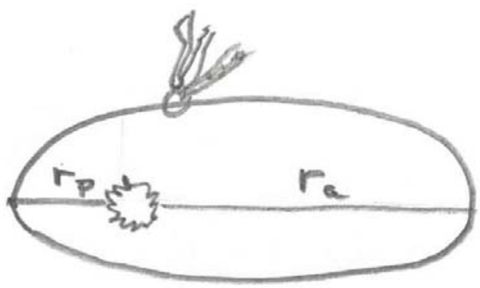
a) What is the total work done by the friction force on the child?

b) What is the speed of the child at the bottom of the slide?

c) How long does the child take to slide down the ramp?

Problem 5: ***(Planetary Orbits)***

Comet Encke was discovered in 1786 by Pierre Mechain and in 1822 Johann Encke determined that its period was 3.3 years. It was photographed in 1913 at the aphelion distance, $r_a = 6.1 \times 10^{11} m$, (furthest distance from the sun) by the telescope at Mt. Wilson. The distance of closest approach to the sun, perihelion, is $r_p = 5.1 \times 10^{10} m$. The universal gravitation constant $G = 6.7 \times 10^{-11} N \cdot m^2 \cdot kg^{-2}$. The mass of the sun is $m_s = 2.0 \times 10^{30} kg$.



a) Explain why angular momentum is conserved about the focal point and then write down an equation for the conservation of angular momentum between aphelion and perihelion.

b) Explain why mechanical energy is conserved and then write down an equation for conservation of energy between aphelion and perihelion.

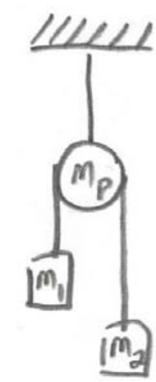
c) Find the velocities at perihelion and aphelion.

Problem 6: ***escape speed of moon***

Find the escape speed of a rocket from the moon. Ignore the rotational motion of the moon. The mass of the moon is $m = 7.36 \times 10^{22}\,kg$. The radius of the moon is $R = 1.74 \times 10^{6}\,m$.

Problem 7: **(*Torque and angular acceleration)***

A pulley of mass m_p, radius R, and moment of inertia $I_{cm} = (1/2)m_p R^2$ about the center of mass is hung from a ceiling with a massless string. A massless inextensible rope is wrapped around the pulley an attached on one side to an object of mass m_1 and on the other side to an object mass $m_2 > m_1$. At time $t = 0$, the objects are released from rest.



a) Draw the free body diagram on the pulley and the two objects.

b) Write down Newton's Second Law for the pulley and the two objects.

c) Write down the rotational equation of motion for the pulley.

d) Find the direction and magnitude of the translational acceleration of the two objects.

e) How long does it take for the object of mass m_2 to fall a distance d?

f) What is the tension on the two sides of the rope?

Problem 8: Projectile Motion

A bat hits a baseball into the air with an initial speed, $v_0 = 5.0 \times 10^{1}\,m/s$, and makes an angle $\theta = 3.0 \times 10^{1}\deg$ with respect to the horizontal. How high does it go from the point where it was hit? How far does the ball travel if it is caught at exactly the same height that it is hit from? When the ball is in flight, ignore all forces acting on the ball except for gravitation.