

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01T

Fall Term 2004

Exam 3: Solutions

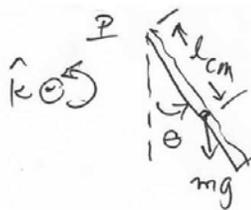
Problem 1a: (5 points)

A physical pendulum consists of a ruler of mass m oscillating about a fixed point. The ruler is pulled to one side and allowed to oscillate. The pendulum has moment of inertia I_p about its pivot point and the distance from the pivot point to the center of mass is l_{cm} .



- a) The period of the pendulum is equal to $2\pi\sqrt{I_p/mgl_{cm}}$.
- b) The period of the pendulum is greater than $2\pi\sqrt{I_p/mgl_{cm}}$.
- c) The period of the pendulum is less than $2\pi\sqrt{I_p/mgl_{cm}}$.

Answer B.

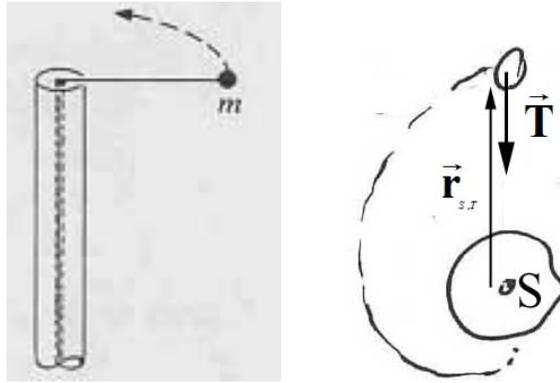


the torque about the pivot point is $\vec{\tau}_p = -mgl_{cm} \sin\theta \hat{k}$
If we make the small angle approximation $\sin\theta \approx \theta$

then we over estimate the torque
 $\vec{\tau}_{SHO} \approx -mgl \theta \hat{k}$ with period $T = 2\pi\sqrt{\frac{I_p}{mgl_{cm}}}$
 This means that we overestimate the angular acceleration since $\vec{\tau} = I_p \vec{\alpha}$ thus the pendulum takes longer to reach the bottom than the period for a 'pure' simple harmonic oscillator.

Problem 1b: (5 points)

A tetherball is attached to a post by a string. The string passes through a hole in the center of the post at the top. The string is gradually shortened by drawing it through the hole. Ignore gravity.



Until the ball hits the post,

- The energy and angular momentum about the center of the post are constant.
- The energy of the ball is constant but the angular momentum about the center of the post changes.
- Both the energy and the angular momentum about the center of the post, change.
- The energy of the ball changes but the angular momentum about the center of the post is constant.

Explain your reasoning. The tension force points radially in; so torque about central point is zero, $\vec{\tau}_S = \vec{r}_{s,r} \times \vec{T} = \vec{0}$. Therefore the angular momentum about central point is constant, $\vec{L}_{S,f}^{system} = \vec{L}_{S,0}^{system}$. All radial forces, for example gravitation, have angular momentum about central point constant.

Small displacement has radially component inward so work done by tension is not zero, $dW^{ext} = \vec{T} \cdot d\vec{r} \neq 0$. Mechanical energy is not constant, $W_{ext} = \Delta K \neq 0$

Problem 1c: (5 points)

A cart on a horizontal table is connected to one end of a spring. The other end of the spring is attached to a wall. The cart is set in motion perpendicular to wall so that it oscillates about its equilibrium point. There is negligible friction between the cart and the table.



A lump of sticky putty is dropped vertically onto the cart and lands the instant the cart has its maximum velocity. The putty sticks without bouncing. During the collision

- a) the velocity of the cart changes; the mechanical energy of the system consisting of the putty and the cart is constant.
- b) the velocity of the cart is constant; the mechanical energy of the system consisting of the putty and the cart is constant.
- c) the velocity of the cart changes; the mechanical energy of the system consisting of the putty and the cart changes.
- d) the velocity of the cart is constant; the mechanical energy of the system consisting of the putty and the cart changes.

Explain your reasoning. Answer C. There is an external spring force acting on the system consisting of the putty and the cart so if the collision takes place over a finite non-zero interval of time, the velocity will decrease. If the collision between the putty and the cart is instantaneous so we can ignore the impulse from the external spring force, the momentum of the system in the horizontal direction is constant. The initial momentum is just the mass of the cart times the initial velocity of the cart. The final momentum is the mass of the putty and cart times the final velocity of the system. Since the mass increases the velocity for the system decrease.

The collision is completely inelastic so the mechanical energy is not conserved. Suppose you consider the putty and cart as the system, assume the collision is instantaneous so that the system does not move during the collision. Then the spring force does no external work on the system. Some of the initial mechanical energy is converted into heat. Some of this heat dissipates into the atmosphere while the rest flows into the cart and putty. Thus the energy of the system decreases due to the loss of heat to the atmosphere.

Problem 1d: (5 points)

A figure skater is spinning with her arms held close to her body. She relaxes her arm muscles and her arms move outward. Compared to her initial rotational kinetic energy, her rotational kinetic energy after she her arms move outward is

- a) the same.
- b) larger.
- c) smaller.

Explain your reasoning. Answer C. We assume that there is negligible frictional torque acting on the skater while she relaxes her arm muscles and her arms move outward. Therefore the angular momentum about the central axis is constant, $I_{cm,0}\omega_0 = I_{cm,f}\omega_f$. The final angular velocity is therefore $\omega_f = (I_{cm,0} / I_{cm,f})\omega_0$. The final rotational kinetic energy is $K_f^{rot} = (1/2)I_{cm,f}\omega_f^2$. Using the result for the final angular velocity, the final kinetic energy becomes

$$K_f^{rot} = (1/2)I_{cm,f}(I_{cm,0} / I_{cm,f})^2 \omega_0^2 = (1/2)(I_{cm,0} / I_{cm,f})\omega_0^2 = (I_{cm,0} / I_{cm,f})K_0$$

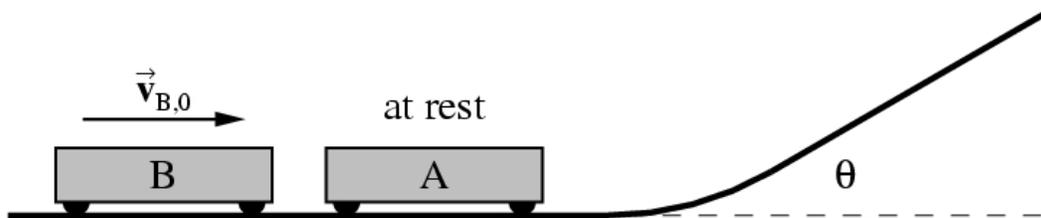
Since the moment of inertia increases as she moves her arms out, the ratio $(I_{cm,0} / I_{cm,f}) < 1$. Thus the kinetic energy decreases,

$$K_f^{rot} = (I_{cm,0} / I_{cm,f})K_0 < K_0.$$

Note her arms move outward because there is not enough force pulling them inward and so they actually move in a curved trajectory but not in circular motion. The arms appear to move radially outward to the skater. Remember that the skater is not a rigid body. In order to prevent her arms from departing from her body, she must apply a force to slow the arms down and stop them. This work is dissipative changing kinetic energy into heat.

Problem 2: (25 points)

In the lab frame, Cart B has mass m_B and is moving to the right with initial velocity $\vec{v}_{B,0}$. Cart A has mass $m_A = 2m_B$ and is at rest. Cart B collides *elastically* with cart A. After the collision, Cart A moves up an incline plane that makes an angle θ with respect to the horizontal. You may assume that the track is frictionless and the acceleration due to gravity is g .



- What are the directions and velocities of Cart A and Cart B immediately after the collision?
- What distance does Cart A move along the incline plane when it comes to rest?

Initial picture $\boxed{B} \rightarrow v_{B,0} \quad \boxed{A} \quad v_{A,0} = 0 \quad \rightarrow \hat{c}$
 $m_A = 2m_B$

final picture $v_{B,f} \leftarrow \boxed{B} \quad \boxed{A} \rightarrow v_{A,f}$

no external forces so momentum is constant

$$\hat{c}: m_B v_{B,0} = -m_B v_{B,f} + 2m_B v_{A,f}$$

$$\Rightarrow v_{B,0} = -v_{B,f} + 2v_{A,f} \quad (1)$$

collision is elastic so energy is constant

$$\frac{1}{2} m_B v_{B,0}^2 = \frac{1}{2} m_B v_{B,f}^2 + \frac{1}{2} (2m_B) v_{A,f}^2$$

$$\Rightarrow v_{B,0}^2 = v_{B,f}^2 + 2v_{A,f}^2 \quad (2)$$

$$\text{eq (1)} \Rightarrow v_{B,f} = -v_{B,o} + 2v_{A,f} \quad (3)$$

Substitute this into eq (2) yielding

$$(v_{B,o})^2 = (v_{B,o} - 2v_{A,f})^2 + 2v_{A,f}^2$$

$$(v_{B,o})^2 = v_{B,o}^2 - 4v_{A,f}v_{B,o} + 4v_{A,f}^2 + 2v_{A,f}^2$$

$$\Rightarrow 0 = -4v_{A,f}v_{B,o} + 6v_{A,f}^2$$

$$\Rightarrow \frac{2}{3}v_{B,o} = v_{A,f} \quad (4)$$

eq (3) then becomes

$$v_{B,f} = -v_{B,o} + (2)\left(\frac{2}{3}v_{B,o}\right)$$

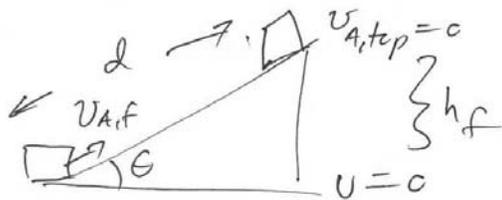
$$v_{B,f} = \frac{1}{3}v_{B,o} \quad (5)$$

treating cart A has a system, energy is constant

$$E_o = E_f \Rightarrow$$

$$K_o = m_A g h_f \Rightarrow$$

$$\frac{1}{2}m_A v_{A,f}^2 = m_A g d \sin \theta$$

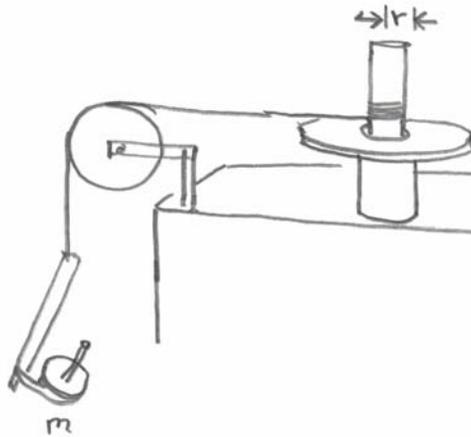


solve for the distance d , and use eq (5)

$$d = \frac{v_{A,f}^2}{2g \sin \theta} = \frac{2}{9} \frac{v_{B,o}^2}{g \sin \theta}$$

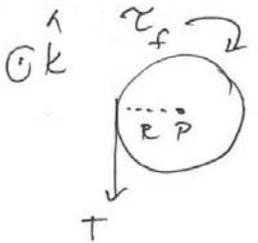
Problem 3: (30 points)

A steel washer, is mounted on a cylindrical rotor of radius r . A massless string, with a mass m attached to the other end, is wrapped around the side of the rotor and passes over a massless pulley. Assume that there is a constant frictional torque about the axis of the rotor. The mass is released and falls. As the mass falls, the rotor undergoes an angular acceleration of magnitude α_1 . After the string detaches from the rotor and the rotor coasts to a stop with an angular acceleration of magnitude α_2 . Let g denote the gravitational constant.



- What is the moment of inertia I_R of the rotor assembly (including the washer) about the rotation axis? Show all your work. Express your answer in terms of the given quantities r , α_1 , α_2 , m , and g .
- Suppose the rotor has an angular velocity ω_0 , when a second washer of moment of inertia I_W is dropped on top of the first washer. It takes a time Δt for the dropped washer to move at the same angular velocity as the rotor. What is the angular velocity ω_f of the assembly immediately after the collision is finished and the dropped washer and rotor move at the same angular velocity ω_f ? Express your answer in terms of the quantities I_R , I_W , α_2 , Δt , and ω_0 as needed.

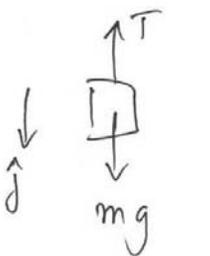
a) The torque diagram for the rotor (seen from overhead) is as shown



$$\vec{\tau}_P = I_R \vec{\alpha}_P$$

$$\hat{k}: RT = I_R \alpha_1 \quad (1)$$

The force diagram on the mass is



$$\vec{F} = m\vec{a}$$

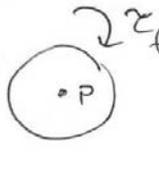
$$\hat{j}: mg - T = ma \quad (2)$$

$$\Rightarrow mg - ma = T \quad (2a)$$

The linear acceleration and angular acceleration

$$a = R\alpha_1 \quad (3)$$

After the string is detached, the only torque left is the frictional torque



$$\vec{\tau}_P = I_R \vec{\alpha}$$

$$-\tau_f = I_R \alpha_2 \quad (4)$$

eq (1) becomes (with eq (4))

$$RT + I_R \alpha_2 = I_R \alpha_1 \quad (5)$$

Use eq (2a) for the tension in eq (5)

$$R(mg - ma) = I_R (\alpha_1 - \alpha_2)$$

Now use eq (3)

$$R(mg - mR\alpha_1) = I_R (\alpha_1 - \alpha_2) \quad (6)$$

Solve for I_R :

$$I_R = \frac{mR(g - R\alpha_1)}{(\alpha_1 - \alpha_2)} \quad (7)$$

b) During the collision there is a frictional torque so the angular momentum is not constant

$$-\tau_f \Delta t = L_f - L_0 \quad (8)$$

The final angular momentum is

$$L_f = (I_R + I_w) \omega_f$$

The initial angular momentum is

$$L_0 = I_R \omega_0$$

so eq (8) becomes

$$-\tau_f \Delta t = (I_R + I_w) \omega_f - I_R \omega_0 \quad (9)$$

Use $-\tau_f = I_R \alpha_2$ then eq (9) becomes

$$I_R \alpha_2 \Delta t = (I_R + I_w) \omega_f - I_R \omega_0$$

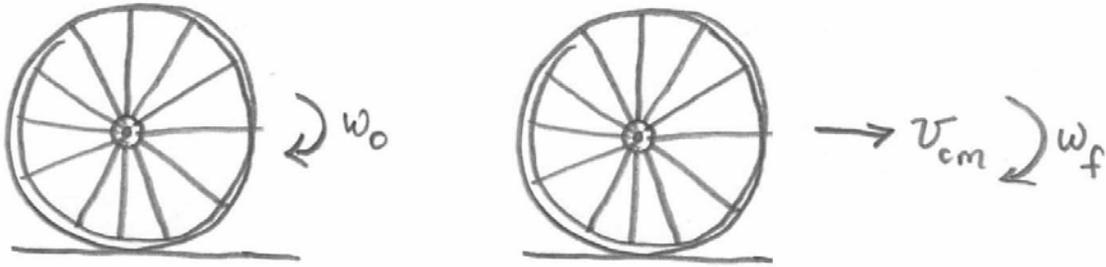
solve for ω_f :

$$\omega_f = \frac{I_R (\omega_0 + \alpha_2 \Delta t)}{I_R + I_w}$$

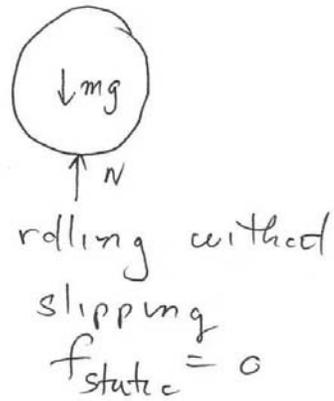
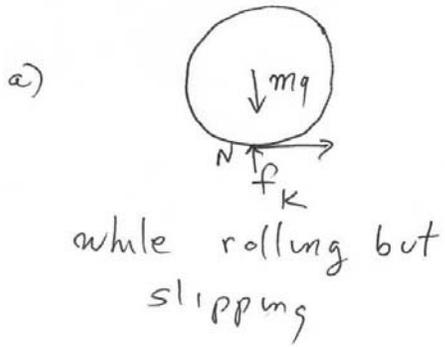
note that $\alpha_2 < 0$.

Problem 4: (25 points)

The center of mass of a bicycle wheel of radius R and mass m is initially spinning with angular velocity ω_0 . The wheel is lowered to the ground without bouncing. As soon as the wheel touches the level ground, the wheel starts to move forward until it begins to roll without slipping with an unknown final angular velocity ω_f and an unknown velocity of the center of mass $v_{cm,f}$. Assume that all the mass of the wheel is located on the rim.

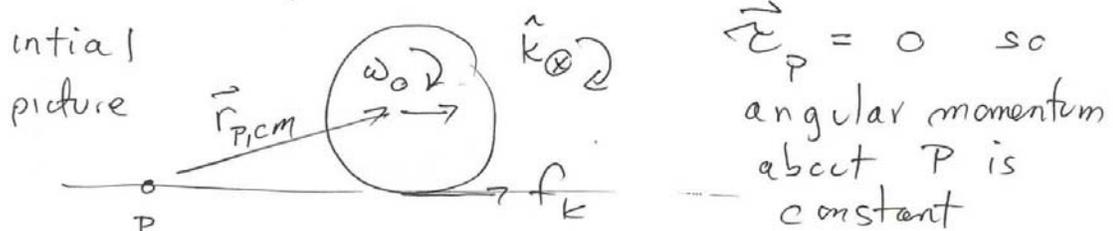


- Draw a free body diagram of all the forces acting on the bicycle wheel while it is moving forward.
- What is the relation between the angular velocity of the wheel ω_f and the velocity of the center of mass $v_{cm,f}$ when it begins to roll without slipping?
- What is the velocity of the center of mass of the wheel when it begins to roll without slipping?



b) when the wheel begins rolling without slipping, $v_{cm} = R \omega_f$. (1)

c) two approaches:



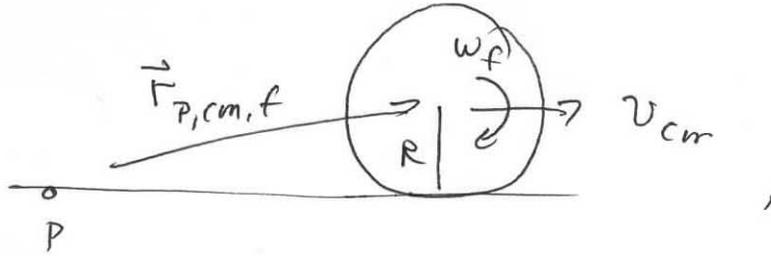
$$\vec{L}_{P,0} = \vec{L}_{P,0}^{orbit} + \vec{L}_{cm,0}^{spin}$$

$$= I_{cm} \omega_0 \hat{k}$$

so $I_{cm} = m R^2$

$$\vec{L}_{P,0} = m R^2 \omega_0 \hat{k}$$

find picture



$$\vec{L}_{P,f} = \vec{F}_{P,cm,t} \times m \vec{v}_{cm} + I_{cm} \omega_f \hat{k}$$

$$= (m R v_{cm} + I_{cm} \omega_f) \hat{k}$$

$$= (m R v_{cm} + m R^2 \frac{v_{cm}}{R}) \hat{k}$$

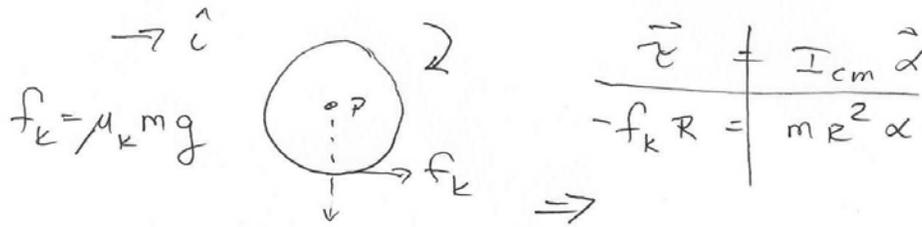
$$= 2 m R v_{cm} \hat{k}$$

$$\vec{L}_{P,o} = \vec{L}_{P,f} \Rightarrow$$

$$\hat{k} : m R^2 \omega_\alpha = 2 m R v_{cm} \Rightarrow$$

$$v_{cm} = \frac{1}{2} R \omega_0$$

2nd approach



$$-\mu_k mg R = m R^2 \alpha \Rightarrow \alpha = -\frac{\mu_k g}{R} \quad (1)$$

$$\begin{array}{l} \vec{F} = m \vec{a} \\ \hline f_k = m a \\ \mu_k mg = m a \Rightarrow a = \mu_k g \quad (2) \end{array}$$

Angular kinematics:

$$\omega_f = \omega_0 + \alpha t = \omega_0 - \frac{\mu_k g}{R} t \quad (3)$$

linear kinematics

$$v_{cm,f} = a t = \mu_k g t \Rightarrow t = \frac{v_{cm,f}}{\mu_k g} \quad (4)$$

eq (3) becomes $\omega_f = \omega_0 - \frac{\mu_k g}{R} \frac{v_{cm,f}}{\mu_k g} \Rightarrow$

$$\omega_f = \omega_0 - \frac{v_{cm,f}}{R}$$

Use $\omega_f = \frac{v_{cm,f}}{R} \Rightarrow \frac{v_{cm,f}}{R} = \omega_0 - \frac{v_{cm,f}}{R}$

$$\Rightarrow v_{cm,f} = \frac{1}{2} R \omega_0$$