

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

Exam 3: Equation Summary

Momentum:

$$\vec{p} = m\vec{v}, \quad \vec{F}_{ave} \Delta t = \Delta\vec{p}, \quad \vec{F}_{ext}^{total} = \frac{d\vec{p}^{total}}{dt}$$

$$\text{Impulse: } \vec{I} \equiv \int_{t=0}^{t=t_f} \vec{F}(t) dt = \Delta\vec{p}$$

$$\text{Torque: } \vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P \quad |\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_{\perp} F = r F_{\perp}$$

Static Equilibrium:

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}; \quad \vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}.$$

$$\text{Rotational dynamics: } \vec{\tau}_S^{total} = \frac{d\vec{L}_S}{dt}$$

$$\text{Angular Velocity: } \vec{\omega} = (d\theta/dt) \hat{k}$$

$$\text{Angular Acceleration: } \vec{\alpha} = (d^2\theta/dt^2) \hat{k}$$

$$\text{Fixed Axis Rotation: } \vec{\tau}_S = I_S \vec{\alpha}$$

$$\tau_S^{total} = I_S \alpha = I_S \frac{d\omega}{dt}$$

$$\text{Moment of Inertia: } I_S = \int_{body} dm (r_{\perp})^2$$

$$\text{Angular Momentum: } \vec{L}_S = \vec{r}_{S,m} \times m\vec{v},$$

$$|\vec{L}_S| = |\vec{r}_{S,m}| |m\vec{v}| \sin \theta = r_{\perp} p = r p_{\perp}$$

Angular Impulse:

$$\vec{J}_S = \int_{t_0}^{t_f} \vec{\tau}_S dt = \Delta\vec{L}_S = \vec{L}_{S,f} - \vec{L}_{S,0}$$

Rotation and Translation:

$$\vec{L}_S^{total} = \vec{L}_S^{orbital} + \vec{L}_{cm}^{spin},$$

$$\vec{L}_S^{orbital} = \vec{r}_{S,cm} \times \vec{p}^{total},$$

$$\vec{L}_{cm}^{spin} = I_{cm} \vec{\omega}_{spin}$$

$$\vec{\tau}_S^{orbit} = \frac{d\vec{L}_S^{orbit}}{dt}, \quad \vec{\tau}_{cm}^{spin} = \frac{d\vec{L}_{cm}^{spin}}{dt}$$

Rotational Energy: $K_{cm} = \frac{1}{2} I_{cm} \omega_{cm}^2$

Rotational Power: $P_{rot} \equiv \frac{dW_{rot}}{dt} = \vec{\tau}_S \cdot \vec{\omega} = \tau_S \omega = \tau_S \frac{d\theta}{dt}$

One Dimensional Kinematics: $\vec{v} = d\vec{r} / dt$, $\vec{a} = d\vec{v} / dt$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

Constant Acceleration:

$$x(t) = x_0 + v_{x,0}(t-t_0) + \frac{1}{2} a_x (t-t_0)^2 \quad v_x(t) = v_{x,0} + a_x(t-t_0)$$

$$y(t) = y_0 + v_{y,0}(t-t_0) + \frac{1}{2} a_y (t-t_0)^2 \quad v_y(t) = v_{y,0} + a_y(t-t_0)$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Newton's Second Law: Force, Mass, Acceleration

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

Newton's Third Law: $\vec{F}_{1,2} = -\vec{F}_{2,1}$

Force Laws:

Universal Law of Gravity: $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$, attractive

Gravity near surface of earth: $\vec{F}_{grav} = m_{grav} \vec{g}$, towards earth

Contact force: $\vec{F}_{contact} = \vec{N} + \vec{f}$, depends on applied forces

Static Friction: $0 \leq f_s \leq f_{s,max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k |\Delta x|$, restoring

Kinematics Circular Motion: arc length: $s = R\theta$; angular velocity: $\omega = d\theta/dt$
 tangential velocity: $v = R\omega$; angular acceleration: $\alpha = d\omega/dt = d^2\theta/dt^2$; tangential acceleration $a_\theta = R\alpha$.

Period: $T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$; **frequency:** $f = \frac{1}{T} = \frac{\omega}{2\pi}$,

Radial Acceleration: $|a_r| = R\omega^2$; $|a_r| = \frac{v^2}{R}$; $|a_r| = 4\pi^2 R f^2$; $|a_r| = \frac{4\pi^2 R}{T^2}$

Center of Mass: $\vec{R}_{cm} = \sum_{i=1}^{i=N} m_i \vec{r}_i / m^{total} \rightarrow \int_{body} dm \vec{r} / m^{total}$;

Velocity of Center of Mass: $\vec{V}_{cm} = \sum_{i=1}^{i=N} m_i \vec{v}_i / m^{total} \rightarrow \int_{body} dm \vec{v} / m^{total}$

Torque: $\vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P$ $|\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_{\perp} F = r F_{\perp}$

Static Equilibrium: $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}$; $\vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}$.

Kinetic Energy: $K = \frac{1}{2}mv^2$; $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

Work: $W = \int_{r_0}^{r_f} \vec{F} \cdot d\vec{r}$; **Work- Kinetic Energy:** $W^{total} = \Delta K$

Power: $P = \vec{F} \cdot \vec{v} = dK/dt$

Potential Energy: $\Delta U = -W_{conservative} = -\int_A^B \vec{F}_c \cdot d\vec{r}$

Potential Energy Functions with Zero Points:

Constant Gravity: $U(y) = mgy$ with $U(y_0 = 0) = 0$.

Inverse Square Gravity: $U_{gravity}(r) = -\frac{Gm_1 m_2}{r}$ with $U_{gravity}(r_0 = \infty) = 0$.

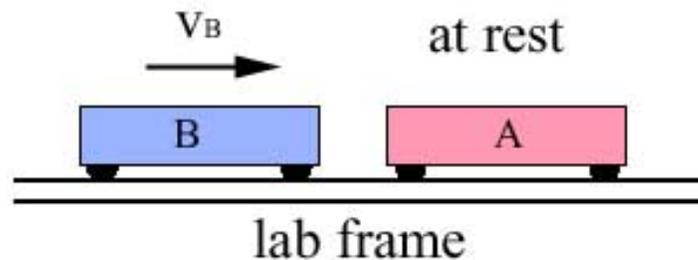
Hooke's Law: $U_{spring}(x) = \frac{1}{2}kx^2$ with $U_{spring}(x = 0) = 0$.

Work- Mechanical Energy: $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech} = (K_f + U_f^{total}) - (K_0 + U_0^{total})$

Practice Exam Three

Problem 1a) (5 points) Collisions and Center of Mass Reference Frame

In the lab frame, Cart A is at rest. An identical cart B, of the same mass, moving to the right, collides *elastically* with cart A. After the collision, describe the motion of the carts in a reference frame moving with the center of mass. (You may find a picture may help clarify your explanation.)



Problem 1b) (5 points) Collisions

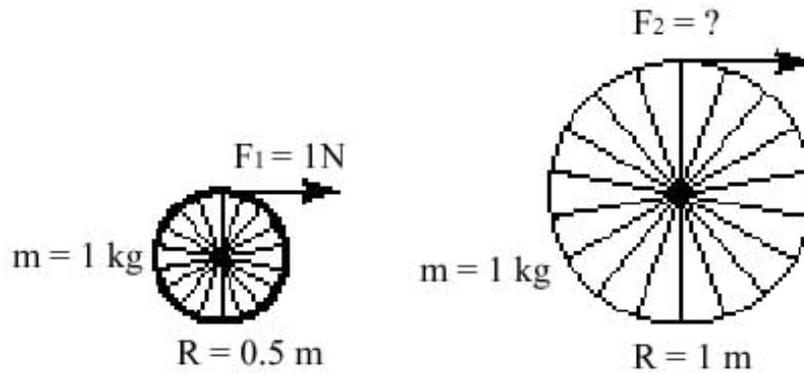
An object P has an initial velocity \vec{v} . It strikes an initially stationary object Q which is attached to a massless spring, as shown.



Assume that the masses of P and Q are equal. At the point of maximum compression of the spring, is the momentum of Q greater, equal, or less than the initial momentum of P . Explain your reasoning.

Problem 1c) (5 points) Torque and Moment of Inertia

Two wheels with fixed hubs, each having a mass of 1 kg , start from rest, and forces are applied: $F_1 = 1\text{ N}$ to the first wheel and F_2 to the second wheel, as shown. Assume the hubs and spokes are massless, so that the moment of inertia about the center of mass is $I_{cm} = mR^2$. How large must the force F_2 be in order to impart identical angular accelerations on each wheel? Explain how you arrived at your result.



Problem 1d) (5 points) Angular Momentum and Rotational Energy

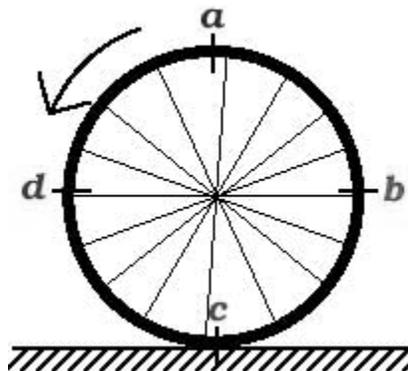
A figure skater stands on one spot on the ice (assumed frictionless) and spins around with her arms extended. When she pulls in her arms, she reduces her moment of inertia about the spin axis passing through her center of mass and her angular speed increases so that her angular momentum is conserved. Compared to her initial rotational kinetic energy, her rotational kinetic energy after she has pulled in her arms must be

1. the same.
2. larger because she's rotating faster.
3. smaller because her moment of inertia is smaller.

Explain your reasoning.

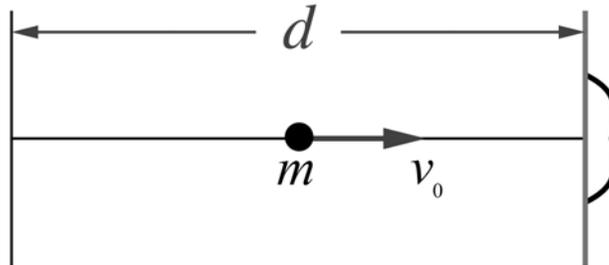
Problem 1e) (5 points) Rolling without slipping

A bicycle tire rolls without slipping across the ground (moving to the left in the figure below). Describe the direction and magnitude of the velocity at each of the four points shown in the figure as seen from an observer at rest on the ground.

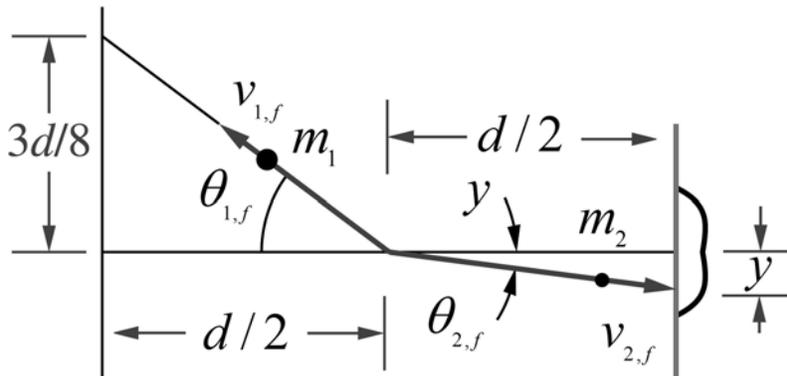


Problem 2: Exploding Puck

A hockey player shoots a “trick” hockey puck along the ice towards the center of the goal from a position d directly in front of the goal. The initial speed of the puck is v_0 and the puck has a mass m .



Half way to the goal the puck explodes into two fragments. One piece of mass $m_1 = (3/5)m$ comes back towards the player and passes $3d/8$ to the side of the spot it was initially shot from with a speed $v_{1,f} = (5/12)v_0$. The other piece of the puck with mass $m_2 = (2/5)m$ continues on towards the goal with a speed $v_{2,f}$.

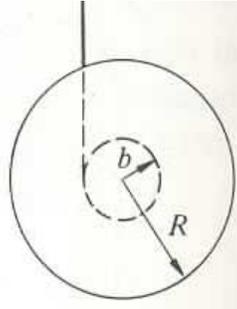


Assume that there is no friction as the puck slides along the ice and that the mass of explosive in the puck is negligible.

- Write down the equations for conservation of momentum of the puck and fragments in terms of the quantities shown in the figure above.
- By what distance, y , does the piece that continues towards the goal miss the center of the goal? Express your answer in terms of d .

Problem 3: Torque, Rotation and Translation

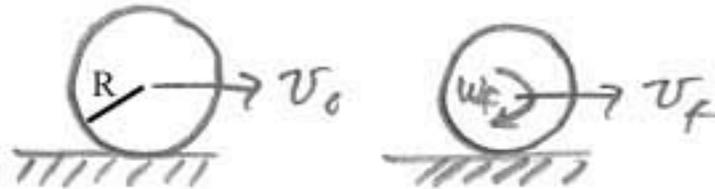
A Yo-Yo of mass m has an axle of radius b and a spool of radius R . Its moment of inertia can be taken to be $I = (1/2)mR^2$ and the thickness of the string can be neglected. The Yo-Yo is released from rest.



- What is the tension in the cord as the Yo-Yo descends?
- Use conservation of energy to find the angular velocity of the Yo-Yo when it reaches the bottom of the string.
- What happens to the Yo-Yo at the bottom of the string?

Problem 4: Rolling Cylinder

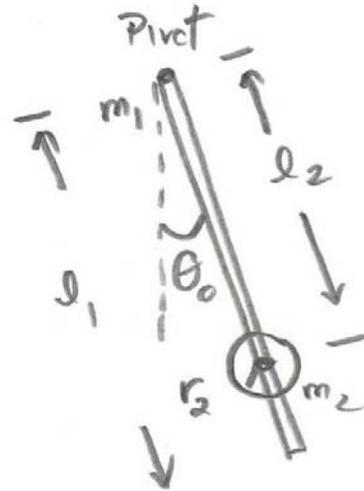
A solid cylinder of mass m and radius R is initially thrown along a wooden floor hallway with an initial speed v_0 and zero initial angular velocity $\omega_0 = 0$ as shown in the figure below.



- What is the moment of inertia of the cylinder about the axis of rotation of the cylinder?
- What effects does friction have on the cylinder before it reaches its final speed? Is the friction static or kinetic? Which direction does it point?
- What is the angular speed, ω_f , of the cylinder when it just starts to roll without slipping? Express your answer in terms of m , R , and v_0 .

Problem 5: Experiment 7: Physical Pendulum

A physical pendulum consists of a rod of mass m_1 pivoted at one end. The rod has length l_1 and moment of inertia I_1 about the pivot point. A disc of mass m_2 and radius r_2 with moment of inertia I_{cm} about its center of mass, is rigidly attached a distance l_2 from the pivot point. The pendulum is initially displaced to an angle θ_0 and then released from rest.



- What is the moment of inertia of the physical pendulum about the pivot point?
- What is the angular acceleration of the pendulum about the pivot point?
- What is the angular velocity of the pendulum when the pendulum is at the bottom of its swing?
- If the disc is moved further from the pivot, will the period of the pendulum increase or decrease or stay the same?

Problem 6: Experiment 11 Angular Momentum

A steel washer, is mounted on the shaft of a small motor. The moment of inertia of the motor and washer is I_0 . Assume that the frictional torque on the axle remains the same throughout the slowing down. The washer is set into motion. When it reaches an initial angular velocity ω_0 , at $t = 0$, the power to the motor is shut off, and the washer slows down until it reaches an angular velocity of ω_b at time t_b . At that instant, a second steel washer with a moment of inertia I_w is dropped on top of the first washer. Assume that the second washer is only in contact with the first washer. The collision takes place over a time $\Delta t = t_a - t_b$. Assume the frictional torque on the axle remains the same. The two washers continue to slow down until at they stop at t_f .

- Describe how you measured the moment of inertia in the experiment.
- What is the angular deceleration while the first washer and motor is slowing down?

- c) What is the angular velocity of the two washers immediately after the collision is finished?
- d) What is the angular deceleration after the collision?