

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Physics

Physics 8.01 TEAL

Fall Term 2004

**Exam 3: Equation Summary**

**Momentum:**

$$\vec{p} = m\vec{v}, \quad \vec{F}_{ave} \Delta t = \Delta\vec{p}, \quad \vec{F}_{ext}^{total} = \frac{d\vec{p}^{total}}{dt}$$

$$\text{Impulse: } \vec{I} \equiv \int_{t=0}^{t=t_f} \vec{F}(t) dt = \Delta\vec{p}$$

$$\text{Torque: } \vec{\tau}_S = \vec{r}_{S,P} \times \vec{F}_P \quad |\vec{\tau}_S| = |\vec{r}_{S,P}| |\vec{F}_P| \sin \theta = r_{\perp} F = r F_{\perp}$$

**Static Equilibrium:**

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \dots = \vec{0}; \quad \vec{\tau}_S^{total} = \vec{\tau}_{S,1} + \vec{\tau}_{S,2} + \dots = \vec{0}.$$

$$\text{Rotational dynamics: } \vec{\tau}_S^{total} = \frac{d\vec{L}_S}{dt}$$

$$\text{Angular Velocity: } \vec{\omega} = (d\theta/dt) \hat{k}$$

$$\text{Angular Acceleration: } \vec{\alpha} = (d^2\theta/dt^2) \hat{k}$$

$$\text{Fixed Axis Rotation: } \vec{\tau}_S = I_S \vec{\alpha}$$

$$\tau_S^{total} = I_S \alpha = I_S \frac{d\omega}{dt}$$

$$\text{Moment of Inertia: } I_S = \int_{body} dm (r_{\perp})^2$$

$$\text{Angular Momentum: } \vec{L}_S = \vec{r}_{S,m} \times m\vec{v},$$

$$|\vec{L}_S| = |\vec{r}_{S,m}| |m\vec{v}| \sin \theta = r_{\perp} p = r p_{\perp}$$

**Angular Impulse:**

$$\vec{J}_S = \int_{t_0}^{t_f} \vec{\tau}_S dt = \Delta\vec{L}_S = \vec{L}_{S,f} - \vec{L}_{S,0}$$

**Rotation and Translation:**

$$\vec{L}_S^{total} = \vec{L}_S^{orbital} + \vec{L}_{cm}^{spin},$$

$$\vec{L}_S^{orbital} = \vec{r}_{S,cm} \times \vec{p}^{total},$$

$$\vec{L}_{cm}^{spin} = I_{cm} \vec{\omega}_{spin}$$

$$\vec{\tau}_S^{orbit} = \frac{d\vec{L}_S^{orbit}}{dt}, \quad \vec{\tau}_{cm}^{spin} = \frac{d\vec{L}_{cm}^{spin}}{dt}$$

**Rotational Energy:**  $K_{cm} = \frac{1}{2} I_{cm} \omega_{cm}^2$

**Rotational Power:**  $P_{rot} \equiv \frac{dW_{rot}}{dt} = \vec{\tau}_S \cdot \vec{\omega} = \tau_S \omega = \tau_S \frac{d\theta}{dt}$

**One Dimensional Kinematics:**  $\vec{v} = d\vec{r} / dt$  ,  $\vec{a} = d\vec{v} / dt$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt' \quad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

**Constant Acceleration:**

$$x(t) = x_0 + v_{x,0}(t-t_0) + \frac{1}{2} a_x (t-t_0)^2 \quad v_x(t) = v_{x,0} + a_x(t-t_0)$$

$$y(t) = y_0 + v_{y,0}(t-t_0) + \frac{1}{2} a_y (t-t_0)^2 \quad v_y(t) = v_{y,0} + a_y(t-t_0)$$

where  $x_0, v_{x,0}, y_0, v_{y,0}$  are the initial position and velocities components at  $t = t_0$

**Newton's Second Law: Force, Mass, Acceleration**

$$\vec{F} \equiv m\vec{a} \quad \vec{F}^{total} = \vec{F}_1 + \vec{F}_2 \quad F_x^{total} = ma_x \quad F_y^{total} = ma_y \quad F_z^{total} = ma_z$$

**Newton's Third Law:**  $\vec{F}_{1,2} = -\vec{F}_{2,1}$

**Force Laws:**

Universal Law of Gravity:  $\vec{F}_{1,2} = -G \frac{m_1 m_2}{r_{1,2}^2} \hat{r}_{1,2}$  , attractive

Gravity near surface of earth:  $\vec{F}_{grav} = m_{grav} \vec{g}$  , towards earth

Contact force:  $\vec{F}_{contact} = \vec{N} + \vec{f}$  , depends on applied forces

Static Friction:  $0 \leq f_s \leq f_{s,max} = \mu_s N$  direction depends on applied forces

Kinetic Friction:  $f_k = \mu_k N$  opposes motion

Hooke's Law:  $F = k |\Delta x|$  , restoring

**Kinematics Circular Motion:** arc length:  $s = R\theta$  ; angular velocity:  $\omega = d\theta/dt$   
 tangential velocity:  $v = R\omega$  ; angular acceleration:  $\alpha = d\omega/dt = d^2\theta/dt^2$  ; tangential acceleration  $a_\theta = R\alpha$  .

**Period:**  $T = \frac{2\pi R}{v} = \frac{2\pi}{\omega}$ ; **frequency:**  $f = \frac{1}{T} = \frac{\omega}{2\pi}$ ,

**Radial Acceleration:**  $|a_r| = R\omega^2$ ;  $|a_r| = \frac{v^2}{R}$ ;  $|a_r| = 4\pi^2 R f^2$ ;  $|a_r| = \frac{4\pi^2 R}{T^2}$

**Center of Mass:**  $\vec{R}_{cm} = \sum_{i=1}^{i=N} m_i \vec{r}_i / m^{total} \rightarrow \int_{body} dm \vec{r} / m^{total}$ ;

**Velocity of Center of Mass:**  $\vec{V}_{cm} = \sum_{i=1}^{i=N} m_i \vec{v}_i / m^{total} \rightarrow \int_{body} dm \vec{v} / m^{total}$

**Kinetic Energy:**  $K = \frac{1}{2}mv^2$ ;  $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

**Work:**  $W = \int_{r_0}^{r_f} \vec{F} \cdot d\vec{r}$ ; **Work- Kinetic Energy:**  $W^{total} = \Delta K$

**Power:**  $P = \vec{F} \cdot \vec{v} = dK/dt$

**Potential Energy:**  $\Delta U = -W_{conservative} = -\int_A^B \vec{F}_c \cdot d\vec{r}$

**Potential Energy Functions with Zero Points:**

Constant Gravity:  $U(y) = mgy$  with  $U(y_0 = 0) = 0$ .

Inverse Square Gravity:  $U_{gravity}(r) = -\frac{Gm_1 m_2}{r}$  with  $U_{gravity}(r_0 = \infty) = 0$ .

Hooke's Law:  $U_{spring}(x) = \frac{1}{2}kx^2$  with  $U_{spring}(x = 0) = 0$ .

**Work- Mechanical Energy:**  $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech} = (K_f + U_f^{total}) - (K_0 + U_0^{total})$

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**Department of Physics**

Physics 8.01T

Fall Term 2004

**Exam 3**

Name \_\_\_\_\_

Section:    Morning    Pritchard/Surrow    10 - 12    \_\_\_\_\_

                  Afternoon    Dourmashkin    1-3    \_\_\_\_\_

Table and Group: \_\_\_\_\_

The following exam consists of four problems. Answers without work shown will not be given any credit. Good luck!

**Problem 1    (20 points)**

**Part a)        (5 points)**                    \_\_\_\_\_

**Part b)        (5 points)**                    \_\_\_\_\_

**Part c)        (5 points)**                    \_\_\_\_\_

**Part d)        (5 points)**                    \_\_\_\_\_

**Problem 2    (25 points)**                    \_\_\_\_\_

**Problem 3    (30 points)**                    \_\_\_\_\_

**Problem 4    (25 points)**                    \_\_\_\_\_

**Total         (100 points)**                    \_\_\_\_\_

**Problem 1a: (5 points)**

A physical pendulum consists of a ruler of mass  $m$  oscillating about a fixed point. The ruler is pulled to one side and allowed to oscillate. The pendulum has moment of inertia  $I_p$  about its pivot point and the distance from the pivot point to the center of mass is  $l_{cm}$ .

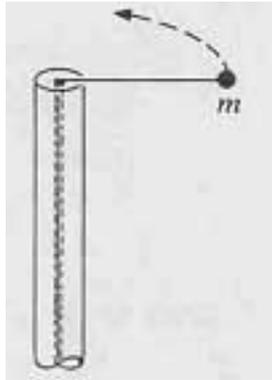


- a) The period of the pendulum is equal to  $2\pi\sqrt{I_p / mgl_{cm}}$ .
- b) The period of the pendulum is greater than  $2\pi\sqrt{I_p / mgl_{cm}}$ .
- c) The period of the pendulum is less than  $2\pi\sqrt{I_p / mgl_{cm}}$ .

**Explain your reasoning:**

**Problem 1b: (5 points)**

A tetherball is attached to a post by a string. The string passes through a hole in the center of the post at the top. The string is gradually shortened by drawing it through the hole. Ignore gravity.



Until the ball hits the post,

- a) The energy and angular momentum about the center of the post are constant.
- b) The energy of the ball is constant but the angular momentum about the center of the post changes.
- c) Both the energy and the angular momentum about the center of the post, change.
- d) The energy of the ball changes but the angular momentum about the center of the post is constant.

**Explain your reasoning.**

**Problem 1c: (5 points)**

A block on a horizontal table is connected to one end of a spring. The other end of the spring is attached to a wall. The block is set in motion perpendicular to wall so that it oscillates about its equilibrium point.



A lump of sticky putty is dropped vertically onto the block and lands the instant the block has its maximum velocity. The putty sticks without bouncing. During the collision

- a) the velocity of the block changes; the mechanical energy of the system consisting of the putty and the block is constant.
- b) the velocity of the block is constant; the mechanical energy of the system consisting of the putty and the block is constant.
- c) the velocity of the block changes; the mechanical energy of the system consisting of the putty and the block changes.
- d) the velocity of the block is constant; the mechanical energy of the system consisting of the putty and the block changes.

**Explain your reasoning.**

**Problem 1d: (5 points)**

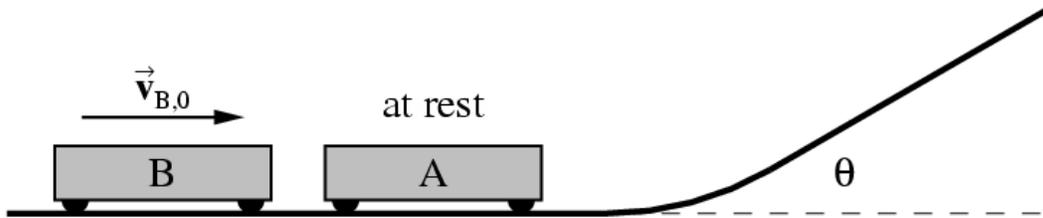
A figure skater is spinning with her arms held close to her body. She relaxes her arm muscles and her arms move outward. Compared to her initial rotational kinetic energy, her rotational kinetic energy after she her arms move outward is

- a) the same.
- b) larger.
- c) smaller.

**Explain your reasoning.**

**Problem 2: (25 points)**

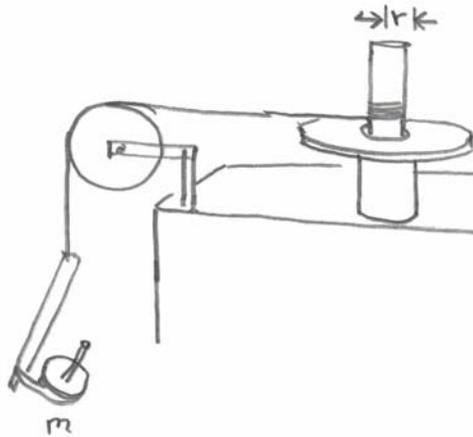
In the lab frame, Cart B has mass  $m_B$  and is moving to the right with initial velocity  $\vec{v}_{B,0}$ . Cart A has mass  $m_A = 2m_B$  and is at rest. Cart B collides *elastically* with cart A. After the collision, Cart A moves up an incline plane that makes an angle  $\theta$  with respect to the horizontal. You may assume that the track is frictionless and the acceleration due to gravity is  $g$ .



- a) What are the directions and velocities of Cart A and Cart B immediately after the collision?
- b) What distance does Cart A move along the incline plane when it comes to rest?

### Problem 3: (30 points)

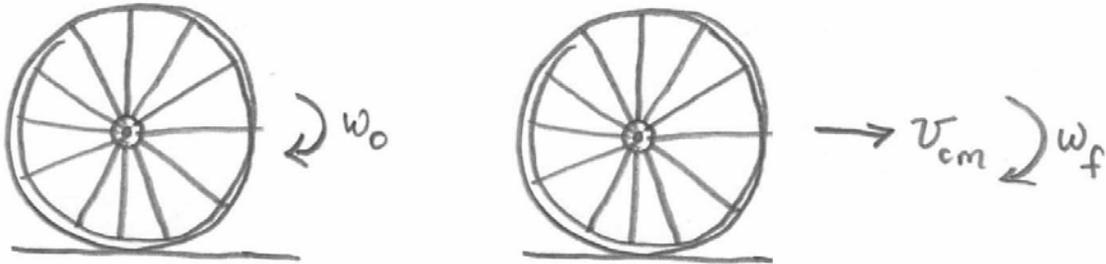
A steel washer, is mounted on a cylindrical rotor of radius  $r$ . A massless string, with a mass  $m$  attached to the other end, is wrapped around the side of the rotor and passes over a massless pulley. Assume that there is a constant frictional torque about the axis of the rotor. The mass is released and falls. As the mass falls, the rotor undergoes an angular acceleration of magnitude  $\alpha_1$ . After the string detaches from the rotor and the rotor coasts to a stop with an angular acceleration of magnitude  $\alpha_2$ . Let  $g$  denote the gravitational constant.



- What is the moment of inertia  $I_R$  of the rotor assembly (including the washer) about the rotation axis? Show all your work. Express your answer in terms of the given quantities  $r$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $m$ , and  $g$ .
- Suppose the rotor has an angular velocity  $\omega_0$ , when a second washer of moment of inertia  $I_W$  is dropped on top of the first washer. It takes a time  $\Delta t$  for the dropped washer to move at the same angular velocity as the rotor. What is the angular velocity  $\omega_f$  of the assembly immediately after the collision is finished and the dropped washer and rotor move at the same angular velocity  $\omega_f$ ? Express your answer in terms of the quantities  $I_R$ ,  $I_W$ ,  $\alpha_2$ ,  $\Delta t$ , and  $\omega_0$  as needed.

### Problem 4: (25 points)

The center of mass of a bicycle wheel of radius  $R$  and mass  $m$  is initially spinning with angular velocity  $\omega_0$ . The wheel is lowered to the ground without bouncing. As soon as the wheel touches the level ground, the wheel starts to move forward until it begins to roll without slipping with an unknown final angular velocity  $\omega_f$  and an unknown velocity of the center of mass  $v_{cm,f}$ . Assume that all the mass of the wheel is located on the rim.



- Draw a free body diagram of all the forces acting on the bicycle wheel while it is moving forward.
- What is the relation between the angular velocity of the wheel  $\omega_f$  and the velocity of the center of mass  $v_{cm,f}$  when it begins to roll without slipping?
- What is the velocity of the center of mass of the wheel when it begins to roll without slipping?