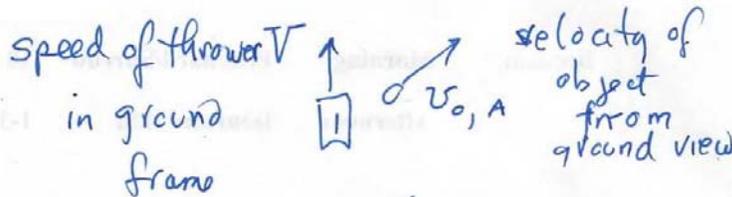
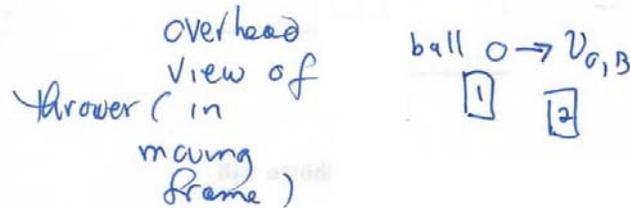


Exam 1 Solutions

**Problem 1: (10 points) Concept Questions: Briefly answer the following questions.**

a) (5 points) Two people are running forward at the same speed. One person throws a ball sideways to the other person at a constant speed  $v$  as seen by the thrower. Describe the motion of the ball as seen by a person standing on the ground. In particular according to the person on the ground is the ball moving faster, slower, or the same speed compared to the speed of the ball  $v$  as seen by the thrower?

**Explain your answer:**



In the ground frame, ball has a forward velocity component as well as sideways velocity component so speed is greater

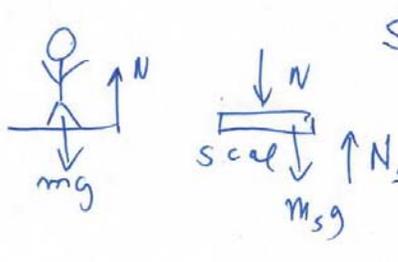
$$v_{0,A} = ((v_{0,B})^2 + V^2)^{1/2}$$

b) (5 points) You are standing on a spring bathroom scale in an elevator and look at the scale while the elevator is at rest with respect to the ground. In between floors, when the elevator is moving at a constant speed with respect to the ground, you look at the scale again. Does the scale show a greater value, the same value, or a lower value?

Explain your answer.

When elevator is moving at constant speed  $N - mg = 0 \Rightarrow N = mg$ .

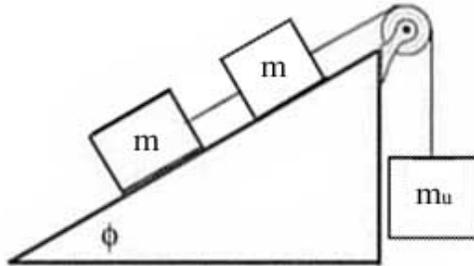
Scale is measuring normal force  $N$  (compresses spring) hence scale shows same value.



The image contains two free-body diagrams. The first diagram shows a stick figure representing a person standing on a scale. An upward-pointing arrow is labeled 'N', representing the normal force exerted by the scale on the person. A downward-pointing arrow is labeled 'mg', representing the person's weight. The second diagram shows a rectangular scale. A downward-pointing arrow is labeled 'N', representing the normal force exerted by the person on the scale. Another downward-pointing arrow is labeled 'm\_s g', representing the weight of the scale. An upward-pointing arrow is labeled 'N\_{s, floor}', representing the normal force exerted by the floor on the scale.

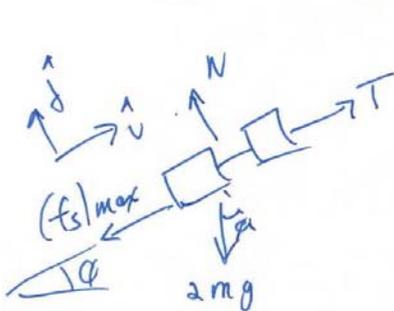
**Problem 2: (30 points)** Two identical blocks of given mass  $m$ , are attached together by a massless string and constrained to move along a plane that is inclined at a given angle  $\phi$  to the horizontal. The upper block is connected via a second massless inextensible string that passes over a massless pulley, to a third block of unknown mass  $m_u$ . Assume the coefficient of static friction between the block and the inclined plane is given as  $\mu_s$  and the coefficient of kinetic friction is given as  $\mu_k$ . Assume the gravitational constant is given as  $g$ .

- What is the mass of the third block such that the blocks **just start slipping up** the inclined plane? Include in your answer a briefly explanation of how you intend to model this problem?
- Now suppose the string between the two identical blocks is cut. How does your model change? What are the accelerations of the lower block and the upper block on the inclined plane?



**Answer:**

model: masses on incline plane just slipping,  $a = 0$ ,  $f_s = (f_s)_{max} = \mu_s N$ . System:  
 a) two blocks    b) block with unknown mass



$$\vec{F} = m \vec{a}$$


---


$$T - (f_s)_{max} - 2mg \sin \phi = 0 \quad (1)$$

$$N - 2mg \cos \phi = 0 \quad (2)$$

$$(f_s)_{max} = \mu_s N = 2\mu_s mg \cos \phi \quad (3)$$

$$\vec{F} = m\vec{a}$$

$$m_u g - T = 0 \Rightarrow T = m_u g \quad (3)$$

Eq (1) with eq (2), (3), and (4) becomes

$$m_u g - \mu_s 2mg \cos \phi - 2mg \sin \phi = 0$$

$$\Rightarrow m_u = 2m(\mu_s \cos \phi + \sin \phi)$$

part 5): model: lower block may slide:  
 three individual objects

$$\vec{F} = m\vec{a}$$

$$f - mg \sin \phi = m a_{\text{lower}} \quad (5)$$

$$N - mg \cos \phi = 0 \quad (6)$$

Just slipping  $f = (f_s)_{\text{max}} = \mu_s N = \mu_s mg \cos \phi$   
 $a_{\text{lower}} = 0$

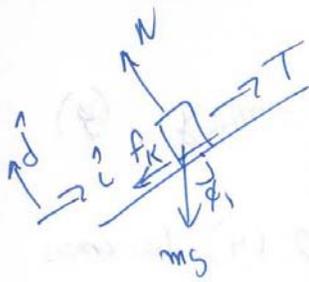
$$\Rightarrow \mu_s mg \cos \phi = mg \sin \phi$$

$$\Rightarrow \mu_s = \tan \phi \quad \text{if } \phi > \tan^{-1}(\mu_s)$$

slips. Then  $f = f_k = \mu_k N = \mu_k mg \cos \phi$

$$\mu_k mg \cos \phi - mg \sin \phi = m a_{\text{lower}}$$

$$\Rightarrow a_{\text{lower}} = g(\mu_k \cos \phi - \sin \phi) < 0 \quad \text{accelerates down inclined plane.}$$



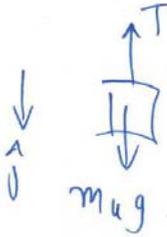
upper block will slip up

$$\vec{F} = m\vec{a}$$

$$T - f_k - mg \sin \theta = m a_{\text{upper}}$$

$$N - mg \cos \theta = 0$$

$$f_k = \mu_k N = \mu_k mg \cos \theta$$



$$\vec{F} = m_u \vec{a}_u$$

$$m_u g - T = m_u a_u$$

constraint  $a_u = a_{\text{upper}} \equiv a$

$$T - \mu_k mg \cos \theta - mg \sin \theta = m a$$

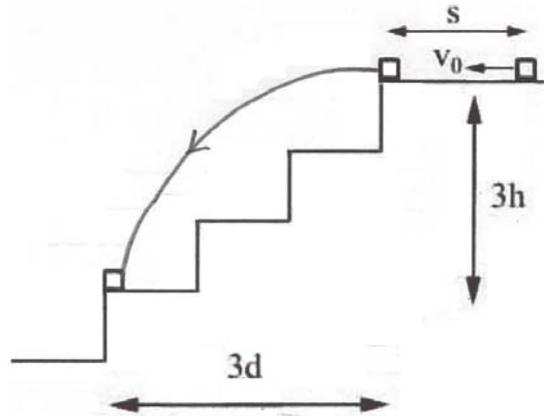
$$m_u g - T = m_u a$$

add equations

$$m_u g - mg(\mu_k \cos \theta + \sin \theta) = (m + m_u) a$$

$$\Rightarrow a = \frac{m_u g - mg(\mu_k \cos \theta + \sin \theta)}{m + m_u}$$

**Problem 3: (30 points)** An object of given mass  $m$  starts with a given velocity  $v_0$  and slides an unknown distance  $s$  along a floor and then off the top of a staircase. The goal of this problem is to find the distance  $s$ . The coefficient of kinetic friction between the object and the floor is given by  $\mu_k$ . The object strikes at the far end of the third stair. Each stair has a given rise of  $h$  and a given run of  $d$ . You may neglect air resistance and assume the gravitational constant is given as  $g$ .

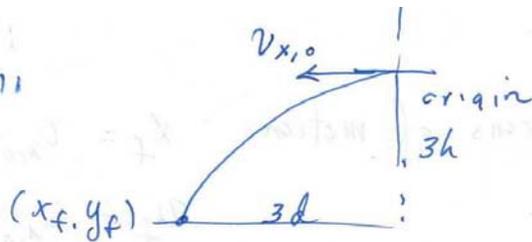


- Briefly describe how you intend to model the motion of the object? What are the given quantities in this problem?
- What is the velocity of the object just as it leaves the top of the staircase? Express your answer in terms of the given quantities only.
- What is the acceleration of the object while it slides on the floor? Express your answer in terms of the given quantities only.
- What is the unknown distance  $s$  that the object slides along the floor? Express your answer in terms of the given quantities only.

**Answer:**

model : two stages  
 stage (1) : horizontal motion with deceleration due to friction  
 stage (2) : projectile motion:  $a_{hor} = 0$ ,  $a_{ver} = -g$   
 with pos. vertical direction upward  
 givens:  $m, v_0, \mu_k, h, d, g$

projectile motion:



initial conditions:  $x_0 = 0, y_0 = 0, v_{y,0} = 0$   
 $v_{x,0}$  unknown

final conditions:  $x_f = 3d, y_f = -3h$

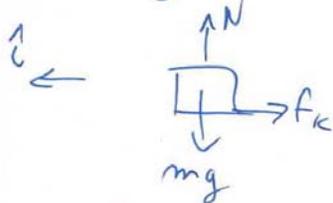
equations of motion:  $x_f = v_{x,0} t_f$   
 $y_f = -\frac{1}{2} g t_f^2$

become (1)  $3d = v_{x,0} t_f$

(2)  $-3h = -\frac{1}{2} g t_f^2 \Rightarrow t_f = \sqrt{\frac{6h}{g}}$

eq (1)  $\Rightarrow v_{x,0} = \frac{3d}{t_f} = \frac{3d}{\sqrt{6h/g}} = d \sqrt{\frac{3g}{2h}}$

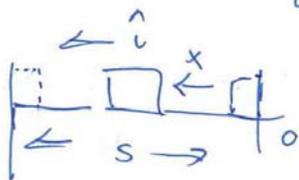
horizontal motion:



$$\begin{array}{c|c} \vec{F} & m\vec{a} \\ \hline \hat{i} & -f_c = ma \\ \hat{j} & N - mg = 0 \end{array}$$

$f_k = \mu_k N = \mu_k mg \Rightarrow -\mu_k mg = ma \Rightarrow$

$a = -\mu_k g$



initial conditions:  $x_0 = 0, v_{x,0} = v_0$

final conditions:  $x_f = s, v_f = d \sqrt{\frac{3g}{2h}}$

equations of motion:  $x_f = v_{x,0} t_f + \frac{1}{2} a t_f^2$  (3)

$$v_f = v_{x,0} + a t_f \quad (4)$$

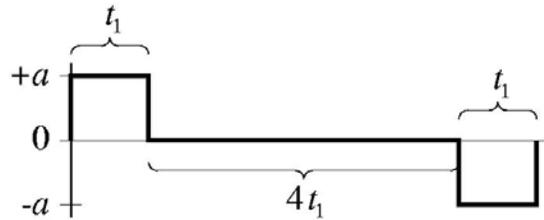
$$\text{eq (4)} \Rightarrow t_f = \frac{v_f - v_{x,0}}{a}$$

$$\begin{aligned} \text{eq (3)} \Rightarrow x_f &= v_{x,0} \left( \frac{v_f - v_{x,0}}{a} \right) + \frac{1}{2} a \left( \frac{v_f - v_{x,0}}{a} \right)^2 \\ &= \frac{v_{x,0} v_f}{a} - \frac{v_{x,0}^2}{a} + \frac{1}{2a} (v_f^2 + v_{x,0}^2 - 2v_f v_{x,0}) \end{aligned}$$

$$x_f = \frac{1}{2a} (v_f^2 - v_{x,0}^2)$$

$$s = \frac{1}{(2)(-14 \text{ kg})} \left( d^2 \frac{3}{2} \frac{g}{h} - v_0^2 \right)$$

**Problem 4: (30 points)** A person of given mass  $m$  is standing on a scale in an elevator in Building 24. Initially the elevator is at rest. The elevator then begins to ascend to the sixth floor that is a given distance  $h$  above the starting point. The elevator undergoes an unknown constant acceleration  $a$  for a given time  $t_1$ . Then the elevator moves at a constant velocity for a time  $t_2 = 4t_1$ . Finally the elevator brakes with a deceleration of the same magnitude as the initial acceleration for a time  $t_3 = t_1$  until stopping at the sixth floor. Assume the gravitational constant is given as  $g$ . The goal of this problem is to find the magnitude of the acceleration and the readings on the scale.



- Briefly explain how you intend to model this problem.
- What is the magnitude of the acceleration and deceleration of the elevator? Express your answer in terms of the given quantities.
- When the elevator is accelerating upwards, what is the magnitude of the contact force between the person's feet and the scale?
- When the elevator is braking, what is the magnitude of the contact force between the person's feet and the scale?

**Answer:**

model: three stages of motion

stage (1): constant acceleration, starting from rest  
 distance:  $y_1 = \frac{1}{2} a t_1^2$ ,  $v_1 = a t_1$

stage (2) constant velocity with  $v_0 = v_1 = a t_1$   
 distance:  $y_2 = v t_2 = 4 a t_1^2$  since  $t_2 = 4 t_1$

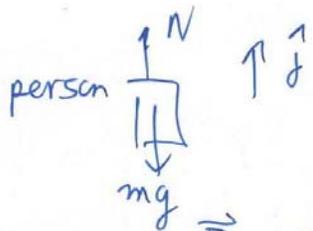
stage (3) constant deceleration with  $v_0 = v_1 = a t_1$   
 distance  $y_3 = v_1 t_3 - \frac{1}{2} a t_3^2 = v_1 t_1 - \frac{1}{2} a t_1^2 = a t_1^2 - \frac{1}{2} a t_1^2$

total distance:  $y^T = y_1 + y_2 + y_3$

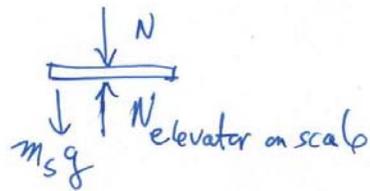
$$h = \frac{1}{2} a t_1^2 + 4 a t_1^2 + a t_1^2 - \frac{1}{2} a t_1^2 = 5 a t_1^2$$

$$\Rightarrow a = \frac{h}{5 t_1^2}$$

The scale indicates normal force  $N$



scale



$$\uparrow \downarrow: \frac{\vec{F}}{N - mg} \quad \left| \quad \frac{m \vec{a}}{m a} \right.$$

$$\Rightarrow N = mg + m a$$

stage (1)  $N = m \left( g + \frac{h}{5 t_1^2} \right) > m g$  goes up (increases)

stage (2)  $N = m g$  doesn't change

stage (3)  $N = m \left( g - \frac{h}{5 t_1^2} \right) < m g$  goes down (decreases)