

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Physics

Physics 8.01T

Fall Term 2004

Practice Exam 1 Solutions

**Concept Questions: Briefly answer the following questions.**

1. When a car accelerates forward, which force is responsible for this acceleration? State clearly which body exerts this force, and on which body the force acts.

**Answer:** The friction force of the road acting on the wheels accelerates the car forward.

2. A weightlifter and a barbell are both at rest on a large scale. The weightlifter begins to lift the barbell, ultimately holding it motionless above her head. Does the scale reading ever differ from the combined weight of the two bodies at any time during the lift? Explain.

**Answer:** Choose positive direction upwards. The motion of the barbells is as follows:

**Stage 1:** Barbells are being accelerated upwards as weightlifter jerks them from the floor. In this stage,  $N - m^{total}g = m^{total}a_y$ , Since acceleration is positive upwards,  $N = m^{total}g + m^{total}a_y > m^{total}g$  so scale shows increase.

**Stage 2:** Barbells move at constant velocity. Since acceleration is zero,  $N = m^{total}g$  so scale doesn't change.

**Stage 3:** Barbells are being decelerated as the weight lifter brings them to rest above head. Since acceleration is negative upwards,  $N = m^{total}g + m^{total}a_y < m^{total}g$  so scale shows decrease.

3. As a space shuttle burns up its fuel after take-off, it gets lighter and lighter and its acceleration larger and larger. Between the moment it takes off and the time at which it has consumed nearly all of its fuel, is its average speed larger than, equal to, or smaller than half its final speed? Explain why.

**Answer:** In figure 1, the graph of velocity vs. time for both cases are shown, constant acceleration and non-constant acceleration.

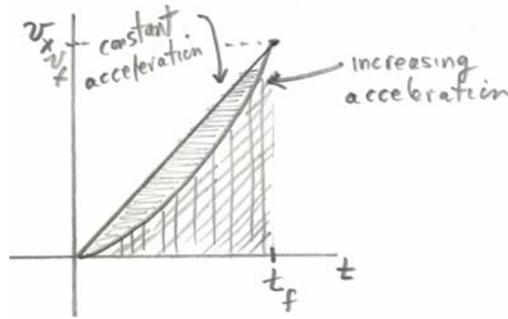


Figure 1: velocity graphs for constant and non-constant acceleration

When the acceleration is constant, the velocity is a linear function of time. Therefore the average velocity is given by (see figure 1)

$$\bar{v}_x = \frac{1}{2}(v_x(t) + v_{x,0}) = \frac{1}{2}v_{x,f}.$$

Hence for an object starting at rest, the average velocity is one half of the final velocity.

Suppose the space shuttle reaches the same final velocity but it's acceleration is increasing. In figure 1, Since the average velocity is always less in the non-constant acceleration case than the constant acceleration case, the average velocity for the non-constant acceleration case must be less than one half the final velocity.

Alternatively, we also defined average velocity as  $\bar{v}_x \equiv \Delta x / t_f$ . Since the displacement is the area under graph of velocity vs. time, we see from figure 1, that for the case of non-constant acceleration the displacement is less than the constant acceleration case, hence the average velocity for the non-constant acceleration case is less than one half the final velocity (which is the average velocity for the constant acceleration case.)

4. Can astronauts floating in orbit tell which objects within their space-ship would be heavy or light on Earth even though everything in the ship is effectively weightless? Explain.

**Answer:** Yes. From the equivalence principle, gravitational mass is equal to inertial mass,

$$m_{grav} = m_{in}.$$

Therefore if the astronauts apply the same force to two different objects, and measure the accelerations.

$$F = m_1 a_1 = m_2 a_2$$

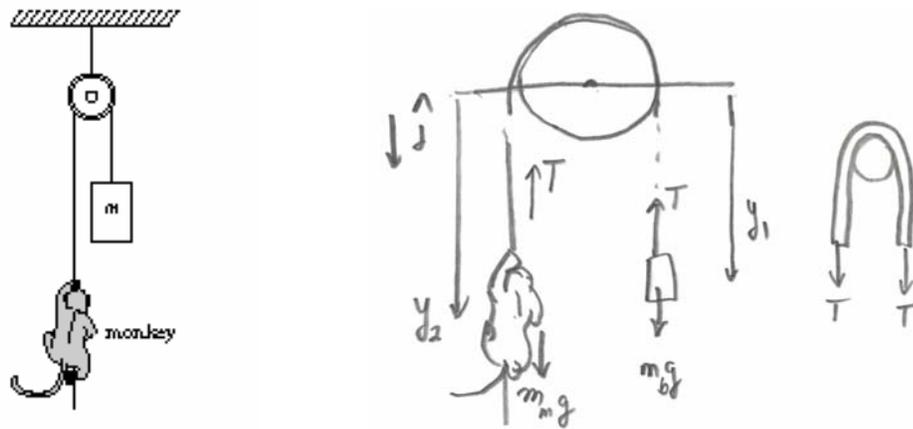
From these measurements, they can determine the inertial mass ratio,

$$\frac{m_1}{m_2} = \frac{a_2}{a_1},$$

The object that has the smaller acceleration has the greater inertial mass, and hence would be heavier on earth.

5. A monkey clings to a rope that passes over a pulley. The monkey's weight is balanced by the mass  $m$  of a block hanging at the other end of the rope; both monkey and block are motionless. In order to get to the block, the monkey climbs a distance  $L$  (measured along the rope) up the rope.

- (a) Does the block move as a result of the monkey's climbing?  
 (b) If so, in which direction and by how much?



**Answer:** The force diagrams are shown in the above figure. As the monkey pulls up on the rope, there is a force down on the rope (tension). This tension is transmitted through the rope (assume a massless rope). This is also the force on the object. Both Both the monkey and object satisfy Newton's Second Law

$$mg - T = ma_y.$$

So both objects accelerate upwards with the same acceleration.

Note that the length of the rope is

$$l = y_1 + \pi R + y_2.$$

As the monkey climbs, the rope is getting shorter at a rate,

$$\frac{d^2 l}{dt^2} = a_{y,1} + a_{y,2} = 2a_y.$$

Since the accelerations are both negative, the length of the rope obeys,

$$l(t) = l_0 + \frac{1}{2}(2a_y)t^2.$$

Each object moves half the distance that the rope decreases in length!

6. A basketball player is jumping vertically upward in order to land a shot. Her legs are flexed and pushing on the floor so that her body is accelerated upward.

- Draw free-body diagrams of the player's body and Earth. Show the relative magnitudes of the various forces and describe each in words (i.e., contact, gravitational, etc., and indicate which object exerts that force and on what). Identify the action-reaction pairs.
- Repeat this exercise for the situation immediately after the player's body breaks contact with the floor.
- Finally, consider, in the same manner, the situation at the top of the jump.

a) *force on basketball player* *force diagram on earth*

when in contact with earth

b) *force on basket player* when no longer in contact with earth

*force diagram on earth*

c) exactly same as in part b)

**Analytic Questions: Show all your work.**

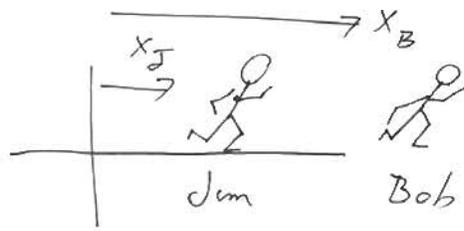
**Problem 1 *One dimensional kinematics: track event***

During a track event two runners, Bob, and Jim, round the last turn and head into the final stretch with Bob a distance  $d$  in front of Jim. They are both running with the same velocity  $v_0$ . When the finish line is a distance  $s$  away from Jim, Jim accelerates at a constant  $a_j$  until he catches up to Bob and passes him. Jim then continues at a constant speed until he reaches the finish line.

- a) How long did it take Jim to catch Bob?
- b) How far did Jim still have to run when he just caught up to Bob?
- c) How long did Jim take to reach the finish line after he just caught up to Bob?

Bob starts to accelerate at a constant  $a_B$  at the exact moment that Jim catches up to him, and accelerates all the way to the finish line and crosses the line exactly when Jim does. Assume Bob's acceleration is constant.

- d) What is Bob's acceleration?
- e) What is Bob's velocity at the finish line? Who is running faster?



choose  $t=0$ , when Jim starts to accelerate. choose origin at Jim.

then

$$x_J = v_{0,J} t + \frac{1}{2} a_J t^2 \quad \text{Jim accelerates}$$

$$x_B = x_{B,0} + v_{0,B} t \quad \text{Bob constant velocity}$$

with  $x_{B,0} = d$

Jim catches Bob when at  $t = t_1$

$$x_J = x_B$$

$$v_{0,J} t_1 + \frac{1}{2} a_J t_1^2 = d + v_{0,B} t_1$$

$$\frac{1}{2} a_J t_1^2 + (v_{0,J} - v_{0,B}) t_1 - d = 0$$

$$t_1^2 + \frac{2}{a_J} (v_{0,J} - v_{0,B}) t_1 - \frac{2d}{a_J} = 0$$

$$\text{Solve for } t_1 = \frac{-2(v_{0,J} - v_{0,B}) \pm \left( \left( \frac{2}{a_J} (v_{0,J} - v_{0,B}) \right)^2 + \frac{8d}{a_J} \right)^{1/2}}{2}$$

Choose positive square root so that  $t_1 > 0$

$$t_1 = \frac{-\frac{2}{a_J}(v_{0,J} - v_{0,B}) + \left( \frac{2}{a_J}(v_{0,J} - v_{0,B})^2 + \frac{8d}{a_J} \right)^{1/2}}{2}$$

calculate  $x_B = d + v_{0,B} t_1 = x_J(t_1)$

finish line is a distance  $S - x_B(t_1)$

Jim is running at a constant velocity

$$s < x \quad v_J = v_{J,0} + a_J t_1$$

so it takes Jim a time

$$t_2 = \frac{S - x_B(t_1)}{v_J} = \frac{S - d - v_{0,B} t_1}{v_{J,0} + a_J t_1}$$

to reach finish line.

Now Bob accelerates and reaches the finish line in the time  $t_2$

$$S - x_B(t_1) = v_B t_2 + \frac{1}{2} a_B t_2^2$$

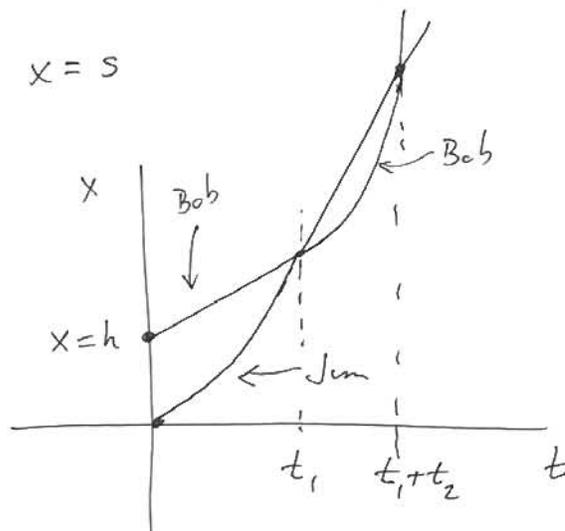
Solve  $a_B = \frac{2(S - x_B(t_1) - v_B t_2)}{t_2^2}$

where  $x_B(t_1) = d + v_{0,B} t_1$

$t_1$  and  $t_2$  are above

The final velocity of Bob is

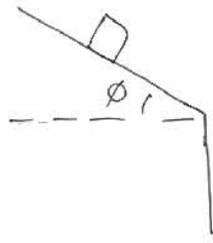
$$v_B(\text{final}) = v_{B,0} + a_B t_2$$



From the graph, the slope of Bob's position function at the finish line is greater than Jim's, Bob is running faster at the finish.

**Problem 2: *Second Law and projectile motion***

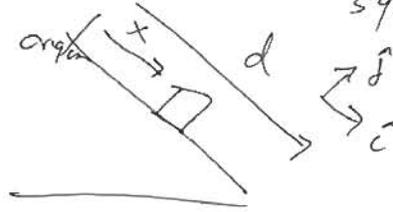
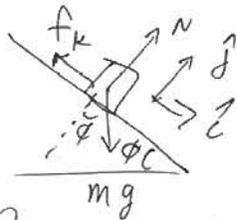
An object with mass  $m$  slides down a roof a distance  $d$  that is inclined at an angle of  $\phi$  to the horizontal. The contact surface between the roof and object has a coefficient of kinetic friction  $\mu_k$ . The edge of the bottom of the roof is at a height  $h$  above the ground. What is the horizontal displacement from the edge of the roof to the point where the object hits the ground?



two motions

- 1) sliding down roof
- 2) free fall in air

i) sliding down roof: choose coordinate system



Free body diagram

$$\vec{F} = m a$$

$$\hat{c}: mg \sin \phi - f_k = m a$$

$$\hat{s}: N - mg \cos \phi = 0 \Rightarrow N = mg \cos \phi$$

$$\text{kinetic friction: } f_k = \mu_k N = \mu_k mg \cos \phi$$

$$\Rightarrow mg \sin \phi - \mu_k mg \cos \phi = m a$$

$$a = g(\sin \phi - \mu_k \cos \phi)$$

$$x = x_0 + v_{x,0} t + \frac{1}{2} a t^2, \quad v = a t$$

$$x_0 = 0 \text{ choice, } v_{x,0} = 0 \text{ starts from rest}$$

$x = d$ , edge of roof

$$d = \frac{1}{2} a t^2 \Rightarrow t = \sqrt{\frac{2d}{a}}$$

$$v = a t = a \sqrt{\frac{2d}{a}} = \sqrt{2da}$$

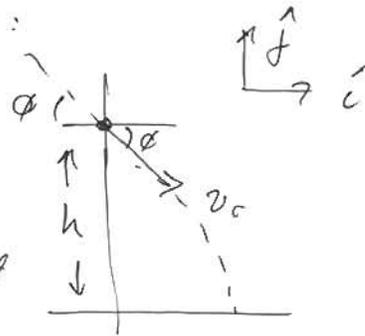
$$= \sqrt{2d(g \sin \phi - \mu_k \cos \phi)}$$

stage 2) free fall:

critical conditions

coordinate system:

origin at point object  
leaves roof



critical conditions:  $x_0 = y_0 = 0$

$$v_{x,0} = v_0 \cos \phi, \quad v_{y,0} = -v_0 \sin \phi$$

where  $v_0 = \sqrt{2d(g \sin \phi - \mu_k \cos \phi)}$

$$y_f = v_{y,0} t_f - \frac{1}{2} g t_f^2$$

$$x_f = v_{x,0} t_f$$

$$v_{y,0} = -v_0 \sin \phi, \quad y_f = -h$$

$$\frac{1}{2} g t_f^2 - v_{y,0} t_f + y_f = 0$$

$$t_f^2 + \frac{2}{g} v_0 \sin \phi t_f - \frac{h}{g} = 0$$

$$t_f = \frac{1}{2} \left( -\frac{2}{g} v_0 \sin \phi \pm \left( \frac{4v_0^2 \sin^2 \phi + 8h}{g^2} \right)^{1/2} \right)$$

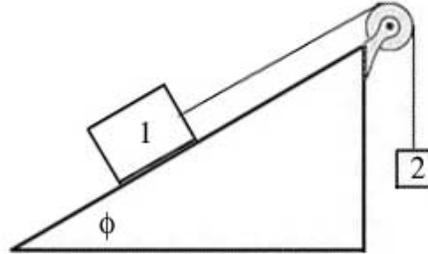
$t_f > 0 \Rightarrow$  positive root

$$t_f = \frac{1}{2} \left( -\frac{2}{g} v_0 \sin \phi + \left( \frac{4v_0^2 \sin^2 \phi + 8h}{g^2} \right)^{1/2} \right)$$

$$x_f = v_0 \cos \phi t_f$$

$$x_f = \frac{v_0 \cos \phi}{2} \left( -\frac{2}{g} v_0 \sin \phi + \left( \frac{4v_0^2 \sin^2 \phi + 8h}{g^2} \right)^{1/2} \right)$$

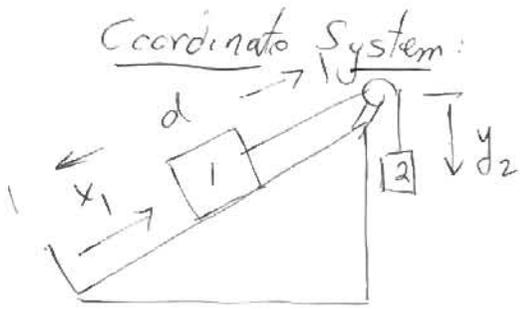
**Problem 3** A block 1 of mass  $m_1$ , constrained to move along a plane inclined at angle  $\phi$  to the horizontal, is connected via a massless inextensible string that passes over a massless pulley, to a second block 2 of mass  $m_2$ . Assume the coefficient of static friction on the between the block and the inclined plane is  $\mu_s$  and the coefficient of kinetic friction is  $\mu_k$ . Assume the gravitational constant is  $g$ .



- a) What is the relation between the masses of block 1 and block 2 such that the system just starts to slip?

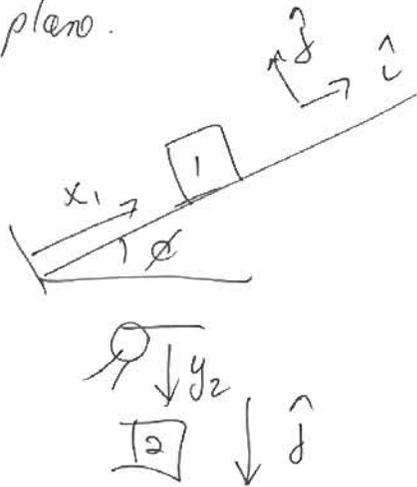
For the following questions suppose block 2 has a mass greater than the value you found in part a).

- b) Calculate the acceleration of the blocks.
- c) Calculate the tension in the rope.
- d) If the block 2 starts out at a height  $h$  above the bottom of the inclined plane and is released at rest. How long does it take to fall a distance  $s$ ? Note that block 1 starts off a distance greater than  $s$  from the pulley.



We choose different coordinate systems for each object. For the object 1 on the inclined plane, we

choose axis aligned with the inclined plane.



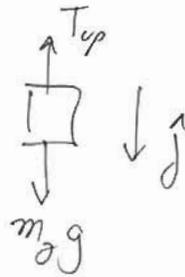
For the object suspended we choose vertical axis and we have positive direction downward.

Constraint: Object 1 and object 2 move in the same direction with respect to positive choice so

$$a_{x,1} = a_{y,2} = a$$

## Free Body Diagrams:

Object 2:



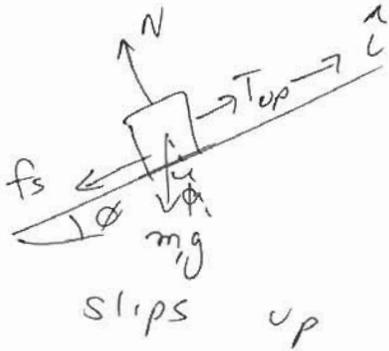
$$\vec{F}_s^{\text{total}} = m_2 \vec{a}_2$$


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$$m_2 g - T = \frac{m_2 a_2}{2, 2}$$

$$\Rightarrow m_2 g = T$$

If the object 1 starts to slip up the inclined plane, then static friction points down the inclined plane:



$$\vec{F}_i^{\text{total}} = m_1 \vec{a}_1$$


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$$\begin{aligned} \hat{i}: T_{up} + f_s - m_1 g \sin \phi &= m_1 a_{x,1} = 0 \\ \hat{j}: N - m_1 g \cos \phi &= 0 \\ N &= m_1 g \cos \phi \end{aligned}$$

$(f_s) = \mu_s N$  } maximum value for static friction

$a = 0$  } just slipping condition

Summary:  $T_{up} - \mu_s N - m_1 g \sin \phi = 0$  slips up  
 $N - m_1 g \cos \phi = 0$   
 $m_2 g = T_{up}$

Slips up:  $m_2 g - \mu_s (m_1 g \cos \phi) - m_1 g \sin \phi = 0$   
 $\Rightarrow (m_2)_{\text{up}} = m_1 (\mu_s \cos \phi + \sin \phi)$

Suppose  $m_2 > (m_2)_{\text{up}}$ . Then object 1  
 slips up, and equations of motion are

object 1:  $T_{\text{up}} - f_k - m_1 g \sin \phi = m_1 a_{\text{up}}$

object 2:  $m_2 g - T_{\text{up}} = m_2 a_{\text{up}}$

add these equations:

$$m_2 g - f_k - m_1 g \sin \phi = (m_1 + m_2) a_{\text{up}}$$

use the force law for kinetic friction

$$f_k = \mu_k N = \mu_k m_1 g \cos \phi$$

to get  $m_2 g - \mu_k m_1 g \cos \phi - m_1 g \sin \phi = (m_1 + m_2) a_{\text{up}}$

solve  $a_{\text{up}} = \frac{(m_2 - \mu_k m_1 \cos \phi - m_1 \sin \phi) g}{m_1 + m_2}$

In order to calculate the tension

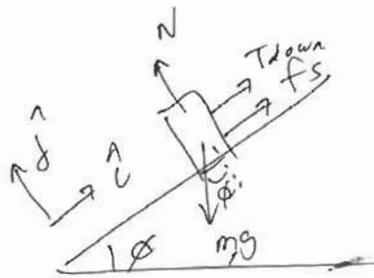
use

$$T_{up} = m_2 g - m_2 a_{up}$$

$$T_{up} = m_2 g - \frac{m_2 (m_2 - \mu_k m_1 \cos \phi - m_1 \sin \phi)}{m_1 + m_2} g$$

$$T_{up} = \frac{m_1 m_2 (1 + \mu_k \cos \phi + \sin \phi) g}{m_1 + m_2}$$

if the object 1 starts to slip down the inclined plane, then static friction points up the inclined plane:



slips down

$$\vec{F}_1^{total} = m_1 \vec{a}_1$$

$$T_{down} + f_s - m_1 g \sin \phi = 0$$

$$N - m_1 g \cos \phi = 0$$

$$m_2 g = T_{down}$$

slips down :-  $m_2 g + \mu_s (m_1 g \cos \phi) - m_1 g \sin \phi = 0$

$$(-m_2)_{down} = m_1 (-\mu_s \cos \phi + \sin \phi)$$

For slipping down: if

$$m_2 < (m_2)_{\text{down}} = m_1(-\mu_s \cos \phi + g \sin \phi)$$

kinetic friction:  $f_k = \mu m_1 g \cos \phi$

force equations:  $T_{\text{down}} + f_k - m_1 g \sin \phi = m_1 a_{\text{down}}$

add  $m_2 g - T_{\text{down}} = m_2 a_{\text{down}}$

$$f_k + m_2 g - m_1 g \sin \phi = (m_1 + m_2) a_{\text{down}}$$

$$m_2 g + \mu_k m_1 g \cos \phi - m_1 g \sin \phi = (m_1 + m_2) a_{\text{down}}$$

$$a_{\text{down}} = \frac{(m_2 + m_1 (\mu_k \cos \phi - \sin \phi)) g}{m_1 + m_2}$$

The tension is given by  $T_{\text{down}} = m_2 g - m_2 a_{\text{down}}$

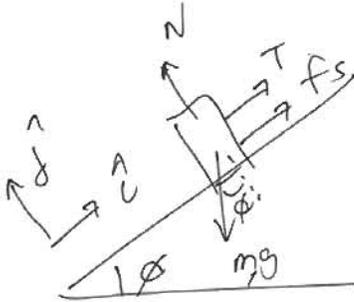
$$T_{\text{down}} = m_2 g - \frac{m_2 (m_2 + m_1 (\mu_k \cos \phi - \sin \phi)) g}{m_1 + m_2}$$

$$T_{\text{down}} = \frac{m_1 m_2 (1 - \mu_k \cos \phi + \sin \phi) g}{m_1 + m_2}$$

note that  $T_{\text{up}} > T_{\text{down}}$ . if the object 2 moves down, the tension in the rope is less than if object 2 moves up.

$$(-m_2)_{\text{down}} < m_2 < (m_2)_{\text{up}} \quad \text{with} \quad m_2 g = T$$

force diagram on object 2: friction can point other way. Choose one direction. solve for friction.



$$\begin{array}{l} \vec{F}_i^{\text{Total}} = m_i \vec{a}_i \\ \hline T + f_s - m_1 g \sin \phi = 0 \\ N - m_1 g \cos \phi = 0 \end{array}$$

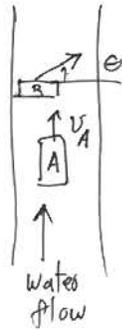
$$f_s = m_1 g \sin \phi - m_2 g$$

When friction is negative, the friction points down the inclined plane, preventing the object from sliding up. when friction is zero, object will stay put even if there is not friction. When friction is positive, friction prevents the object from sliding down.

**Problem 4:** Suppose a MIT student wants to row across the Charles River. Suppose the water is moving downstream at a constant rate of 1.0 m/s. A second boat is floating downstream with the current. From the second boat's viewpoint, the student is rowing perpendicular to the current at 0.5 m/s. Suppose the river is 800 m wide.

- a) What is the direction and magnitude of the velocity of the student as seen from an observer at rest along the bank of the river?
- b) How far down river does the student land on the opposite bank?
- c) How long does the student take to reach the other side?

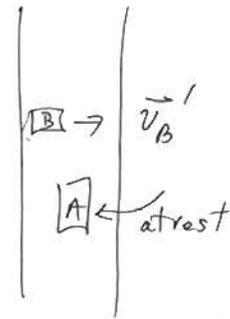
from  
observer  
on  
land



Boat B moves at  
an angle with respect  
to land



from  
observer  
on  
boat A



Boat B moves horizontally  
to the current.

$$\vec{v}_B = \vec{v}_B' + \vec{v}$$

$$\vec{v} = v_A \hat{j}, \quad \vec{v}_B' = v_B' \hat{i}$$

$$\vec{v}_B = v_B' \hat{i} + v_A \hat{j}$$

$$|\vec{v}_B| = ((v_B')^2 + (v_A)^2)^{1/2}$$

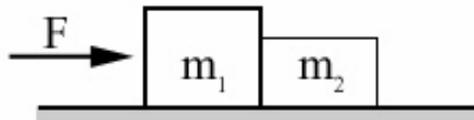
$$v_A = 1 \text{ m} \cdot \text{s}^{-1}, \quad v_B' = 0.5 \text{ m} \cdot \text{s}^{-1}$$

$$|\vec{v}_B| = ((1 \text{ m} \cdot \text{s}^{-1})^2 + (0.5 \text{ m} \cdot \text{s}^{-1})^2)^{1/2} = 1.12 \text{ m} \cdot \text{s}^{-1}$$

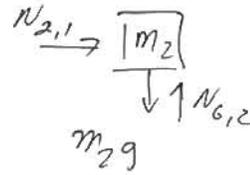
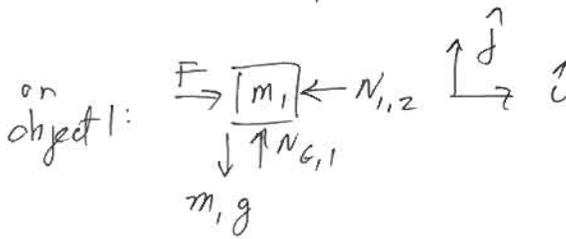
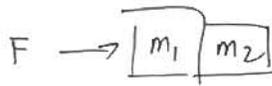
$$\tan \theta = v_A / v_B' = 1.0 \text{ m} \cdot \text{s}^{-1} / 0.5 \text{ m} \cdot \text{s}^{-1} = 2$$

$$\theta = \tan^{-1}(2) = 63.4^\circ$$

**Problem 5:** Two blocks sitting on a frictionless table are pushed from the left by a horizontal force, as shown below.



- Draw a free-body diagram for each of the blocks.
- What is the acceleration of the blocks?
- Express, in terms of the quantities given in the figure, the force of contact between the two blocks.



$$\vec{F} = m_1 \vec{a}_1$$

$$\hat{x}: F - N_{1,2} = m_1 a_1$$

$$\hat{z}: N_{c,1} - m_1 g = 0$$

$$\vec{F}_2 = m_2 \vec{a}_2$$

$$N_{2,1} = m_2 a_2$$

$$N_{c,2} - m_2 g = 0$$

Action-reaction pair:  $N \equiv N_{1,2} = N_{2,1}$  magnitude  
 (note: the sign was put in by hand in the free diagrams)

Constraint:  $a \equiv a_1 = a_2$

Summary:

$$F - N = m_1 a$$

$$N = m_2 a$$

add

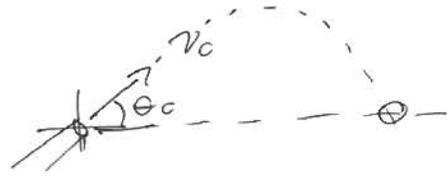
$$F = (m_1 + m_2) a \Rightarrow a = \frac{F}{m_1 + m_2}$$

Contact force

$$N = m_2 a = \frac{m_2 F}{m_1 + m_2}$$

**Problem 6:** In the projectile motion experiment, a ball of diameter  $D$  is projected upwards at an angle  $\theta$  with respect to the horizontal. The ball leaves the tube at a height  $h$  above the ground. Just as it leaves the tube, the ball passed a photogate which measures the voltage across a photo-transistor. The width of the graph of voltage vs. time at half maximum is  $\Delta T$ . The ball has diameter  $D$ . How long will it take the ball to return to the same height that it left the tube? How far in the horizontal direction has it traveled at that point. What is the direction and magnitude of the velocity at that point?

photo gate  $v_0 = \frac{D}{\Delta T}$



Choose origin at height point ball left tube. Then

$$y(t) = v_0 \sin \theta_0 t - \frac{1}{2} g t^2$$

$$x(t) = v_0 \cos \theta_0 t$$

Set  $t = t_f$  when  $y(t_f) = 0$ . Then

$$0 = v_0 \sin \theta_0 t_f - \frac{1}{2} g t_f^2 \quad (1)$$

$$x_f = v_0 \cos \theta_0 t_f \quad (2)$$

$$\text{eq(1)} \Rightarrow \frac{1}{2} g t_f^2 = v_0 \sin \theta_0 t_f \quad \text{or}$$

$$t_f = \frac{2 v_0 \sin \theta_0}{g} = \frac{2 D \sin \theta_0}{\Delta T g} \quad (3)$$

$$x_f = v_0 \cos \theta_0 t_f = v_0 \frac{\cos \theta_0 2 v_0 \sin \theta_0}{g}$$

$$= \frac{2 v_0^2 \sin(2 \theta_0)}{g}$$

$$x_f = \frac{2 D^2 \sin(2 \theta_0)}{g \Delta T^2}$$

$$\vec{v}(t_f) = v_x(t_f) \hat{i} + v_y(t_f) \hat{j}$$

$$v_x(t_f) = v_{x,i} = v_0 \cos \theta_0$$

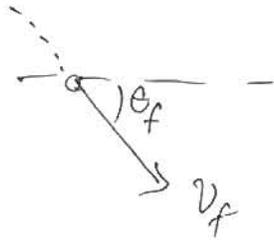
$$v_y(t_f) = v_0 \sin \theta_0 - g t_f$$

$$= v_0 \sin \theta_0 - 2g \frac{v_0 \sin \theta_0}{g} = -v_0 \sin \theta_0$$

$$|\vec{v}(t_f)| = (v_{x,f}^2 + v_{y,f}^2)^{1/2}$$

$$= (v_0^2 \cos^2 \theta_0 + v_0^2 \sin^2 \theta_0)^{1/2} = v_0$$

velocity has same magnitude as  
exit velocity but direction changes



$$\tan \theta_f = \frac{v_{y,f}}{v_{x,f}} = \frac{-v_0 \sin \theta_0}{v_0 \cos \theta_0}$$

$$\tan \theta_f = -\tan \theta_0$$

$$\theta_f = -\theta_0$$