

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01T

Fall Term 2004

Problem Set 10: Torque, Rotational Dynamics, Physical Pendulum, Angular Momentum

Available on-line November 5; Due: November 16 at 4:00 p.m.

Please write your name, subject, lecture section, table, and the name of the recitation instructor on the top right corner of the first page of your homework solutions. Please place your solutions in your lecture section table box.

Nov 5

Hour One: Problem Solving Session 15: Moment of Inertia and Rotational Energy

Readings: YF 9.1-9.6

Problem Set 9: Due Tues Nov 9 at 4:00 pm.

Nov 8

Hour One: Torque, Rotational Dynamics.

Readings: YF 10.1-10.4

Hour Two: Problem Solving Session 16: Rotational Dynamics, Simple Harmonic Motion: Physical Pendulum

Nov 10

Hour One: Experiment 8: Physical Pendulum

Hour Two: Conservation Laws: Angular Momentum

Readings: YF 10.5-10.6

Nov 12

Hour One: Problem Solving Session 17: Angular Momentum

Readings: YF 10.1-10.6

Problem Set 10: Due Tues Nov 16 at 4:00 pm.

Nov 15

Hour One: Experiment 9: Angular Momentum

Reading: Experiment 9

Hour Two: Planetary Motion Reading Class Notes: Planetary Orbits: The Kepler Problem, Energy Diagrams Readings: YF 7.1, 7.5, 12.3-12.5

Nov 17

Hour One: Problem Solving Session 18: Galactic Black Hole

Reading: YF 12.5, 12.8

Hour Two: Test Review

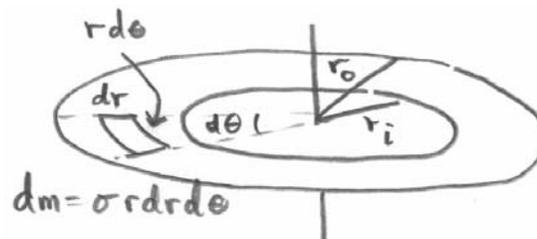
Nov 18 QUIZ THREE: Energy, Momentum, and Rotational Motion 7:30-9:30 pm.

Nov 19 No Class

Problem Set 11: Due Tues Nov 23 at 4:00 pm.

Problem 1: (Moment of Inertia)

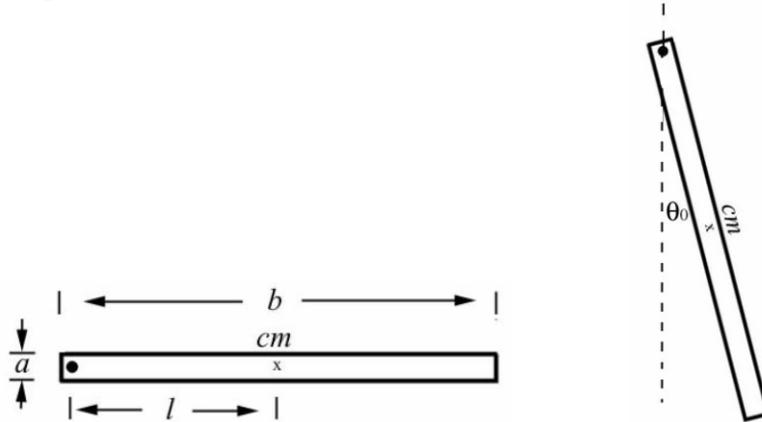
A 1" US Standard Washer has inner radius $r_1 = 1.35 \times 10^{-2} m$ and an outer radius $r_2 = 3.10 \times 10^{-2} m$. The washer is approximately $d = 4.0 \times 10^{-3} m$ thick. The density of the washer is $\rho = 7.8 \times 10^3 kg/m^3$. Calculate the moment of inertia of the washer about an axis passing through the center of mass and show that it is equal to $I_{cm} = \frac{1}{2} m_w (r_0^2 + r_i^2)$.



Problem 2: Experiment 09 Physical Pendulum

Part One: Ruler Pendulum

The ruler has a mass $m_r = 0.159$ kg, a width $a = 0.028$ m, a length $b = 1.00$ m, and the distance from the pivot point to the center of mass is $l = 0.479$ m.



Enter your measured period into the T_{meas} column of the table below and calculate the other entries using the formulas

$$T_{\text{ideal}} = 2\pi\sqrt{l/g}$$

and

$$T_{\text{theory}} = 2\pi\sqrt{l/g} \sqrt{\frac{I}{ml^2} \left(1 + \frac{\theta_0^2}{16}\right)}$$

with $g = 9.805 \text{ ms}^{-2}$.

θ_0	T_{meas}	T_{theory}	T_{ideal}
0.10			
0.52			

Part Two: Added Mass

Consider the effect of a brass weight clipped to the ruler. The weight is shaped like a washer with an outer radius $r_o = 0.016$ m and an inner radius $r_i = 0.002$ m; it has a mass $m_w = 0.050$ kg. It is clipped to the ruler so that the inner hole is over the 0.500 m mark on the ruler, or $l = 0.479$ m from the pivot point. The clip has a mass $m_c = 0.0086$ kg and you may assume its center of mass is also over the 0.500 m mark on the ruler. If you treat the washer and clip as point masses,

then, as was discussed in the notes for Experiment 08, the combined unit (ruler, weight and clip) has a moment of inertia about the pivot point

$$I_p = \frac{m_r}{12}(a^2 + b^2) + m_r l^2 + (m_c + m_w)d^2$$

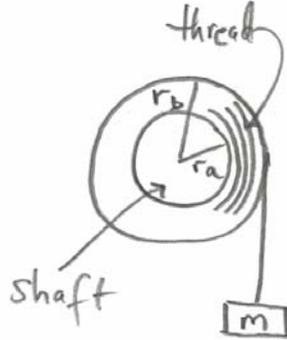
where $d = l$ for this situation. The restoring torque that tries to return the pendulum to a vertical position will be

$$\tau = (m_r l + m_c d + m_w d)g \sin \theta \approx (m_r l + m_c d + m_w d)g \theta$$

1. Use these two expressions to derive an equation of motion for the pendulum and calculate its period T in the small amplitude ($\sin \theta \cong \theta$) approximation. Express your answer algebraically in terms of the variables a , b , d , l , m_r , m_w , m_c , and g .
2. Evaluate your result numerically and compare with the value you measured in your experiment.
3. If you treat the brass object washer as a point mass, its moment of inertia about the pivot point P is $I_{w,P} = m_w l^2$. If the brass object is a washer with an inner radius r_i and outer radius r_o , then moment of inertia about its center of mass given by $I_w = \frac{1}{2} m_w (r_o^2 + r_i^2)$. If the washer is a solid disc with radius r , the moment of inertia about its center of mass given by $I_w = \frac{1}{2} m_w r^2$. When this is taken into account, what is the new (and more accurate) expression for $I_{w,P}$? How many percent does this differ from the simpler expression $I_{w,P} = m_w l^2$?

Problem 3: Stall Torque of Motor

The following simple experiment can measure the stall torque of a motor. (See sketch.) A mass m is attached to one end of a thread. The other end of the thread is attached to the motor shaft so that when the motor turns, the thread will wind around the motor shaft. The motor shaft without thread has radius $r_0 = 1.2 \times 10^{-3} \text{ m}$. Assume the thread winds evenly effectively increasing the radius of the shaft. Eventually the motor will stall.



a) Suppose a mass $m = 5.0 \times 10^{-2} \text{ kg}$ stalls the motor when the wound thread has an outer radius of $r_f = 2.4 \times 10^{-3} \text{ m}$. Calculate the stall torque.

b) Suppose the motor has an unloaded full throttle angular frequency of $\omega_0 = 2\pi f_0 = 2\pi(6.0 \times 10^1 \text{ Hz})$ (unloaded means that the motor is not applying any torque). Suppose the relation between angular frequency ω and the applied torque τ of the motor is given by the relation

$$\omega = \omega_0 - b\tau$$

where b is a constant with units $\text{s/kg} - \text{m}^2$. Using your result from part a), calculate the constant b . Make a graph of ω vs. τ .

c) Graph the power output of the motor vs. angular frequency ω . At what angular frequency is the power maximum? What is the power output at that maximum? Briefly explain the shape of your graph. In particular, explain the power output at the extremes $\tau = 0$ and $\tau = \tau_{stall}$.

d) What torque does the motor put out at its maximum power output?

Problem 4 (Conservation of Angular Momentum)

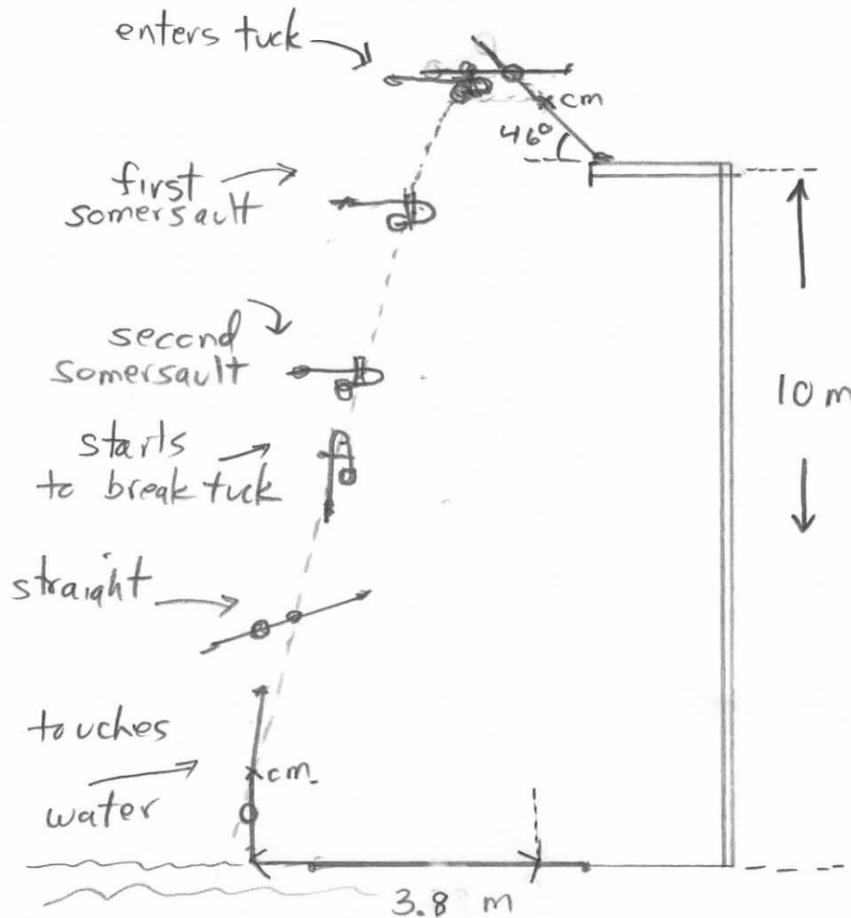
A meteor of mass $m = 2.1 \times 10^{13} \text{ kg}$ is approaching earth as shown on the sketch. The radius of the earth is $r_e = 6.37 \times 10^6 \text{ m}$. The mass of the earth is $m_e = 5.98 \times 10^{24} \text{ kg}$. Suppose the meteor has an initial speed of $v_0 = 1.0 \times 10^1 \text{ m/s}$. Assume that the meteor started very far away from the earth. Suppose the meteor just grazes the earth. The initial moment arm of the meteor (h on the sketch) is called the impact parameter. You may ignore all other gravitational forces except the earth. The effective scattering area for the meteor is the area πh^2 . This is the effective target size of the earth as initially seen by the meteor.



- Draw a force diagram for the forces acting on the meteor.
- Can you find a point about which the gravitational torque of the earth's force on the meteor is zero for the entire orbit of the meteor?
- What is the initial angular momentum and final angular momentum (when it just grazes the earth) of the meteor?
- Apply conservation of angular momentum to find a relationship between the meteor's final velocity and the impact parameter h .
- Apply conservation of energy to find a relationship between the final velocity of the meteor and the initial velocity of the meteor.
- Use your results in parts d) and e) to calculate the impact parameter and the effective scattering cross section.

Problem 5: Platform Diving

In the 2002 World Cup Trials, Kyle Prandi set up a diving record with a back $3\frac{1}{2}$ somersault pike from the 10 m board. He pushed off from the board at an angle of $\theta_0 = 46^\circ$ with an initial speed $v_0 = 3.3\text{m/s}$. You may assume that his body was completely straight with his arms stretched above his head when he jumped.



He took .33 seconds to enter into a tuck after completing $\frac{1}{2}$ a rotation. At the .49 second mark he returned to his starting height. Once in a full tuck, he completed 2 revolutions at the 1.1 second mark. At that point he began to straighten out which he finished at the 1.47 second mark after making $\frac{1}{4}$ rotation. He made 1 more rotation, when his fingers touched the water 1.65 seconds after he left the platform. When he touched the surface his legs were bent but his center of mass was 1.3 m above the surface of the water. Kyle is 1.7 m long and when his arms are straight out above his head, his length is 2.2 m. His mass is 63 kg. His center of mass is 0.9 m above his soles. You may see the jump at

http://www.usadiving.org/USD_03redesign/media/video.htm

Somersaults	Dive details	time	Vertical distance starting point
zero	start	0	0
.25	Enters full pike	.33	
.75	Returns to starting height	.49	
1.25	Completes first turn in full pike	.73	
2.25	Completes second turn in full pike	1.1	
2.5	Starts to straighten	1.2	
3.4	Completely straight	1.47	
3.5	Fingers touch water	1.65	

- a) Based on the above information, make a graph of his angular velocity as a function of time. Indicate any assumptions that you have made for the various stages of his motion.
- b) Based on the initial conditions, calculate his vertical distance from his starting point at the various times indicated in the table.
- c) Explain whether or not you think his final angular velocity is equal to his initial angular velocity?
- d) Let I_0 denote his moment of inertia about his center of mass just after he left the board. Let I_1 denote his moment of inertia about his center of mass when he is in a full tuck. Let After he pulled his body into a tuck, by what fraction, $(I_1 - I_0)/I_0$, did his moment of inertia change?
- e) Suppose when he goes into a tuck, he has reduced his length by a factor of 2. Does the ratio, I_1 / I_0 , agree reasonably with your angular velocity data? Explain your answer.